MATHEMATICS 6 MYP 1 third edition

Chapter summaries

Haese Mathematics

June 1, 2022

CHAPTER 1: WHOLE NUMBERS

A Place value
B Number lines
C Big numbers
D Rounding numbers

Keywords:

- ascending
- billion
- counting number
- descending
- digit
- expanded form
- greater than
- Hindu-Arabic number system
- less than
- million
- natural number
- number line
- number system
- numeral
- numeral form
- place value
- place value chart
- round
- round down
- round up
- trillion
- whole number
- zero

In this edition, this chapter focuses on the structure of whole numbers, rather than performing operations with them. The operations work has been moved to its own chapter.

We describe the expanded form of the number 6794 as simply $6000+700+90+4$ rather than $6\times1000+7\times100+9\times10+4$. This is in part because the students have not done order of operations yet, so the second expression could be confusing. There is a greater emphasis on students converting numbers between numeral form and word form.

In Section B (Number lines), the emphasis is on using the number line to order numbers, rather than to perform operations with the numbers. We take the opportunity to introduce students to the $<$ and $>$ signs.

In Section C (Big numbers), students are introduced to millions, billions, and trillions at the same time. We feel that they are not conceptually different, and that it is easier for the student to see them together.

In Section D (Rounding numbers), the term *multiple* is used, even though multiples are not covered until Chapter 4. Here, we only consider multiples of 10, 100, and 1000, and the terms are explained as they are needed.

CHAPTER 2: OPERATIONS

A Addition
B Subtraction
C Multiplication
D Column multiplication
E Division
F Problems with multiple operations
G Exponent notation
H Order of operations

Keywords:

- add
- addition
- base
- BEDMAS
- difference
- divide
- dividend
- division
- divisor
- exponent
- exponent notation
- index
- multiplication
- multiply
- operations
- power
- product
- quotient
- remainder
- subtract
- subtraction
- sum

This chapter comprises the operations work which was previously in the chapter titled Whole numbers.

Rather than simply presenting strategies for performing operations, we have presented investigations aimed at getting students to discover the strategies for themselves. We think this will help them appreciate and understand why the strategies are helpful. We have also presented the strategies *before* giving the standard techniques using columns, as students should first try to use the strategies to perform operations quickly. If no such strategy is appropriate, only then should they use a column method.

The multiplication and division by powers of 10 have been separated into the multiplication and division sections, since they are important multiplication and division strategies in their own right. In particular, the ease of multiplying by a power of 10 forms the basis for other multiplication strategies.

CHAPTER 3: LINES AND ANGLES

A Lines
B Angles
C Measuring angles
D Calculating angles
E Vertically opposite angles

Keywords:

- acute angle
- angle
- arms
- bracket notation
- degrees
- intersecting lines
- line
- line segment
- obtuse angle
- parallel lines
- point
- point of intersection
- protractor
- ray
- reflex angle
- revolution
- right angle
- straight angle
- three point notation
- vertex
- vertically opposite angles

We have organised the sections of this chapter into a more logical structure. The opening section is primarily about lines rather than points, so we have moved the material about points before Section A, and Section A is now named "Lines" rather than "Points and lines".

We feel it is better that students are familiar with how to describe and classify an angle before they measure angles, so Section B now only asks students to name and classify angles. Measuring by protractor is then covered in Section C.

Section D (Calculating angles) incorporates what was previously in Section 3C (Angles at a point or on a line), as well as the questions from Section 3B (Angles) that dealt with finding unknown angles by addition or subtraction of known angles. We feel these questions fit well together under the title of "Calculating angles", as the strategies employed are very similar. Notice that some questions require students to find x, but we are not expecting students to formally solve equations at this stage. The students should find x intuitively, using the information given about right angles, angles on a line, or angles at a point.

CHAPTER 4: NUMBER PROPERTIES

- **A** Zero and one
- **B** Square numbers
- **C** Cubic numbers
- **D** Divisibility
- **E** Divisibility tests
- **F** Factors
- **G** Prime and composite numbers
- **H** Highest common factor
- **I** Multiples

Keywords:

- composite number
- cubic number
- divisible
- even
- factor
- factor pair
- highest common factor
- identity
- multiple
- odd
- one
- prime number
- square number
- undefined
- zero

In the Discussion in Section A, students should find that $0 + 12 = 12$, $0 \times 12 = 0$, and $1 \times 12 = 12$. Subtraction and division do not have this property, since for example $5 - 3$ does not equal $3 - 5$, and $5 \div 3$ does not equal $3 \div 5$. The special name for this property is commutativity.

We have introduced highest common factor so we can use it for describing what happens when reducing fractions to lowest terms. We talk about multiples, but we will not introduce lowest common multiple yet, since addition and subtraction of fractions is only done where one denominator is a multiple of the other. Lowest common multiples will be covered in MYP 2.

CHAPTER 5: GEOMETRIC SHAPES

- **A** Polygons
- **B** Triangles
- **C** Quadrilaterals
- **D** Circles
- **E** Solids
- **F** Drawing solids
- **G** Nets of solids

Keywords:

- apex
- circle
- closed figure
- compass
- cone
- cross-section
- cube
- cylinder
- edge
- equilateral triangle
- face
- irregular polygon
- isosceles triangle
- kite
- net
- parallelogram
- polygon
- prism
- pyramid
- quadrilateral
- radius
- rectangle
- regular polygon
- rhombus
- scalene triangle
- solid
- solid of uniform cross-section
- sphere
- square
- surface
- three-dimensional
- trapezium
- triangle
- two-dimensional
- vertex

In this edition, we have moved Circles from Section B to Section D. This allows us to consider the polygons together, before moving on to circles, and then to solids.

The work on constructing triangles using a compass, protractor, and ruler has been moved to an Activity at the end of Section B.

In Section E (Solids), we have taken more care to define terms such as face, edge, and vertex of a solid. This will help students talk sensibly about the solids.

In the Discussion at the end of Section E, students should find that pyramids and cones are called tapered solids because, as we move up the solid, the cross-section tapers to a point. Students should also recognise that the solids given are pyramids, and it should be emphasised that the apex need not be directly above the centre of the base.

In Section F, we have taken the opportunity here to introduce the word *ellipse*, to describe the shape of a circle viewed from an angle. This shape is used when drawing cones, spheres, and cylinders.

CHAPTER 6: FRACTIONS

- **A** Fractions
- **B** Fractions as division
- **C** Proper and improper fractions
- **D** Fractions on a number line
- **E** Equal fractions
- **F** Lowest terms
- **G** Comparing fractions
- **H** Adding and subtracting fractions
- **I** Multiplying a fraction by a whole number
- **J** A fraction of a quantity

Keywords:

- bar
- denominator
- equal fractions
- fraction
- improper fraction
- lowest terms
- mixed number
- numerator
- proper fraction
- simplest form

In Section C, we have simplified the process of converting between improper fractions and mixed numbers.

Having introduced highest common factor in Chapter 4, we can now use this term when describing how to reduce fractions to lowest terms.

We have tried to avoid the term "simplest form" as much as possible in this new series, as it can be quite ambiguous. The simplest form to work with quite often depends on the context, and what you are trying to do. So, we have predominantly used "lowest terms" to describe fractions. However, we have kept the simplest form usage occasionally, as we understand this is in common use.

In the Discussion of an alternative method of adding and subtracting fractions in Section H, students should find that this approach is most useful when proper fraction parts sum to less than one. They should also notice that care must be taken when using this method to subtract fractions, since the fraction part of the subtracted fraction must also be subtracted.

We have added Section I (Multiplying a fraction by a whole number), so that students can find fractions of quantities more sensibly using fraction multiplication, rather than division. All that is required is for students to recognise that "of" means that we multiply. This will also allow students to multiply decimal numbers by a whole number by converting the decimal number to a fraction.

CHAPTER 7: DECIMALS

- **A** Decimal numbers
- **B** Decimal numbers on a number line
- **C** Ordering decimal numbers
- **D** Rounding decimal numbers
- **E** Converting decimals to fractions

F Converting fractions to decimals
G Adding and subtracting decimal numbers
H Multiplying by powers of 10
I Dividing by powers of 10
J Multiplying decimals by a whole number
K Dividing decimals by a whole number

Keywords:

- decimal number
- decimal point
- hundredth
- place value
- tenth
- thousandth

In the Discussion in Section A, students should recognise that including a "0" before the decimal point in 0.37 makes the decimal point stand out more. Students should see that 1.5 and 1.50 have the same value, and there are some situations, such as dealing with currency, where we would write 1.50 rather than 1.5 . It may be worth returning to this Discussion once students have completed the section on rounding decimal numbers, and have students think about the difference between rounding 1.5032 to 1.5 (one decimal place) or 1.50 (2 decimal places). Is the second approximation more accurate, even though both approximations have the same value?

In Section C, we extend the work in Section B and use a number line to order decimal numbers, in line with what has been done with whole numbers and fractions. We then look at a more formal method for ordering decimals.

In the Discussion in Section D, students should indeed find that the same rule for rounding whole numbers can also be used to round decimal numbers.

When multiplying and dividing decimal numbers by powers of 10, students should understand that each digit moves a particular number of places in the place value chart. This has the *effect* of moving the decimal point.

We have improved the method used to multiply a decimal number by a whole number, by converting the decimal number to a fraction with a denominator that is a power of 10.

CHAPTER 8: MEASUREMENT: INTRODUCTION

A Units
B Reading scales
C Mass

Keywords:

- gram
- kilogram
- mass
- milligram
- scale
- tonne
- units

We wanted to give students a more general introduction to measurement, so we have added this chapter, which talks students through the different units used to measure things, and how to read scales. This chapter contains plenty of Activities for students to explore different measuring devices, and provides students the opportunity to think about when we measure things in our daily lives, and why different units of measurement are used.

The Discussion in Section A is a prelude to the distinction between discrete and continuous variables in later years. Students should find that is does not make sense to use fractions or decimals when counting items (you cannot have $\frac{1}{2}$ a car or 4.5 people), but it does make sense when measuring quantities (you can have $\frac{1}{2}$ a metre or 4.5 kg).

CHAPTER 9: MEASUREMENT: LENGTH

A Units of length
B Operations with lengths
C Perimeter
D Scale diagrams

Keywords:

- centimetre
- kilometre
- length
- metre
- millimetre
- perimeter
- scale
- scale diagram
- scale factor

We have separated the work which was previously a single Section "Length" into two sections. In Section A students solely deal with converting between units of length, and in Section B students perform operations with lengths. This separation gives us the chance to highlight the idea that two lengths must be written in the same units before we can perform operations with them.

In the Discussion at the start of Section A, students should conclude that using parts of the body to measure length has problems because these body parts will differ in length from person to person.

In the Discussion at the start of Section B, it should be emphasised to students that we need to add units of the same type in order to obtain a meaningful answer. A good way to illustrate this would be to consider adding 2 metres and 10 centimetres. If we simply add the 2 and the 10, in what units should the answer be written?

CHAPTER 10: MEASUREMENT: AREA, VOLUME, AND CAPACITY

A Area
B The area of a rectangle
C The area of a triangle
D Volume
E The volume of a rectangular prism
F Capacity

Keywords:

- area
- capacity
- cubic centimetre
- cubic metre
- cubic millimetre
- kilolitre
- litre
- megalitre
- millilitre
- square centimetre
- square kilometre
- square metre
- square millimetre
- volume

In this edition, we have minimised the amount of work done on "square units", in favour of getting students more quickly into using the metric units they would see around them such as square centimetres and square metres.

In the Discussion at the end of Section A, students should find that they can more quickly find the total area by multiplying the number of rows by the number of columns, rather than counting individual squares. This leads into the formula for finding the area of a rectangle in the next section.

We have removed the area of a parallelogram in this edition, to give a more steady progression of difficulty through the years. There is an Investigation about the area of a parallelogram at the end of Section C, and students will encounter it more formally in MYP 2.

We have added capacity to the chapter in this addition. At MYP 1, students will simply be introduced to the units of capacity, and converting between them. The connection between capacity and volume will be explored in MYP 2.

CHAPTER 11: TIME

A Time lines
B Units of time
C The calendar year
D Time calculations
E 24-hour time
F Timetables

Keywords:

- 12-hour time
- 24-hour time
- calendar year
- day
- hour
- leap year
- minute
- month
- second
- time line
- timetable
- week
- year

In this edition, we have introduced a new Section C (The calendar year), which covers the number of days in each month, and explains the conditions for a leap year. This allows us to separate the time calculations involving dates, such as "How many days is it from April 24th to July 17th?", from the calculations involving times of the day such as "Find the time which is 3 hours 40 minutes after 10:45 am". We feel this is appropriate, as they are conceptually quite different.

In the Discussion at the start of Section B, students should discuss how people talk about time differently for longer periods such as days, weeks, or years. When we talk about the duration of events using these units, we usually do not mean that the task will be performed during this *entire* time period, rather that the time actually describes how long it will be between the task starting and finishing.

In Section D, we have slightly amended the method used to find the difference between two times. Rather than counting to the nearest hour, counting the number of complete hours, then counting the remaining minutes, we first count the number of full hours, and then find the remaining minutes. This is more in line with how we think about duration, and is especially easier for time differences with a small number of minutes left over. For example, to find the time difference from 9:10 am to 11:20 am, rather than counting from 9:10 am to 10 am, 10 am to 11 am, then 11 am to 11:20 am, we count 2 hours from 9:10 am to 11:10 am, then another 10 minutes from 11:10 am to 11:20 am.

In Section E, students should be reminded to take care with times between 12 pm and 1 pm. These are pm times, but do not "change" when converted to 24-hour time.

CHAPTER 12: PERCENTAGE

- **A** Percentage
- **B** Converting percentages into fractions
- **C** Converting fractions into percentages
- **D** Converting percentages into decimals
- **E** Converting decimals into percentages
- **F** Number lines
- **G** Expressing one quantity as a percentage of another
- **H** Finding a percentage of a quantity
- **I** Percentage increase or decrease

Keywords:

- percent
- percentage

The Investigation at the end of Section A gives students an opportunity to find some common percentages, before formally considering conversions between percentages and fractions in Sections B and C.

In Section H, students should be reminded of the method used to find fractions of quantities in Chapter 6. Students should remember that "of" means that we multiply, and the only difference here is that we are working with a percentage, which we convert to a decimal.

Percentage increase and decrease has been included in this edition. This is effectively an application of Section H, where we first find the size of the change by finding a percentage of the quantity, then apply that change to the original amount by addition or subtraction.

CHAPTER 13: POSITIVE AND NEGATIVE NUMBERS

- **A** The number line
- **B** Ordering numbers
- **C** Words indicating positive and negative
- **D** Addition and subtraction on the number line

E Adding and subtracting negative numbers
F Multiplying negative numbers
G Dividing negative numbers

Keywords:

- negative number
- negative sign
- opposite
- positive number
- positive sign

This chapter has been significantly restructured from the previous edition.

We have introduced the number line from the start of the chapter, as we feel that this will help students understand the relationship between positive and negative numbers.

We have also made clearer the distinction between talking about *position* (for example, representing 3 levels below ground level as -3), and *movements* (for example, representing going down 3 levels as "subtract 3"). In the contextual problems, students should understand that we start from a position on a number line, described by a positive or negative number, interpreted by the context. Then, there is a series of movements, corresponding to operations (addition or subtraction) involving positive quantities. We then end up at our final position, which may be positive or negative, to be interpreted in context. Understanding of all of this is set up by the Activity which distinguishes between positions and directions.

In the Discussion in Section A, students should find that the opposite of zero is zero.

In the Discussion in Section B, students should see that terms such as *larger* and *smaller* are potentially confusing when discussing positive and negative numbers, because the terms have an association with *size*. For example, -5 is less than -3, however some may argue that -5 is a *larger* number than -3, because it is further from 0.

In Section D, students use a number line to answer problems which either start at a negative value (such as $-4 + 7$) or which have a negative answer (such as $3 - 8$). In Section E, students perform additions and subtractions with negative numbers, such as $9 + -6$ or $2 - -8$.

Multiplication by negative numbers can be conceptually difficult for many students, especially the idea of multiplying two negative numbers together. The Discussion at the start of Section F should be used to lead students in the right direction. By considering 3×-5 as "3 lots of -5", students should be able to see that $3 \times -5 = -15$. From there students could be asked to consider -3×-5 as "the opposite of 3×-5", which is the opposite of -15, which is 15. This should give students a justification that the product of two negative numbers is positive.

CHAPTER 14: SEQUENCES

A Generating a sequence
B Finding a rule for a sequence
C Patterns

Keywords:

- pattern
- rule
- sequence

This chapter has been added for this edition. This will give students an opportunity to investigate patterns (on which an emphasis is placed in the MYP framework) using words only, before describing such relationships using algebraic variables and formulae in later years.

When generating sequences involving fractions (such as "Start at 2, and add $\frac{3}{8}$ each time"), students should be reminded that it is not always sensible to write the fractions in the sequence in lowest terms immediately, as it will be easier to generate the terms if the fraction is left with the same denominator as the fraction specified in the rule. The terms of the sequence may be written in lowest terms as the final answer.

When trying to find a rule for a given sequence, students should make sure their rule holds for *all* terms of the sequence, not just the first two terms. For example, for a sequence starting $2, 6, ...$, the rule may involve adding 4 each time, or it may involve multiplying by 3 each time. Subsequent terms of the sequence will need to be considered to determine the correct rule.

In the Global Context about honeycombs at the end of the chapter, there are two versions of Question **6**. The harder version divides the cells in each layer into "light" and "dark" cells. This approach is conceptually more challenging, but offers a better insight into *why* the number of walls in each layer follows the required rule. Before the students begin the task, the teacher should look at each version of Question **6**, and decide which version is most appropriate for their students.

CHAPTER 15: LOCATION

- **A** Grid references
- **B** Locating points
- **C** Coordinates
- **D** Positive and negative coordinates
- **E** Compass points

Keywords:

- axes
- coordinates
- east
- grid reference
- map
- north
- northeast
- northwest
- number plane
- ordered pair
- origin
- south
- southeast
- southwest
- west
- x-axis
- x-coordinate
- y-axis
- y-coordinate

When dealing with coordinates, we have removed the idea of "movement from the origin" in order to plot points. Students should instead be encouraged to think of the axes as number lines, and to plot points on the number plane in the same way they would plot numbers on a number line. This is especially helpful when dealing with negative coordinates in Section D, as it removes the need to think about an x-coordinate of -2 as "moving 2 to the left".

In the Discussion in Section C, students could consider that using numbers makes it easier to compare the position of points relative to each other than with letters. A possible disadvantage of using numbers in both axes is that there is the potential to swap the coordinates, so the position $(3, 5)$ could be mistaken for $(5, 3)$.

The Discussion at the end of the chapter is intended to highlight the distinction between how directions have been used in this chapter, and how they are used in everyday language. The directions in this chapter have been used very precisely, so as to allow students to describe the exact coordinates of locations. However, in everyday language we might say that B is "to the south of A", even if it is not *due* south of A.

CHAPTER 16: LINE GRAPHS

- **A** Line graphs
- **B** Travel graphs
- **C** Conversion graphs

Keywords:

- conversion graph
- decreasing
- increasing
- line graph
- travel graph

In Section A we have included questions asking students to construct their own line graph from data. We have also omitted reference to point graphs as an intermediate step as we feel it is not helpful for students.

In the Discussion in Section A, students should think about the advantages and disadvantages of using straight line segments or curves between the known points. It would be useful to consider situations where we are confident of how the quantities behave between the known data points, or which it is clear from the given data that the quantities do not change in a linear manner. In these situations, it may be preferable to use a curve to more accurately describe what is happening between the given data points. However, if we are not sure what would happen between the given data points, using straight line segments is likely to give us a good estimate of what is happening.

We have also added some questions asking students to construct their own travel graph from data in Section B. For straight line travel graphs, students should be able to determine the speed of the object. The Discussion at the start of Section B should prompt students to realise that a straight line travel graph indicates constant speed, and that a steeper graph indicates greater speed. This will lay the foundation for interpreting the gradient of a travel graph as the speed of the object in later years.

CHAPTER 17: PROBABILITY

- **A** Describing probability
- **B** Using numbers to describe probabilities
- **C** Outcomes
- **D** Calculating probabilities

Keywords:

- 50-50 chance
- certain
- equally likely
- event
- impossible
- likely
- outcome
- probability
- unlikely

In the Discussion at the end of Section A, students should be encouraged to think about what it means for an event to be *impossible* to occur, as opposed to "highly unlikely". Many events are extremely unlikely to occur, but unless we can definitively rule out the possibility of them occurring, we cannot classify them as impossible.

In Section D, a Discussion regarding equally likely events has been moved to MYP 2. Instead, question **4** has been added to help students see why we must require the events to be equally likely.

Section D ends with an Activity about experimental probability. This experiment has been deliberately chosen to be one for which theoretical probabilities can be calculated. Students should see that, as the number of trials of the experiment increases, the experimental probabilities get closer to the theoretical probabilities. This concept will be explored further in subsequent years.

CHAPTER 18: STATISTICS

- **A** Categorical data
- **B** Dot plots
- **C** Pictograms
- **D** Column graphs
- **E** Pie charts
- **F** Numerical data
- **G** Measuring the centre of a data set

Keywords:

- average
- categorical data
- column graph
- data
- dot plot
- frequency
- mean
- mode
- numerical data
- pictogram
- pie chart
- statistics
- tally
- tally and frequency table

In this edition, each type of graph for categorical data gets its own section. Pictograms are new in this edition and come right after dot plots.

Activity 1 allows students to practise recording their own data, organising it, and then drawing graphs to display their data. Please note that the bird watching simulation shows the birds in a random order, so each student will end up with a different data set. As an extension, you can ask the students to draw different types of graphs to display their data and discuss the effectiveness of each graph for conveying information.

Activity 2 invites the student to use technology to generate graphs for the previous exercises. We have provided a link to our statistics software, but have not provided instructions for generating graphs with spreadsheets. While knowing how to use spreadsheets is an important skill, we feel that it is not our place to teach students how to use spreadsheets, as there are many different spreadsheet programs available.

CHAPTER 19: TRANSFORMATIONS

- **A** Translations
- **B** Reflections
- **C** Rotations
- **D** Combinations of transformations

Keywords:

- anticlockwise
- centre of rotation
- clockwise
- image
- mirror line
- object
- reflection
- rotation
- transformation
- translation

In this edition, we have made the shadings and outlines of the object and the image the same, rather than having them be different colours. We have done this to make sure that the only difference between the object and image is the transformation it has undergone.

In the Discussion at the start of Section C, students should find that, when a point is rotated about O, its distance from O does not change. This should help students when performing rotations of their own.

In the Discussion at the end of Section C, students should find that a translation only changes the position of an object, while a reflection and rotation also change the orientation of the object. Most importantly, none of these transformations change the size or shape of the object. This will be emphasised further in coming years as we start to talk about congruent figures.

In this edition, enlargements and reductions have been moved up to MYP 2. We felt that it was more sensible that students first explore transformations which do not change the size of the object, before extending to enlargements and reductions in MYP 2. Instead, a section on combinations of transformations has been included. The more able students should be encouraged to think about whether the order in which transformations are performed is important.