Mathematics for Australia Year 10A 2nd edition

Chapter summaries

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CHAPTER 1: EXPONENTS

A Exponent laws
B Rational exponents
C Scientific notation

Keywords:

- base
- exponent
- exponent laws
- exponent notation
- index
- power
- rational exponent
- scientific notation
- standard form

This chapter has been renamed from "Indices" due to the change in terminology in the most recent syllabus update.

The material in Section A should be familiar to students from Years 8 and 9, so it should be a gentle introduction to the year.

Section B gives students their first look at rational exponents. This may be quite a conceptual leap for many students, since it is not immediately clear what it means to raise a number to a fractional power. Students should be reminded that they have already encountered such a leap when defining the zero and negative exponents in previous years – what we are doing is giving a meaningful interpretation of these exponents such that the existing exponent laws still hold true. In the first Investigation, we establish that the only meaningful interpretation of $a^{\frac{1}{n}}$ is the nth root of a.

In Question **4** of Exercise 1B.1, students write algebraic expressions involving roots as powers of x. This will be useful when studying calculus in later years.

In Section C, we have adjusted our approach to explaining scientific notation, to give students more guidance in how to choose their values of a and k in $a \times 10^k$. This section extends what was done in Year 9 by also considering the addition and subtraction of numbers in scientific notation. Students should write the terms with the same power of 10 to perform the addition or subtraction. Through practice, students should see that it is better to adjust the smaller value so that its power of 10 matches that of the larger value. This is because the power of 10 of the larger value is more likely to be the correct power of 10 required to write the final answer in scientific notation.

CHAPTER 2: ALGEBRA: EXPANSION

A The distributive law
B The product $(a+b)(c+d)$
C The difference between two squares
D The perfect squares expansion
E Further expansion
F The binomial expansion

Keywords:

- binomial
- binomial expansion
- difference between two squares
- distributive law
- expansion
- factorisation
- FOIL rule
- perfect squares expansion

In this edition, we have split algebraic expansion and factorisation into two separate chapters. This gives us the opportunity to provide a more complete coverage of algebraic expansion than what was given in the previous edition. Given that students will have only seen most of this material for the first time in Year 9, it seems reasonable to spend a little more time in Year 10 to reinforce this work.

In Section C, some of the expansions involve working with surds. Although this comes before Chapter 8 (Surds and other radicals), it only involves very intuitive calculations such as $(\sqrt{2})^2 = 2$ based on the definition of a surd, which students should be familiar with from previous years.

In Section E (Further expansion), it may help students to recognise that they have already encountered the idea of "multiplying each term in the first bracket by each term in the second bracket", since that is what happens when applying the FOIL rule to $(a+b)(c+d)$ in Section B.

Section F (The binomial expansion) will now be new to students, since the binomial expansion was removed from Year 9 in this edition. The Investigation on Pascal's triangle at the end of the section gives students a hint towards generating a general binomial expansion, which students may explore further in Years 11 and 12.

CHAPTER 3: ALGEBRA: FACTORISATION

- **A** Common factors
- **B** Difference between two squares factorisation
- **C** Perfect squares factorisation
- **D** Expressions with four terms
- **E** Factorising $x^2 + bx + c$
- **F** Factorising $ax^2 + bx + c,\ a \neq 1$
- **G** Miscellaneous factorisation

Keywords:

- common factors
- difference between two squares
- factorisation
- fully factorised
- linear factors
- perfect squares
- quadratic trinomial
- splitting the middle term
- sum and product method

In this edition, algebraic factorisation has been separated from expansion into its own chapter. As with expansion, this will allow us to provide a more complete coverage of the factorisation material.

To emphasise that factorisation is the reverse process of expansion, students should be reminded that they can check their factorisations by expanding their answer.

Students should be familiar with the material in Sections A to C from Year 9, so these sections can be worked through quickly if needed.

Section D (Expressions with four terms) is primarily a lead-in to factorising by "splitting" the middle term in Section F. It should be made clear to students that most expressions with four terms cannot be factorised in this way.

In the Discussion in Section E, students should find that:

- If the sum and product of two numbers are both positive, the numbers must both be positive.
- If the sum and product of two numbers are both negative, the numbers are opposite in sign, and the negative number has the largest absolute value.
- If the sum is positive and the product is negative, the numbers are opposite in sign, and the positive number has the largest absolute value.
- If the sum is negative and the product is positive, the numbers must both be negative.

In Section F, students are introduced to "splitting" the middle term. It may help students to see that this is a more general approach to the method used in Section E, since when $a = 1$ the method is essentially reduced to the sum and product method. The only difference is that the factorisation must be completed using the technique studied in Section D. Students should convince themselves that the order in which they write the split terms does not matter, since the resulting factorisation will be the same in either case.

In Section G, students will need to choose which factorisation method to use. Students may wish to produce their own summaries describing when each method is suitable.

CHAPTER 4: SETS

A Sets
B Complement of a set
C Intersection and union
D Special number sets
E Interval notation

Keywords:

- complement
- complementary sets
- disjoint
- element
- empty set
- equal sets
- finite set
- infinite set
- integers
- intersection
- interval notation
- irrational numbers
- member
- natural numbers
- negative integers
- positive integers
- rational numbers
- real numbers
- set
- subset
- union
- universal set

This chapter has been added in this edition. The chapter contains some useful work regarding special types of numbers and interval notation, which are used frequently throughout the book, so it is helpful to address this work explicitly.

This material is often presented alongside Venn diagrams as a single chapter, however, as was the case in Year 9, we felt the best approach was to split Sets and Venn diagrams into two chapters, with "Linear equations and inequalities" between them. This way, interval notation can be introduced in Sets before it is used in linear inequalities, and linear equations can be used to solve problems involving Venn diagrams.

When discussing the union "A or B", it is important to emphasise that elements in both A and B are included in the union. This is a good opportunity to discuss how words can be used differently in mathematics than they are in everyday use, as "or" is often used to mean "one or the other, but not both" in everyday use.

In this edition, we have changed the definition of natural numbers to include 0. Although either interpretation is accepted in the Australian Curriculum glossary, this appears to be the most commonly used definition in other curriculums around the world.

We have endeavoured to extend what was done in the Year 9 sets chapter by giving more opportunities to explore the concepts of finite and infinite sets. In the Discussion at the end of Section D, students should consider that, when we have two *finite* sets A and B where A is a subset of B, it is clear that there are more elements in B than in A. However, it is less clear when A and B are both infinite sets! It is tempting to say that there are more elements in $\mathbb{Z}$ than in $\mathbb{Z}^+$, since $\mathbb{Z}$ contains all the elements of $\mathbb{Z}^+$, as well as some extra elements. But does it make sense to say that one infinite set has *more* elements than another infinite set? A potentially more illuminating example may be if A is the set of even integers, and B is the set of positive integers. Clearly A is a subset of B, but one could easily generate each element of A by multiplying each element of B by 2! Does this mean that, in some sense, they have the same number of elements?

A similar question occurs in the second dot point. Since the interval of numbers from 0 to 1 appears smaller than the interval of numbers greater than 1, it seems logical that there are more numbers greater than 1 than there are numbers between 0 and 1. However, for each number greater than 1, there is a corresponding number between 0 and 1, which is found by taking the reciprocal of the original number. Does this correspondence mean that there are the same number of real numbers between 0 and 1 as there are greater than 1?

When dealing with these questions, students should be reminded that rules that apply to finite sets do not necessarily extend to infinite sets.

CHAPTER 5: ALGEBRAIC FRACTIONS

A Evaluating algebraic fractions
B Simplifying algebraic fractions
C Multiplying algebraic fractions
D Dividing algebraic fractions
E Adding and subtracting algebraic fractions

Keywords:

- algebraic fraction
- evaluate
- lowest common denominator
- lowest terms
- rational expression
- reciprocal

The structure of this chapter remains largely unchanged from what occurred in the previous edition. The main difference is that multiplication and division of algebraic fractions have been split into separate sections.

Placing this chapter after we have studied factorisation allows us to factorise the numerator and denominator of an algebraic fraction. This helps in cancelling common factors when simplifying, multiplying, or dividing algebraic fractions.

In Section E, we have avoided using the term "simplify" as much as possible, as this may be ambiguous. For example, in some questions students must write the sum of two fractions as a single fraction, whereas in other questions students must take a single fraction such as $\frac{x+9}{3}$, and write it as the sum of two parts. It is therefore unclear which form is the "simplest" in this case. Instead we have been more explicit about what the student should do in each question.

This may be a good opportunity to discuss the merits of the term "simplify", and to help students understand that when we manipulate an algebraic expression, we are turning it into a different form. Whether this new form is "better" or "simpler" may depend on what we are trying to do with the expression.

CHAPTER 6: LINEAR EQUATIONS AND INEQUALITIES

A Linear equations
B Equations with fractions
C Problem solving
D Linear inequalities
E Problem solving with inequalities

Keywords:

- algebraic equation
- double inequality
- equal sign
- equation
- inequality
- inverse operations
- left hand side
- linear equation
- linear inequality
- lowest common denominator
- right hand side

Section A is largely revision of the work done on solving linear equations in previous years, and may be skipped through more quickly if students are comfortable with the material.

In Section B, students use the work done in the previous chapter to solve equations involving algebraic fractions.

In Section C (Problem solving), we have added some questions where it is useful to present the information in a table, in order to translate the problem into an equation.

Since linear inequalities are now introduced in Year 9, there is less background information about linear inequalities in Section D than there was in the previous edition, and we move more quickly towards solving linear inequalities. When solving linear inequalities, students can use the work done in Chapter 4 to write the solutions in interval notation, and to graph the solution on a number line.

In this edition, we include double inequalities, such as $5 < 3x - 4 < 17$. When solving double inequalities, students must remember to write the expressions in the solution from smallest to largest. So, if students have obtained the solution $8 > x > 5$, they must write this as $5 < x < 8$. Students should not think of this step as "reversing the inequality signs" in the same way as when they multiply or divide by a negative number. Instead, it is more about rewriting the solution in the correct format.

In the Discussion at the end of Section D, students should find that double inequalities such as $x + 1 \leqslant 3x - 5 < 10$ can be solved by treating them as two separate inequalities, and then combining the solutions. For example, the solution to $x + 1 \leqslant 3x - 5$ is $x \geqslant 3$, and the solution to $3x - 5 < 10$ is $x < 5$. So, the solution to the double inequality occurs where *both* of these are true, which is the interval $3 \leqslant x < 5$.

The Puzzle at the end of Section E can be solved intuitively or algebraically. Intuitively, students should see that the number of "Yes" responses on Island C will be minimised if we assume all of the migration to the other two islands came from Island C. In this case, 35 people initially on Island C have moved to another island (30 to Island A, 5 to Island B). So, there would be $100 - 35 = 65$ people originally on Island C who are still on Island C.

Also, the number of "Yes" responses on Island C will be maximised if we maximise the amount of migration between the other two islands. If all 5 "No" responses on Island B came from Island A, and 15 "No" responses from Island A came from Island B (the maximum amount possible), then only 15 people initially on Island C moved to another island (to Island A, in this case). So, $100 - 15 = 85$ people originally on Island C would still be on Island C. So, the number of "Yes" responses on Island C could range from 65 to 85 (inclusive).

CHAPTER 7: VENN DIAGRAMS

- **A** Venn diagrams
- **B** Venn diagram regions
- **C** Numbers in regions
- **D** Problem solving with Venn diagrams

Keywords:

- complement
- disjoint
- intersection
- set identity
- subset
- union
- Venn diagram

This chapter has been added in this edition. We felt that, in order to use Venn diagrams in the Probability chapter, it would be preferable to introduce Venn diagrams in their own right earlier in the book. As was done in Year 9, Venn diagrams has been separated from the Sets chapter, and placed after linear equations, so that students can use their linear equation solving skills to find unknown numbers in regions on Venn diagrams.

In Year 10, the work is extended to consider more Venn diagrams with three sets, and to prove some set identities.

In the Puzzle at the end of Section B, students should be encouraged to place numbers in the Venn diagram, starting with 1, 2, 3, ... and so on, until each region contains at least one element. The smallest element in each region then forms the universal set.

When problem solving with three set Venn diagrams in Section D, students should be aware that the difficulty of the problems depend very much on the combination of information given. For example, Worked Example 8 shows the most difficult case, in which the number of elements in all three sets is unknown, since this means that all of the known information must be expressed in terms of this unknown value x. However, Questions **9** and **10** immediately following are the easiest case, in which the number of elements in all three sets is known, and the rest of the regions can be deduced quite easily from the remaining information. Question **11** is slightly harder, and Question **12** is the most difficult form outlined in the worked example.

CHAPTER 8: SURDS AND OTHER RADICALS

- **A** Radicals
- **B** Properties of radicals
- **C** Simplest surd form
- **D** Power equations
- **E** Operations with radicals
- **F** Division with surds

Keywords:

- cube root
- integer denominator
- nth root
- power equation
- radical
- radical conjugate
- rationalising the denominator
- simplest surd form
- square root
- surd

In Section A, students perform calculations based purely on the definition of the square root, such as $\sqrt{5} \times \sqrt{5} = 5$. In Section B, students use properties of square roots, such as $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$, to perform calculations.

In Section C, more able students may be asked to consider situations where simplest surd form is useful, and situations where it is better to leave the surd in its original form. For example, when comparing which value is larger, it is easier to compare $\sqrt{27}$ and $\sqrt{28}$ than to compare $3\sqrt{3}$ and $2\sqrt{7}$.

In this edition, we have added Section D (Power equations). We feel that it is important to consider power equations, as they are useful for solving problems in topics such as measurement, proportion, and similarity, but are rarely addressed in their own right.

In the Discussion at the end of Section D, students should find that, when both sides of an equation are squared, we often introduce an extra solution. There are often good reasons to square both sides of an equation, but we need to be aware that this may create extra solutions, and check whether the solutions we obtain all satisfy the original equation.

The sections “Adding and subtracting radicals” and “Multiplications involving radicals” in the previous edition have been combined into the single section “Operations with radicals”. Since the rules for operating with radicals are identical to those for algebra, we felt there was little to be gained by keeping these sections separate.

In Section F, we give a more general definition of “radical conjugate”, which means we do not need to go through all the cases individually in the exercise.

CHAPTER 9: PYTHAGORAS’ THEOREM

- **A** Pythagoras’ theorem
- **B** Pythagorean triples
- **C** Problem solving
- **D** The converse of Pythagoras’ theorem

Keywords:

- converse of Pythagoras’ theorem
- Pythagoras’ theorem
- Pythagorean triple

In this edition, we have changed the order of the sections, so that the converse of Pythagoras’ theorem is done at the end. This allows us to consider the non-contextual and contextual applications of Pythagoras’ theorem first, before considering the converse.

The section on circle problems has been removed in this edition, and these problems have been moved to the appropriate sections of Chapter 18 (Circle geometry). This approach means that we do not need to present the relevant circle theorems twice.

The questions involving three-dimensional objects have been absorbed into Section C (Problem solving), rather than being presented as a section on their own. We feel that presenting them in a section of their own makes them seem more difficult than they really are.

CHAPTER 10: FORMULAE

- **A** Formula construction
- **B** Substituting into formulae
- **C** Rearranging formulae
- **D** Rearrangement and substitution
- **E** Predicting formulae

Keywords:

- formula
- inverse operations
- rearrange
- subject
- substitution

In this edition, we have moved the Formulae chapter before the Measurement chapter, as we felt that students will benefit from doing this work before encountering the measurement formulae. As a result, we have removed any questions referring to measurement formulae, as these will be addressed in the Measurement chapter. In their place, we have included more questions based on physics formulae, as specified in the new Australian Curriculum.

We have also included more questions in which students must solve a power equation to find an unknown variable. The work done in Chapter 8 should help with this.

In Section D, students are asked to rearrange the formula to make a particular variable the subject, and then use substitution to evaluate that variable in different circumstances. The students are always asked to evaluate the variable multiple times, as this serves to highlight the advantage of rearranging the formula first. If the formula is rearranged first, the rearrangement only needs to be done once, rather than for each time the variable must be evaluated.

In Section E, students observe the first few numbers in a sequence, identify the pattern in the sequence, and use this information to generate a formula for the nth term.

CHAPTER 11: MEASUREMENT

A Length and perimeter
B Area
C Surface area
D Volume
E Capacity

Keywords:

- arc length
- area
- capacity
- centimetre
- circumference
- hectare
- kilometre
- length
- metre
- millimetre
- perimeter
- surface area
- tapered solid
- volume

Now that this chapter occurs after the Formulae chapter, we have included more questions in which students must find an unknown which is not the subject of the formula, for example, finding the radius of a circle given its circumference.

In this edition, all of the measurement formulae given here, including the surface area and volumes of tapered solids and spheres, were introduced in Year 9. This means that in Year 10 there is less need to derive each of these formulae, as this was largely done in Year 9, where appropriate.

Given that none of these formulae will be new to students, it also means that there is less need to split each section into separate subsections, introducing a new formula in each subsection. All of the formulae are presented at the start of the section, and all of the problems are presented in a single exercise.

This approach should allow classes to move through measurement must faster than they would have in previous years. We feel this is important, as there is a lot of other more rigorous content which must be covered at Year 10.

CHAPTER 12: FINANCIAL MATHEMATICS

A Business calculations
B Appreciation and depreciation
C Simple interest
D Compound interest

Keywords:

- appreciation
- break even
- compound interest
- compound interest formula
- cost price
- depreciation
- discount
- future value
- goods and services tax
- inflation
- interest
- interest rate
- loss
- marked price
- mark-up
- multiplier
- per annum
- present value
- principal
- profit
- selling price
- simple interest
- simple interest formula

In this chapter, students apply their knowledge of percentages in financial contexts such as discount and mark-up, appreciation and depreciation, and simple and compound interest.

Section A has undergone a significant restructure from the previous edition. Whereas Section A was split into 5 subsections in the previous edition, here we have presented mark-up, discount, profit, loss, and tax in a single section, noting that each is an application of percentage change.

In Activity 1, we ask students to take the very general percentage change formula "old amount $\times$ multiplier $=$ new amount", and specify what "old amount" and "new amount" refer to in each context. For example, when considering mark-up, we have "selling price $=$ marked price $\times$ multiplier", and when considering loss, we have "selling price $=$ cost price $\times$ multiplier". We hope that this approach helps students see that each context involves the same processes, and that students should not feel that they need to memorise a distinct set of formulae for each context.

In Section C, given that the students have only recently completed the Formulae chapter, we do not feel it is necessary to present a worked example on each of the rearrangements of the simple interest formula, especially since each rearrangement is essentially equivalent. Instead, students are asked in Question **8** to rearrange the simple interest formula to make each other variable the subject, and then the remaining questions in the exercise require one of those rearrangements.

CHAPTER 13: QUADRATIC EQUATIONS

A Equations of the form $x^2 = k$
B The null factor law
C Solving by factorisation
D Completing the square
E The quadratic formula
F Problem solving

Keywords:

- completed square form
- completing the square
- discriminant
- null factor law
- quadratic equation
- quadratic formula

In Section A (Equations of the form $x^2 = k$), students may notice that simple versions of these equations were solved in Section 8D (Power equations). Here we use the same principle to solve more complicated equations such as $(3x-2)^2 = 10$.

In Section C.2, students should recognise that equations such as $x^2 - 9 = 0$, which can be solved by difference between two squares factorisation, could also be solved by rearranging it to $x^2 = 9$. However, for more complicated equations such as $(2x+1)^2 - (x+2)^2 = 0$, it should be clear that using difference between two squares factorisation is more efficient.

Students may have solved quadratic equations with $a = 1$ by completing the square in Year 9. In Year 10 we extend this work to consider completing the square with $a \neq 1$ in Section D.

Students should find that solving quadratic equations by completing the square is quite tedious, and that by applying completing the square to the general quadratic equation $ax^2 + bx + c = 0$, we can obtain the quadratic formula for the solution to the equation in terms of a, b, and c, without having to perform all of the steps each time.

In the Discussion in Section E, students should be able to solve the quadratic equation using factorisation, completing the square, and using the quadratic formula. They should also find that using factorisation is the quickest method in this case. This is a good opportunity to explain that the quadratic formula *can* be used for any quadratic equation, however it is worth checking whether the quadratic equation can be solved by factorisation first, since this method will be much quicker.

In this edition, similarity has been moved later in the book, and now appears after quadratic equations. For this reason, the questions involving similar triangles have been removed from Section F, and now appear in the Congruence and similarity chapter.

CHAPTER 14: COORDINATE GEOMETRY

A The distance between two points
B Midpoints
C Gradient
D Parallel and perpendicular lines
E Using coordinate geometry
F 3-dimensional coordinate geometry

Keywords:

- Cartesian plane
- coordinates
- distance formula
- gradient
- gradient formula
- midpoint
- negative reciprocals
- number plane
- ordered pair
- origin
- parallel lines
- perpendicular lines
- quadrant
- x-axis
- X-axis
- x-coordinate
- y-axis
- Y-axis
- y-coordinate
- Z-axis

In this edition, as with in the Year 9 book, the material involving the equations of lines has been moved to its own chapter "Straight lines" (Chapter 15). In the content that remains, students explore the Cartesian plane, including distance between points, midpoints, and gradients. The work in this chapter provides students the tools to describe straight lines in the Cartesian plane in the following chapter.

In Section D, we first use the converse of Pythagoras' theorem to establish that the gradients of perpendicular lines are negative reciprocals. We then use this fact to show that two parallel lines, which are both cut by the same perpendicular transversal, must have equal gradients.

In the Discussion in Section D, students should find that the rule for gradients of perpendicular lines does not apply to horizontal and vertical lines, since vertical lines have undefined gradient. The idea that vertical and horizontal lines are perpendicular should be intuitive to students, and gradient is not a very useful tool in this case.

Knowledge of the gradient of parallel and perpendicular lines allows us to verify and prove geometric facts in Section E (Using coordinate geometry), which is a new section for this edition.

Section F (3-dimensional coordinate geometry) has also been added in this edition, as it has been included in the Australian Curriculum. Students should be reminded that, although it is harder to visualise coordinates in three dimensions, most of the work that is done (such as finding distances and midpoints) extends fairly logically from what they have seen in two dimensions.

CHAPTER 15: STRAIGHT LINES

A The equation of a line
B Graphing straight lines
C Finding the equation of a line
D Perpendicular bisectors
E Linear inequalities in the Cartesian plane

Keywords:

- axes intercepts
- equation of a line
- general form
- gradient-intercept form
- perpendicular bisector
- point-gradient form
- x-intercept
- y-intercept

This new chapter comprises the material about straight lines that was in Sections E to G of the Coordinate geometry chapter in the previous edition. Given that some new material has also been added in this edition, presenting this work as a chapter in its own right allows us to provide a more complete treatment of this material, without making the chapter too large.

In Section A, we have more explicitly introduced the point-gradient form of the equation of a line. From here the equation can be rearranged into gradient-intercept form, or general form. Students find the gradient of a line in general form by rearranging it to gradient-intercept form. However, with sufficient practice, students should be able to quickly find the gradient of a line from its general form.

We have restructured the text in Section C so that, rather than focusing on the form in which to write the equation, we consider the different combinations of information students may be given about a line, and how to find the equation of the line in each case.

In this edition, we have added Section D (Perpendicular bisectors) since it is a useful application of the work students have been studying.

Section E (Linear inequalities in the Cartesian plane) has also been added in this edition, since it was added to the Australian Curriculum. To graph a region such as $2x + 5y < 10$, students should first graph the line $2x + 5y = 10$, and then use a test point such as $(0, 0)$ to determine which side of the line the required region lies on.

CHAPTER 16: SIMULTANEOUS EQUATIONS

- **A** Graphical solution
- **B** Solution by substitution
- **C** Solution by elimination
- **D** Problem solving
- **E** Non-linear simultaneous equations

Keywords:

- elimination
- simultaneous equations
- simultaneous solution
- substitution

When studying simultaneous equations, it is important that students understand the conceptual shift in that our solution takes the form of a value of x *and* y which make both equations true simultaneously.

In Section A, a graphical approach is used. This should allow students to use what they learnt in the previous chapter, and see that by graphing the line corresponding to each equation, the intersection point gives us the solution to the simultaneous equations.

This approach should illustrate to students why some systems have no solutions or infinitely many solutions. However, reading the solution from a graph makes it difficult to find non-integer solutions accurately. This leads to a need for the algebraic approaches outlined in Sections B and C.

Towards the end of Section D (Problem solving), students may find that the equations they are generating do not appear to be linear simultaneous equations. However, some of the questions require the students to do some rearrangement, and other questions require students to recognise that the equations are inherently linear, even if the variables take forms such as x^2 or y^2.

Section E (Non-linear simultaneous equations) is no longer marked as Extension, as it appears to be a reasonably logical progression for the more advanced students.

CHAPTER 17: CONGRUENCE AND SIMILARITY

- **A** Congruent triangles
- **B** Proof using congruence
- **C** Similarity
- **D** Similar triangles
- **E** Areas and volumes of similar objects

Keywords:

- congruent figures
- congruent triangles
- equiangular
- similar figures
- similar triangles

Since students will have studied congruence in Years 8 and 9, in this edition we have decided to remove the introductory section about congruent figures, and instead move straight into congruence of triangles.

The Investigation in which students discover the information required to uniquely draw a triangle has been moved to Year 9. In Year 10, students are given the criteria under which two triangles can be shown to be congruent.

Students may find Investigation 1 at the end of Section B to be quite subtle. For example, in **2**, students should be reminded that they cannot assume that the sides of the triangle are equal, since that is what we are trying to prove. In part **a**, we can only establish that two pairs of sides are equal, and the equal angles are not between the equal sides. So, we cannot conclude that the triangles formed are congruent. In part **b**, we can use AACorS to prove that the triangles are congruent, and hence the large triangle is isosceles.

In Section C, it should be highlighted that, for figures in general to be similar, both the figures must be equiangular *and* the side lengths must be in proportion. In particular, establishing one of these without the other is not sufficient. Questions **4** and **5** highlight that quadrilaterals that are equiangular may not be similar, and quadrilaterals with side lengths in the same ratio may not be similar. However, *triangles* are special in that, if one of these properties is true, the other must also be true. So, to establish two triangles are similar, only one of these properties need be proven. This is what is explored in Section D.

In Section D, some questions which involve solving a quadratic equation have been added. These questions were in the quadratic equations chapter in the previous edition, but needed to be moved here because the quadratic equations chapter is now placed before similarity.

Whereas as areas and volumes of similar objects were given separate sections in Year 9, they are considered in one section here. This gives more opportunity to explore the relationships between lengths, surface areas, and volumes of similar three-dimensional objects.

CHAPTER 18: CIRCLE GEOMETRY

A Angle in a semi-circle theorem
B Chords of a circle theorem
C Radius-tangent theorem
D Tangents from an external point theorem
E Angle between a tangent and a chord theorem
F Angle at the centre theorem
G Angles subtended by the same arc theorem

Keywords:

- chord
- diameter
- inscribed
- radius
- semi-circle
- subtended
- tangent

In this edition, rather than dividing the circle theorems arbitrarily into a "Circle theorems" Section and a "Further circle theorems" Section, each theorem is given a separate section.

Since the "Circle problems" Section of the chapter on Pythagoras' theorem has been removed, some of the questions that were in that section have been added to this chapter.

In this edition, we have removed the "Geometric proof" Section. The problems in this section have been moved to the section corresponding to the relevant circle theorem. An advantage with this approach is that students can prove each circle theorem at the time that the theorem is used, rather than leaving all of the circle theorem proofs until the end.

Some of the problems that involved trigonometry have been removed in this edition. This is because the circle geometry chapter has been moved earlier in the book, and is now placed before trigonometry.

CHAPTER 19: TRIGONOMETRY

A Labelling right angled triangles
B The trigonometric ratios
C Finding side lengths
D Finding angles
E Problem solving
F True bearings

Keywords:

- adjacent side
- angle of depression
- angle of elevation
- cosine
- hypotenuse
- inverse cosine
- inverse sine
- inverse tangent
- opposite side
- sine
- tangent
- trigonometry
- true bearing
- true north

When introducing the trigonometric ratios in Section B, the aim should be not only to familiarise students with the side lengths involved in each trigonometric ratio, but to help them understand that a ratio such as $\sin 57°$ is not just an abstract term, but an actual number whose value can be determined by measuring sides of right angled triangles.

In Section E (Problem solving), some questions which use circle geometry have been added. This has been done since this chapter now appears after the Circle geometry chapter.

The work on true bearings (Section F) extends what is done in Year 9 to consider multi-leg journeys, in which students must find the bearing of the end point from the starting point. In this chapter, the bearings of each leg are chosen to create a right angled triangle, but this acts as a lead in to the more realistic and complex problems in the following chapter, where the bearings can take any value, and distances and bearings are found using non-right angled triangle trigonometry.

As with the chapter on Pythagoras' theorem, the questions involving three-dimensional objects have been absorbed into the section on problem solving, rather than being presented as a section of their own.

CHAPTER 20: NON-RIGHT ANGLED TRIANGLE TRIGONOMETRY

- **A** Trigonometry with obtuse angles
- **B** The area of a triangle
- **C** The sine rule
- **D** The cosine rule
- **E** Problem solving

Keywords:

- cosine rule
- sine rule
- unit circle

Non-right angled triangle trigonometry is no longer mentioned in the Australian Curriculum at Year 10, and for this reason classes may choose to omit this chapter if they are short on time. However, we would strongly encourage classes to complete this chapter, as it will be extremely beneficial for students to be introduced to non-right angled trigonometry in Year 10, rather than seeing it for the first time in Years 11 and 12. The concept of extending the definition of trigonometric ratios beyond acute angles is likely to be quite challenging for students, and if students can at least be introduced to this concept in Year 10, it will help them for their future study in trigonometric functions and identities.

In Section A, we define the trigonometric ratios for obtuse angles. It may be beneficial to students to see that, although it is not meaningful to talk about the trigonometric ratios for obtuse angles based on our original definition involving right angled triangles, we can extend our definition of trigonometric ratios in a way that is consistent with our original definition, but is also meaningful for a greater range of angles. This is not dissimilar to what we did in Chapter 1, when we extended the definition of exponents so that it is meaningful for rational exponents.

In Section A we also establish the relationship between the trigonometric ratios of supplementary angles, which will be useful when considering the sine and cosine rules later in the chapter.

In Section B (The area of a triangle), Question **2** should help students see that the standard formula for the area of a triangle $A = \frac{1}{2} \times \text{base} \times \text{height}$, is a special case of the formula given here, in which the included angle is a right angle.

In Section C.2, students use the sine rule to find unknown angles, and will encounter the ambiguous case. It may be useful for students to consider a situation where we would use the sine rule to find an angle, in which we know two sides of a triangle, and a non-included angle. From their work in congruence, students should realise that this is not necessarily enough information to describe a triangle uniquely. This should help them understand why the ambiguous case arises.

In Section D (The cosine rule), students may benefit from seeing that Pythagoras' theorem is a special case of the cosine rule, in which the included angle is a right angle.

In Section E (Problem solving), students were introduced to true bearings in Chapter 19, so no further explanation of them is given here. The questions in this chapter allow a greater range of scenarios to be considered, where the cosine and sine rules are used to find distances and bearings.

CHAPTER 21: PROBABILITY

- **A** Sample space and events
- **B** Theoretical probability
- **C** Independent events
- **D** Dependent events
- **E** Experimental probability
- **F** Expectation
- **G** Conditional probability
- **H** Simulations

Keywords:

- 2-dimensional grid
- certain event
- combined events
- complementary events
- compound events
- conditional probability
- dependent events
- event
- expectation
- experimental probability
- frequency
- impossible event
- independent events
- probability
- relative frequency
- sample space
- simulation
- theoretical probability
- tree diagram

The contents of this chapter have been restructured to mirror the structure used in Year 9.

We have added Section A (Sample space and events) in this edition. This section considers not only the different ways to represent the sample space of an experiment, but also the definition of an event connected to an experiment, and how the outcomes of a particular event can be highlighted within the sample space. This serves to give this section some more substance, justifying its inclusion at Year 10.

In Section B (Theoretical probability), the method for finding unknown numbers in Venn diagram regions has been updated to reflect the more formal method described in Chapter 7.

In the Puzzle at the end of Section B, students should be able to recognise that an easy way to solve the puzzle is to add 1 to each face of one die (so it is numbered 2 to 7), and subtract 1 from each face of the other die (so it is numbered 0 to 5). However, students who find this solution should be challenged to find a less trivial solution; one that involves the same number appearing on a die more than once.

Section E (Experimental probability) has been added in this edition. Experimental probabilities are estimated based on repeated experiments, as well as from data given in frequency tables and two-way tables.

Section F (Expectation) is likely to be new to students. It is a reasonably intuitive idea, so students should not have trouble with it, but it is an important first step towards some more complex work involving expected value in Years 11 and 12.

Section G (Conditional probability) has been restructured since the conditional probability formula and its connection with independent events has been removed from the syllabus. This being the case, it made more sense to find conditional probabilities from grids using the more intuitive method used for Venn diagrams and two-way tables in the previous edition, and present this all in a single section. The conditional probability formula has been moved to an Investigation at the end of the section.

Section H (Simulations) has been added to the chapter in this edition. In this section, students use simulations to estimate probabilities in situations where calculating theoretical probabilities is very difficult. Many of the questions in this exercise require students to generate random numbers of their own, so students should be aware that their own answers will not exactly match those given in the back of the book. If students are obtaining answers that are vastly different to those in the back of the book, however, they should consider repeating the question with a new set of random numbers. If they still get very different answers to those in the back of the book, their method may be incorrect.

CHAPTER 22: STATISTICS

Keywords:

- biased sample
- bimodal
- bimodal distribution
- box-and-whisker plot
- box plot
- categorical data
- categorical variable
- census
- class interval

- column graph
- continuous numerical variable
- cumulative frequency
- cumulative frequency graph
- discrete numerical variable
- distribution
- dot plot
- five-number summary
- frequency histogram
- histogram
- interquartile range
- interval midpoint
- lower quartile
- maximum value
- mean
- median
- minimum value
- modal class
- mode
- negatively skewed distribution
- numerical data
- numerical variable
- outlier
- parallel box-and-whisker plot
- percentile
- population
- positively skewed distribution
- range
- sample
- sample size
- standard deviation
- statistics
- survey
- symmetric distribution
- tally and frequency table
- upper quartile
- variable

The chapter opens in much the same way as the first edition. However, we have moved the material on "Describing the distribution of data" to its own section to match what we did for Year 9.

In the "Measures of centre" Section, we have combined the raw data and the frequency table subsections so that it matches the structure in the new Year 9 book.

The "Life expectancy" Activity on page 414 is mostly the same as its previous incarnation in the first edition, but includes updated data and columns for Western Australia.

Cumulative frequency graphs has been moved after box-and-whisker plots as we feel that it is better to introduce quartiles first to provide:

- a gentler introduction to percentiles
- more motivation for *using* cumulative frequency graphs.

The "Comparing distributions of unequal size" subsection is new for this edition. These exercise questions cover the relevant elaboration in Version 9.0 of the Australian Curriculum.

In the Investigation at the end of Section H, students may need extra guidance choosing a report from the ABS website to read.

CHAPTER 23: BIVARIATE STATISTICS

A Association between categorical variables
B Association between numerical variables
C Correlation
D Pearson's correlation coefficient r
E Line of best fit by eye
F Linear regression

Keywords:

- bivariate data
- bivariate statistics
- correlation
- dependent variable
- extrapolation
- independent variable
- interpolation
- least squares regression line
- linear model
- linear regression
- line of best fit by eye
- mean point
- Pearson's correlation coefficient
- pole
- scatter plot
- two-way table

The "Line graphs" Section from the first edition has been removed and in its place is the "Association between categorical variables" Section, which is new for Version 9.0 of the Australian Curriculum. This section focuses on data for two categorical variables organised in a two-way table, which, while similar to the two-way tables covered in probability, considers them from the perspective of data and inference.

The "Association between numerical variables" Section is adapted from the "Scatter plots" Section from the first edition. However there is a greater emphasis on their construction and interpretation of the data as a whole. We felt that identifying individual points did not really fit the overall theme of the chapter, and students should already be familiar with identifying points on a number plane from their studies of coordinate geometry.

For the table guide for interpreting the value of r on page 452, we chose to include absolute value notation to:

- make the table more compact
- more clearly separate the size of r from its sign.

Activity 3 "Weather data" at the end of Section F is new for this edition. This is a good opportunity for students to explore a real data set. However, students may need some guidance to navigate the website. CSV files can be opened in spreadsheet software like Microsoft Excel and LibreOffice. However if such files are not available, copying the table from the website should suffice. It might also help to collate data for several months, rather than analysing data for just one month. In general, the more data you have, the more reliable your inferences will be.

CHAPTER 24: RELATIONS AND FUNCTIONS

A Relations and functions
B Function notation
C Domain and range
D Transformations of graphs

Keywords:

- domain
- function
- function notation
- function value
- image
- interval notation
- range
- reflection
- relation
- scale factor
- stretch
- transformation
- translation

This is intended to be a relatively simple chapter, and students will probably find it a welcome change from the previous few chapters. The purpose of the chapter is to put in place the terminology and structures that will be encountered when studying quadratic functions, exponential functions, and trigonometric functions in later chapters.

In this edition, we have presented domain and range in their own section to give them more emphasis, rather than being part of the Relations section. Section A now focuses on distinguishing between a relation and a function.

In Section D, horizontal stretches of functions have been removed in this edition, since they are not used later in the book. For example, trigonometric functions are stretched horizontally, but this change is described in terms of a change in the period of the function, rather than explicitly in terms of a horizontal stretch. This makes it easier to present all of the transformations as part of a single section, rather than splitting the section into three subsections.

In the Discussion at the end of Section D, students should conclude that:

- No points are invariant when a function is translated, since all points move the same distance in the same direction.
- When a function is stretched vertically or reflected in the x-axis, points on the x-axis are invariant.
- When a function is reflected in the y-axis, points on the y-axis are invariant.

CHAPTER 25: QUADRATIC FUNCTIONS

A Quadratic functions
B Graphs of quadratic functions
C Using transformations to graph quadratics
D Axes intercepts
E Axis of symmetry of a quadratic
F Vertex of a quadratic
G Finding a quadratic function

H Problem solving with quadratic functions

Keywords:

- axis of symmetry
- completed square form
- completing the square
- maximum value
- minimum value
- parabola
- quadratic function
- turning point
- vertex
- x-intercept
- y-intercept

In Section B, we have added an introductory Investigation in which students use the geometric definition to generate a parabola. There is also an Investigation at the end of the section which explores one of the properties of parabolas. Students should find that all of the rays reflected off the parabola pass through a single point, which is the focus of the parabola.

In Section C (Using transformations to graph quadratics), the more involved Investigation which was in the previous edition has been moved to Year 9. Instead, we first establish some basic ideas about the shape and position of $y = (x-h)^2 + k$ and $y = -x^2$ from our work on transformations in the previous chapter. We then investigate the effect of the value of a on the shape of the graph, and the direction in which it opens.

In Questions **7** to **9** of Exercise 25C.1, students must verify the completed square form of each quadratic function, then use this form to sketch the graph. In Section C.2, students must convert the quadratic functions into completed square form themselves.

At the end of Section D, we have added an Activity which links the discriminant of a quadratic function with its number of x-intercepts.

Section G (Finding a quadratic function) has been added in this edition. Finding a function based on information about its graph is an important modelling skill which students may find useful in later years.

The "Quadratic optimisation" Section has been renamed "Problem solving with quadratic functions" (Section H), since students are asked to solve problems other than identifying the optimum point.

CHAPTER 26: EXPONENTIALS AND LOGARITHMS

A Exponential functions
B Graphs of exponential functions
C Exponential equations
D Exponential growth
E Exponential decay
F Logarithms
G Laws of logarithms
H Using logarithms
I Logarithmic scales

Keywords:

- common logarithm
- exponential decay
- exponential equation
- exponential function
- exponential growth
- horizontal asymptote
- logarithm
- logarithmic scale
- logarithm laws
- order of magnitude
- y-intercept

This will be the first time students have encountered exponential functions. Students should understand that, while linear functions are characterised by a quantity changing by a constant *amount* each time period, exponential functions are characterised by a quantity changing by a constant *percentage* each time period.

In this edition, we have moved exponential equations ahead of exponential growth and decay. The exponential equations section now includes practice at solving exponential equations using technology. This can either be done graphically, or by using the Solver function of the calculator. This can then be used in exponential growth and decay to answer questions such as "How long will it take for the population to reach 500?".

The work on logarithms has been expanded from what was in the previous edition. In Section F, students are introduced to the concept of a logarithm, and hopefully get a feel for how logarithms work. In the Discussion at the end of Section F, students should find that $\log(10^n)$ is equal to n for any real n, and that $10^{\log n}$ is equal to n for any $n > 0$. Once the concept of a logarithm is understood, these results follow from the definition of a logarithm. Students should also find that $\log 0$ does not exist, since we cannot raise 10 to any power to obtain the number 0.

Section I (Logarithmic scales) has been added in this edition. Students should be familiar with some of the scales mentioned here, such as the Richter scale. Students should be encouraged to solve these problems by thinking them through intuitively, rather than by algebraic means. For example, in the Richter scale, each increase of 1 in the magnitude means that the intensity is multiplied by 10. If students understand this, then it is fairly straightforward to deduce that, for example, a magnitude 4 earthquake is $10 \times 10 = 100$ times more intense than a magnitude 2 earthquake, and that an earthquake that is 10 times the intensity of a magnitude 4 earthquake has magnitude $4 + 1 = 5$. Trying to solve these problems using the formula $M = \log\left(\frac{I}{I_0}\right)$ is likely to lead to confusion.

CHAPTER 27: ADVANCED TRIGONOMETRY

A The unit circle
B Multiples of $30°$ and $45°$
C The Pythagorean identity
D Trigonometric functions
E Transformations of trigonometric functions
F Trigonometric equations

Keywords:

- amplitude
- cosine
- maximum point
- mean line
- minimum point
- period
- periodic function
- principal axis
- Pythagorean identity
- sine
- tangent
- trigonometric function
- unit circle

We start this chapter by extending the definition of trigonometric ratios for *all* angles. If students have understood how we defined trigonometric ratios for obtuse angles in Chapter 20, then extending the definition further should not be problematic.

In this edition, we have switched the order of Sections B and C. We feel that the trigonometric ratios for multiples of $30°$ and $45°$ follow more logically from the definition of the trignometric ratios in Section A.

In Section D, we have added an Investigation in which students use the trigonometric ratios found in Section B to generate the graphs of $y = \sin x$ and $y = \cos x$ for themselves. This should give students a greater understanding of the graphs' features, including their maximum and minimum values, and their periodic nature.

In Section E, we use the work done in Chapter 24 to generate transformations of $y = \sin x$ and $y = \cos x$. This is not explicitly in the syllabus, but given that we already have the knowledge of transformations, it seems appropriate to apply it here. It also gives scope to consider more realistic modelling situations towards the end of the section.

Given that we have special terminology to describe periodic functions such as amplitude, period, and principal axis, the transformed functions are generally described using this terminology, rather than explicitly referencing the transformations involved.

In Section F, students solve trigonometric equations using their knowledge of the trigonometric ratios for multiples of $30°$ and $45°$. It is important that students understand how to generate all of the solutions in the given domain. For example, they may need to add or subtract multiples of $360°$ to the solutions which can be read from the unit circle.

CHAPTER 28: COUNTING AND PROBABILITY

A The product principle
B The sum principle
C Factorial notation
D Permutations
E Combinations
F Probabilities using permutations and combinations

Keywords:

- combination
- counting problem
- factorial number
- permutation
- product principle
- sum principle

The material in this online chapter is listed under the "optional content for post Year 10 Mathematics pathways", so this chapter may be omitted if time is short. However, it would be a useful chapter for students heading towards Mathematical Methods and Specialist Mathematics in Years 11 and 12.

Sections A and B introduce students to the product and sum principles. Students should be able to identify when the numbers of possibilities should be multiplied, and when they should be added.

In Section D, students are introduced to permutations. These are selections in which the order of selection *is* important. Although the formula for the number of permutations is given in the factorial form $\frac{n!}{(n-r)!}$, students may find that the form $n \times (n-1) \times (n-2) \times .... \times (n-r+1)$ is more useful, and more intuitive as it flows directly from the product principle. The main purpose of the factorial formula comes in Section E, where the formula for the number of combinations is found by dividing the number of permutations by $r!$. This occurs because, for each combination, there are $r!$ corresponding permutations formed by ordering the r selected objects.

In Section F, we find probabilities using permutations and combinations. In this section, we calculate the number of relevant permutations or combinations as in Sections D and E, but then must divide by the total number of possible outcomes to find the relevant probability. Since permutations and combinations are combined here, students must be able to decide which must be used in each question. Some of the questions can be done using either combinations or permutations, however in these cases students must be consistent in their approach when calculating number of outcomes in the event, and the total number of possible outcomes.