

ERRATA

Mathematics for Australia 10A Worked Solutions

2013 First Edition, initial print

The following erratum was made on 02/Jan/2020

page 582 CHAPTER 23 EXERCISE 23D.3, Question 4 a should read:

- 4 a For $H(t) = 4 \sin[45(t - 2)]^\circ + 4$:
- the amplitude is $|4| = 4$
 - the period is $\frac{360}{45} = 8$ seconds
 - the horizontal translation is 2 seconds to the right
 - the principal axis is $H = 4$.

The following errata were made on or before 16/Jul/2015

page 19 CHAPTER 1 PRACTICE TEST 1C, Question 1 a ii should read:

- | | |
|---|---|
| <p>1 a i 1 hour = 60 minutes
 $= 60 \times 60$ seconds
 $\{1 \text{ minute} = 60 \text{ s}\}$
 $= 3600$ seconds
 and $2.998 \times 10^8 \times 3600$
 $= 1.07928 \times 10^{12}$
 $\approx 1.079 \times 10^{12}$
 So, light travels about 1.079×10^{12} m in
 one hour (in a vacuum).</p> | <p>ii 1 day = 24 hours
 $= 24 \times 3600$ seconds {using a i}
 $= 86400$ seconds
 and $2.998 \times 10^8 \times 86400$
 $= 2.590272 \times 10^{13}$
 $\approx 2.590 \times 10^{13}$
 So, light travels about 2.590×10^{13} m in
 one day (in a vacuum).</p> |
|---|---|

page 74 CHAPTER 4 EXERCISE 4B.2, Question 2 h should read:

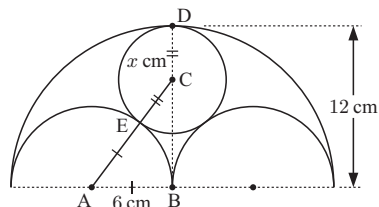
<p>2 g $\frac{3b+9}{6}$ $= \frac{3(b+3)}{6}$ ← HCF is 3 $= \frac{1\cancel{3}(b+3)}{2\cancel{6}}$ $= \frac{b+3}{2}$</p>	<p>h $\frac{8b-12}{6}$ $= \frac{4(2b-3)}{6}$ ← HCF is 4 $= \frac{2\cancel{4}(2b-3)}{3\cancel{6}}$ $= \frac{2(2b-3)}{3}$ $= \frac{4b-6}{3}$</p>
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page 92 CHAPTER 4 PRACTICE TEST 4C, Question 3 c ii should not cancel a division by 0:

3 c Using b, $\left(a - \frac{9}{a}\right) \div \left(1 - \frac{a}{3}\right) = \frac{3(a+3)}{-a}$

<p>i When $a = 1$, $\frac{3(a+3)}{-a} = \frac{3(1+3)}{-1}$ $= \frac{3 \times 4}{-1}$ $= \frac{12}{-1}$ $= -12$</p>	<p>ii When $a = 3$, $1 - \frac{a}{3} = 1 - \frac{3}{3}$ $= 1 - 1$ $= 0$ $\therefore \left(a - \frac{9}{a}\right) \div \left(1 - \frac{a}{3}\right)$ $= \left(a - \frac{9}{a}\right) \div 0$ which is undefined</p>	<p>iii When $a = 5$, $\frac{3(a+3)}{-a} = \frac{3(5+3)}{-5}$ $= \frac{3 \times 8}{-5}$ $= \frac{24}{-5}$ $= -\frac{24}{5}$</p>
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- 14 a ii Consider the 'upper' semi-circle:



Suppose the radius of the smallest circle is x cm and $CB = DB - DC = (12 - x)$ cm.
We construct $[AC]$, which passes through E .

Now, $AE = 6$ cm {from a i}

$$\therefore AC = (6 + x) \text{ cm}$$

$$\therefore (6 + x)^2 = 6^2 + (12 - x)^2 \quad \{\text{Pythagoras in } \triangle ABC\}$$

$$\therefore 36 + 12x + x^2 = 36 + 144 - 24x + x^2$$

$$\therefore 12x = 144 - 24x$$

$$\therefore 36x = 144$$

$$\therefore x = 4$$

\therefore the radius of the smallest circles is 4 cm.

- 10 Let the diagonal $[BD]$ be $2x$ m.
11 Consider the pyramid on top of the tower. Let the diagonal $[AC]$ be $2x$ m.

- 7 Suppose the foot of the ladder is x m from the wall.

\therefore the ladder reaches $2x$ m up the wall.

$$\therefore x^2 + (2x)^2 = 15^2 \quad \{\text{Pythagoras}\}$$

$$\therefore x^2 + 4x^2 = 225$$

$$\therefore 5x^2 = 225$$

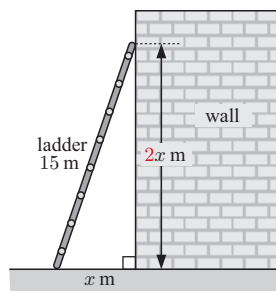
$$\therefore x^2 = 45$$

$$\therefore x = \sqrt{45} \quad \{\text{as } x > 0\}$$

$$\therefore 2x = 2\sqrt{45}$$

$$\therefore 2x \approx 13.42 \text{ m}$$

The ladder reaches about 13.4 m up the wall.



- 1 a $C = 2\pi r$ where $r = 4.2$

$$\therefore C = 2 \times \pi \times 4.2$$

$$\approx 26.4$$

\therefore the circumference is approximately 26.4 cm.

- b $C = 2\pi r$ where $C = 112$

$$\therefore 112 = 2\pi r$$

$$\therefore r = \frac{112}{2\pi}$$

$$\approx 17.8$$

\therefore the radius is approximately 17.8 cm.

- 7 b i $A = 180$, $a = 8$, $b = 6$

$$\therefore c = \frac{180 - 2 \times 8 \times 6}{2(8 + 6)}$$

$$= \frac{84}{2 \times 14}$$

$$= 3$$

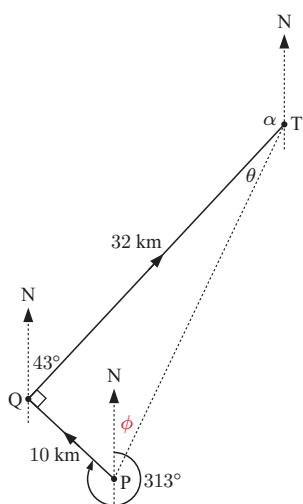
2 g $x^2 + 1 = 3x$
 $\therefore x^2 - 3x + 1 = 0$ which has
 $a = 1, b = -3, c = 1$
 $\therefore x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(1)}}{2(1)}$
 $\therefore x = \frac{3 \pm \sqrt{9 - 4}}{2}$
 $\therefore x = \frac{3 \pm \sqrt{5}}{2}$

h $2x^2 = 2x - 3$
 $\therefore 2x^2 - 2x + 3 = 0$ which has
 $a = 2, b = -2, c = 3$
 $\therefore x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(2)(3)}}{2(2)}$
 $\therefore x = \frac{2 \pm \sqrt{4 - 24}}{4}$
 $\therefore x = \frac{2 \pm \sqrt{-20}}{4}$
 but $-20 < 0$
 \therefore no real solutions exist.

2 d $AB = \sqrt{(-\sqrt{2} - \sqrt{2})^2 + (0 - 0)^2}$
 $= \sqrt{(-2\sqrt{2})^2 + 0^2}$
 $= \sqrt{8}$ units
 $AC = \sqrt{(0 - \sqrt{2})^2 + (-\sqrt{5} - 0)^2}$
 $= \sqrt{(-\sqrt{2})^2 + (-\sqrt{5})^2}$
 $= \sqrt{2 + 5}$
 $= \sqrt{7}$ units
 $BC = \sqrt{(0 - -\sqrt{2})^2 + (-\sqrt{5} - 0)^2}$
 $= \sqrt{(\sqrt{2})^2 + (-\sqrt{5})^2}$
 $= \sqrt{2 + 5}$
 $= \sqrt{7}$ units
 Since $AC = BC$, triangle ABC is isosceles.

e $AB = \sqrt{(-\sqrt{3} - \sqrt{3})^2 + (1 - 1)^2}$
 $= \sqrt{(-2\sqrt{3})^2 + 0^2}$
 $= \sqrt{12}$ units
 $AC = \sqrt{(0 - \sqrt{3})^2 + (-2 - 1)^2}$
 $= \sqrt{(-\sqrt{3})^2 + (-3)^2}$
 $= \sqrt{3 + 9}$
 $= \sqrt{12}$ units
 $BC = \sqrt{(0 - -\sqrt{3})^2 + (-2 - 1)^2}$
 $= \sqrt{(\sqrt{3})^2 + (-3)^2}$
 $= \sqrt{3 + 9}$
 $= \sqrt{12}$ units
 Since $AB = AC = BC$, triangle ABC is equilateral.

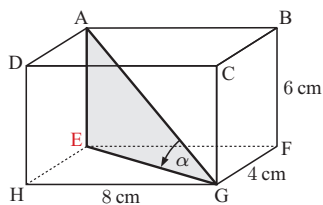
12



$\widehat{NPQ} = 360^\circ - 313^\circ = 47^\circ$ {angles at a point}
 $\widehat{TQP} = 180^\circ - 43^\circ - 47^\circ$ {cointerior angles}
 $= 90^\circ$
a $TP^2 = 10^2 + 32^2$ {Pythagoras}
 $\therefore TP = \sqrt{10^2 + 32^2}$ {as $TP > 0$ }
 $\therefore TP \approx 33.5$
 $\tan \theta = \frac{10}{32}$ { $\tan \theta = \frac{OPP}{ADJ}$ }
 $\therefore \theta = \tan^{-1}\left(\frac{10}{32}\right)$
 $\therefore \theta \approx 17.4^\circ$
 $\alpha = 180^\circ - 43^\circ = 137^\circ$ {cointerior angles}
 $\phi = 180^\circ - \alpha - \theta$ {cointerior angles}
 $\approx 180^\circ - 137^\circ - 17.4^\circ$
 $\approx 25.6^\circ$
 So, the trawler is about 33.5 km from P, on a bearing of $\approx 025.6^\circ$.

b The trawler must sail on a bearing of $360^\circ - 137^\circ - 17.4^\circ \approx 206^\circ$.

10 b



$$(EG)^2 = 4^2 + 8^2 \quad \{\text{Pythagoras}\}$$

$$\therefore EG = \sqrt{80}$$

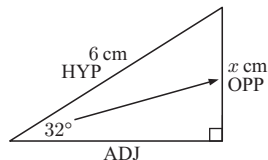
$$\tan \alpha = \frac{6}{\sqrt{80}} \quad \left\{ \tan \alpha = \frac{\text{OPP}}{\text{ADJ}} \right\}$$

$$\therefore \alpha = \tan^{-1}\left(\frac{6}{\sqrt{80}}\right)$$

$$\therefore \alpha \approx 33.9^\circ$$

$$\therefore \widehat{AGE} \approx 33.9^\circ$$

3 a



$$\sin 32^\circ = \frac{x}{6} \quad \left\{ \sin \theta = \frac{\text{OPP}}{\text{HYP}} \right\}$$

$$\therefore 6 \times \sin 32^\circ = x \quad \{\text{multiplying both sides by 6}\}$$

$$\therefore x \approx 3.18 \quad \{\text{calculator}\}$$

4 a

Distance d (m)	Frequency	Interval midpoint	Product
$20 \leq d < 30$	2	25	50
$30 \leq d < 40$	6	35	210
$40 \leq d < 50$	26	45	1170
$50 \leq d < 60$	12	55	660
$60 \leq d < 70$	3	65	195
$70 \leq d < 80$	1	75	75
Total	50		2360

\therefore mean

$$= \frac{\text{sum of data values}}{\text{the number of data values}}$$

$$\approx \frac{2360}{50}$$

$$\approx 47.2 \text{ m}$$

2 a

$$\frac{BC}{\sin 60^\circ} = \frac{12}{\sin 40^\circ} \quad \{\text{sine rule}\}$$

$$\therefore BC = \frac{12 \times \sin 60^\circ}{\sin 40^\circ}$$

$$\therefore BC \approx 16.168 \text{ m}$$

$$\therefore BC \approx 16.2 \text{ m}$$

$$DC = BC - BD$$

$$\approx 16.2 - 6$$

$$\therefore DC \approx 10.2 \text{ m}$$

b Area = $\frac{1}{2}ab \sin C$

$$\therefore 13.5 = \frac{1}{2} \times 6 \times BE \times \sin 40^\circ$$

$$\therefore BE = \frac{9}{2 \times \sin 40^\circ}$$

$$\therefore BE \approx 7.00 \text{ m}$$

10 c

$$0.442 > 0.438 > 0.120$$

\therefore Carina is now most likely to win at 44.2%, followed by Pia at 43.8%, and now Larry is least likely to win at only 12.0%.

4 a

The graph of $y = g(x)$ is obtained by translating $f(x) = -\frac{1}{2}x - 1$ 4 units upwards.

$$\therefore g(x) = f(x) + 4$$

$$= -\frac{1}{2}x - 1 + 4$$

$$= -\frac{1}{2}x + 3$$

b The graph of $y = g(x)$ is obtained by translating $f(x) = \frac{3}{2}x + 1$ 2 units to the right.

$$\therefore g(x) = \frac{3}{2}(x - 2) + 1$$

$$= \frac{3}{2}x - 3 + 1$$

$$= \frac{3}{2}x - 2$$

$$\begin{aligned}
 \mathbf{4} \quad \mathbf{b} \quad & \log 5000 = \log(5 \times 1000) \\
 & = \log 5 + \log 1000 \quad \{\log(ab) = \log a + \log b\} \\
 & = \log 5 + \log 10^3 \\
 & = \log 5 + 3 \log 10 \quad \{\log(a^n) = n \log a\} \\
 & = \log 5 + 3 \quad \{\log 10 = 1\}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{6} \quad \mathbf{a} \quad & 2 + \log 6 \\
 & = 2 \log 10 + \log 6 \quad \{\log 10 = 1\} \\
 & = \log 10^2 + \log 6 \quad \{\log(a^n) = n \log a\} \\
 & = \log 100 + \log 6 \\
 & = \log(100 \times 6) \quad \{\log(ab) = \log a + \log b\} \\
 & = \log 600 \\
 \mathbf{b} \quad & 1 - \log 2 \\
 & = \log 10 - \log 2 \quad \{\log 10 = 1\} \\
 & = \log\left(\frac{10}{2}\right) \quad \{\log\left(\frac{a}{b}\right) \\
 & \quad = \log a - \log b\} \\
 & = \log 5 \\
 \mathbf{c} \quad & \log 80 - 1 \\
 & = \log 80 - \log 10 \quad \{\log 10 = 1\} \\
 & = \log\left(\frac{80}{10}\right) \quad \{\log\left(\frac{a}{b}\right) \\
 & \quad = \log a - \log b\} \\
 & = \log 8 \\
 \mathbf{d} \quad & 1 + 2 \log 5 \\
 & = \log 10 + \log(5^2) \quad \{\log 10 = 1, \\
 & \quad \log(a^n) = n \log a\} \\
 & = \log 10 + \log 25 \\
 & = \log(10 \times 25) \quad \{\log(ab) = \log a + \log b\} \\
 & = \log 250 \\
 \mathbf{7} \quad \mathbf{a} \quad \mathbf{iii} \quad & \log 250 \approx 2.397\,940\,009 \\
 & \quad \approx 2.398 \\
 \mathbf{iv} \quad & \log\left(\frac{1}{250}\right) \approx -2.397\,940\,009 \\
 & \quad \approx -2.398
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{4} \quad \mathbf{c} \quad & \text{When } N = 200, \quad 200 = 10 \times 1.3^t \\
 & \quad \therefore 1.3^t = 20 \\
 & \therefore \log(1.3^t) = \log 20 \quad \{\text{taking the log of both sides}\} \\
 & \therefore t \log 1.3 = \log 20 \quad \{\log(a^n) = n \log a\} \\
 & \quad \therefore t = \frac{\log 20}{\log 1.3} \\
 & \quad \therefore t \approx 11.4
 \end{aligned}$$

It will take about 11.4 weeks for Danielle's blog to have 200 followers.

$$\begin{aligned}
 \mathbf{3} \quad \mathbf{a} \quad & f(x)g(x) \\
 & = (3x - 1)(x + 2) \\
 & = 3x(x + 2) - 1(x + 2) \\
 & = 3x^2 + 6x \\
 & \quad - x - 2 \\
 & = 3x^2 + 5x - 2 \\
 \mathbf{b} \quad & f(x)g(x) \\
 & = (2x^2 - x - 3)(x - 4) \\
 & = 2x^2(x - 4) - x(x - 4) - 3(x - 4) \\
 & = 2x^3 - 8x^2 \\
 & \quad - x^2 + 4x \\
 & \quad - 3x + 12 \\
 & = 2x^3 - 9x^2 + x + 12
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{4} \quad \mathbf{d} \quad & 4q(x) - p(x) = 4(2x^3 - 3x^2 + 6) - (x^4 - 3x^3 + 4x - 1) \\
 & = 8x^3 - 12x^2 + 24 - x^4 + 3x^3 - 4x + 1 \\
 & = -x^4 + 11x^3 - 12x^2 - 4x + 25
 \end{aligned}$$

3 b Let $P(x) = px^4 - 5x^3 - 5x^2 + qx + 9$
 Now $P(-1) = 0$ and $P(3) = 0$ {Factor theorem}
 So, $p(-1)^4 - 5(-1)^3 - 5(-1)^2 + q(-1) + 9 = 0$
 $\therefore p + 5 - 5 - q + 9 = 0$
 $\therefore p - q = -9$ (1)
 and $p(3)^4 - 5(3)^3 - 5(3)^2 + q(3) + 9 = 0$
 $\therefore 81p - 135 - 45 + 3q + 9 = 0$
 $\therefore 81p + 3q = 171$ (2)
 Solving simultaneously: $3p - 3q = -27$ $\{3 \times (1)\}$
 $81p + 3q = 171$ $\{(2)\}$
 Adding, $84p = 144$
 $\therefore p = \frac{12}{7}$
 Substituting $p = \frac{12}{7}$ in (1) gives $(\frac{12}{7}) - q = -9$
 $\therefore \frac{75}{7} = q$
 $\therefore q = \frac{75}{7}$

3

$$\begin{array}{r}
 x^3 - 2x^2 + x + 4 \\
 x - 4 \overline{) x^4 - 6x^3 + 9x^2 + 0x - 22} \\
 \underline{-(x^3 - 4x^3)} \\
 -2x^3 + 9x^2 \\
 \underline{-(-2x^3 + 8x^2)} \\
 x^2 + 0x \\
 \underline{-(x^2 - 4x)} \\
 4x - 22 \\
 \underline{-(4x - 16)} \\
 -6
 \end{array}$$

The quotient is $x^3 - 2x^2 + x + 4$, and the remainder is -6 .
 $\therefore \frac{x^4 - 6x^3 + 9x^2 - 22}{x - 4} = x^3 - 2x^2 + x + 4 - \frac{6}{x - 4}$

10 Let $P(x) = ax^3 + 5x^2 - x + b$
 Now $P(1) = 7$ and $P(-2) = -11$ {Remainder theorem}
 So, $a(1)^3 + 5(1)^2 - (1) + b = 7$
 $\therefore a + 5 - 1 + b = 7$
 $\therefore a + b = 3$ (1)
 and $a(-2)^3 + 5(-2)^2 - (-2) + b = -11$
 $\therefore -8a + 20 + 2 + b = -11$
 $\therefore -8a + b = -33$ (2)
 Solving simultaneously: $a + b = 3$ $\{(1)\}$
 $8a - b = 33$ $\{-1 \times (2)\}$
 Adding, $9a = 36$
 $\therefore a = 4$
 Substituting $a = 4$ in (1) gives $(4) + b = 3$
 $\therefore b = -1$

1 a $f(x) = x^3 + x^2 - 17x + 15$
 $\therefore f(-5) = (-5)^3 + (-5)^2 - 17(-5) + 15$
 $= -125 + 25 + 85 + 15 = 0$
 Since $f(-5) = 0$, $(x + 5)$ is a factor of $f(x)$. {Factor theorem}