HAESE MATHEMATICS
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TEACHER NOTES FOR ADVANCED MATHEMATICS 1 FOR AS AND A LEVEL

This book is designed both as a complete AS Mathematics course, and as the first year of the full A level Mathematics.

The content for A level Mathematics is now prescribed in the syllabus, so the book is suitable for any of the examination bodies. While each examination body may emphasise different aspects of the syllabus, there is sufficient material in the book to provide a solid foundation for study under any of the examination bodies.

As part of the A level Mathematics course, students are required to become familiar with one or more specified large data sets. Our book contains sample questions associated with each examination body's large data sets.

CHAPTER 1: LINES

A	Linear equations	
B	Equations of straight lines	C1
C	Points on lines	
D	Graphing lines	C1
E	Linear relationships	B7
F	Simultaneous linear equations	B4

This opening chapter includes some revision material on straight lines before moving into the specifics of the syllabus. Students revise the graphing of straight lines, as well as solving linear and simultaneous linear equations. Direct proportion is also addressed in this chapter.

CHAPTER 2: RECIPROCAL RELATIONSHIPS

A	Inverse proportion	B7
B	Reciprocal functions	B7
C	Equations involving reciprocals	

After studying direct proportion in Chapter 1, students now encounter inverse proportion in this chapter.

We have included reciprocal functions in this chapter, as they are closely linked to inverse proportion. It also provides an opportunity to introduce the concept of asymptotes.

Rational functions will be addressed in Chapter 4, since we can then introduce them as transformations of the basic reciprocal function $y = \frac{1}{x}$.

CHAPTER 3: QUADRATICS

A	Quadratic equations	B3
B	The discriminant of a quadratic	B3
C	The sum and product of the roots	
D	Quadratic functions	B3
E	Finding a quadratic from its graph	B3
F	Simultaneous equations	B4
G	Problem solving with quadratics	B3
H	Quadratic optimisation	B3

Quadratics are included before considering functions more formally. This is so that, when domain and range are introduced, students have some more interesting functions than straight lines to consider.

This chapter contains a worked example and some questions involving linear inequalities, which are not covered until Chapter 5. If students have not encountered linear equations before, and are struggling with these questions, they should jump ahead to the linear inequalities section of Chapter 5, and then return to these questions.

CHAPTER 4: RELATIONS AND FUNCTIONS

A	Relations and functions	
B	Function notation	
C	Domain and range	
D	Transformations	B9
E	Rational functions	B9

In this chapter, students are introduced to function notation.

In the transformations section, we do not use the same symbol a for each of the transformations as suggested in the syllabus. It appears to be more sensible to use a different letter for each transformation (for example, $af(x)$, $f(bx)$, $f(x-c)$, and $f(x)+d$), so that it is easier to express a combination of these transformations.

We have also used $f(x-c)$ instead of $f(x+c)$, so that a translation to the right is represented by $c>0$, and a translation to the left is represented by $c<0$. This is also consistent with the representation $(x-a)^2+(y-b)^2=r^2$ given for the equation of a circle.

Relations are not explicitly mentioned in the syllabus, however we feel it is important to mention them briefly, as they are instructive for understanding what *is* a function and what *is not* a function.

Rational functions are presented as transformations of reciprocal functions.

CHAPTER 5: INEQUALITIES

A	Linear inequalities	B5
B	Sign diagrams	
C	Quadratic inequalities	B5
D	Rational inequalities (Extension)	

We now deal with inequalities as a group, as students now have the tools required for linear, quadratic, and rational inequalities.

We also introduce sign diagrams which are used later in calculus.

CHAPTER 6: SURDS, INDICES, AND EXPONENTIALS

A	Surds	B2
B	Indices	
C	Index laws	B1
D	Rational indices	B1
E	Algebraic expansion and factorisation	
F	Exponential equations	F5
G	Exponential functions	F1
H	Growth and decay	F7
I	The natural exponential e^x	F1

Sections A to C should be revision for students, and these sections should be skipped through quickly if students are comfortable with the content.

The ability to write expressions involving surds in terms of rational indices is important in the lead-up to calculus, where students must differentiate functions like $f(x) = x\sqrt{x}$.

CHAPTER 7: LOGARITHMS

A	Logarithms in base 10	
B	Logarithms in base a	F3
C	Laws of logarithms	F4
D	Natural logarithms	F3
E	Solving exponential equations using logarithms	F5
F	Growth and decay	F7
G	Logarithmic functions	F3

In this chapter students study logarithms in base 10, and more generally in base a. They are then introduced to the natural logarithm in base e.

The syllabus requires students to "know and use $\ln x$ as the inverse function of e^x". However, inverse functions are not part of the AS course. We therefore outline the algebraic and geometric relationship between logarithmic and exponential functions, without explicit reference to inverse functions. The connection will be highlighted in our A2 book.

CHAPTER 8: REAL POLYNOMIALS

A	Polynomials	
B	Operations with polynomials	B6
C	Zeros, roots, and factors	
D	Polynomial equality	
E	Polynomial division	B6
F	The Remainder theorem	
G	The Factor theorem	B6
H	Sum and product of roots theorem	
I	Graphing cubic functions	B7
J	Graphing quartic functions	B7
K	Polynomial equations	B7

In this chapter we consider the properties, operations, theorems, and graphs associated with real polynomials.

Students are introduced to simple polynomial division by linear expressions only. The division of two polynomials should be familiar to students from their work with rational functions.

CHAPTER 9: THE BINOMIAL THEOREM

A	Factorial notation	D1
B	Counting	
C	Binomial expansions	D1
D	The binomial theorem	D1

The syllabus does not explicitly mention counting. However, it does require factorial notation and the *nCr* notation, and these concepts are hard to motivate without a discussion of counting.

We start by defining *nCr* as the number of ways of choosing *r* objects from *n* objects, and then work towards obtaining a formula for this.

CHAPTER 10: CIRCLES

A	The equation of a circle	C2
B	Tangents and chords	C2

The study of circles provides students with further practice at completing the square. It also reinforces the concept of translations as the circle $(x-a)^2+(y-b)^2=r^2$ is a translation of $x^2+y^2=r^2$.

Section B is a good opportunity to introduce tangents before they are used in calculus.

Studying this chapter after inequalities allows us to talk about ways of defining regions inside and outside of circles.

We have included questions requiring students to prove circle theorems using coordinate geometry, as outlined in the syllabus. These proofs require rather lengthy algebraic manipulation.

CHAPTER 11: TRIGONOMETRY

A	The unit circle	E1, E5
B	The multiples of 30° and 45°	
C	Finding trigonometric ratios	E5
D	Finding angles	
E	The area of a triangle	E1
F	The cosine rule	E1
G	The sine rule	E1
H	Problem solving	

We begin this chapter with a study of the unit circle. While this is not explicitly mentioned in the syllabus, it allows us to give meaning to the trigonometric ratios of obtuse and reflex angles.

The trigonometric ratios for multiples of 30° and 45° have been included, even though they are not in the AS syllabus, as we feel they form an important basis for plotting trigonometric functions and solving simple trigonometric equations.

The trigonometry covered in this chapter is all done in degrees. Students completing the full A level course will encounter radian measure in our A2 book.

CHAPTER 12: TRIGONOMETRIC FUNCTIONS

A	Periodic behaviour	
B	The sine and cosine functions	E3
C	Problem solving with trigonometric functions	E3
D	The tangent function	E3
E	Trigonometric equations	E7

In this chapter we study the graphs, symmetries, and periodicity of trigonometric functions.

We develop the various components of the general sine and cosine functions, using the transformations studied in Chapter 4.

Again, these trigonometric functions are studied in the context of degrees. Students completing the full A level course will study trigonometric functions in radians in our A2 book.

CHAPTER 13: REASONING AND PROOF

A	Logical connectives	A1
B	Proof by deduction	A1
C	Proof by equivalence	A1
D	Definitions	A1
E	Proof by exhaustion	A1
F	Disproof by counter example	A1

We begin the chapter with some general advice for writing proofs. It is hoped that students will find this useful to refer back to as they work through the chapter, and indeed the rest of the course

The logical connectives such as negation and implication are used intuitively, rather than formally with truth tables.

We have introduced some terms to describe processes in algebraic manipulation, such as isolation, collection, and attraction. These terms should help students describe what they are doing in more detail than merely "simplify".

Students should find this chapter interesting and challenging, and should help them think more deeply about exactly what a proof entails.

CHAPTER 14: INTRODUCTION TO DIFFERENTIAL CALCULUS

A	Rates of change	G1
B	Instantaneous rates of change	G1
C	Finding the gradient of the tangent	G1
D	The derivative function	G1

This chapter provides students with their first look at differential calculus. It begins with rates of change, and includes the $\delta y/\delta x$ notation. We use this to motivate an informal study of limits. We only deal with limits where x is finite.

CHAPTER 15: DERIVATIVES AND THEIR APPLICATIONS

A	Simple rules of differentiation	G2
B	Second derivatives	G1
C	Tangents and normals	G3
D	Increasing and decreasing functions	G3
E	Stationary points	G3
F	Shape	G1
G	Optimisation	G3

In this chapter, students are given rules for differentiating simple sums of powers of x.

We then explore the applications of these derivatives in the context of graphs. The first derivative is used to determine where a function is increasing or decreasing, and to solve optimisation problems.

The second derivative is used to analyse the *shape* of the graph.

CHAPTER 16: INTEGRATION

A	The area under a curve	
B	Antidifferentiation	
C	The Fundamental Theorem of Calculus	H1
D	Integration	H2
E	Definite integrals	H3

We begin our study of integration by calculating the area under the curve, using the idea of limits. We feel this approach is consistent with how integral calculus was developed historically. We then move on to consider how the area under a curve relates to antiderivatives of functions.

CHAPTER 17: KINEMATICS

A	Motion in a straight line with constant acceleration	P1, Q1, Q3
B	Kinematic functions	Q1, Q2, Q4

We begin the chapter with equations of motion in a straight line under constant acceleration. We present a series of equations relating the displacement (s), initial velocity (u), final velocity (v), acceleration (a), and time (t), known collectively as the *suvat* equations. These equations are developed without the use of calculus.

We then explore the displacement, velocity, and acceleration functions more generally, and use calculus to explore the relationships between them.

CHAPTER 18: VECTORS

A	Vectors and scalars	J1
B	Geometric operations with vectors	J3
C	Vectors in the plane	J1, J2
D	The magnitude of a vector	J2
E	Operations with plane vectors	J3
F	The vector between two points	J4
G	Parallelism	
H	Problems involving vector operations	J5
I	Constant velocity problems	J5
J	Proof using vector geometry	

Students may be familiar with the work in some of the earlier sections of this chapter. If this is the case, these sections should be worked through quickly.

All of the vectors in this chapter are two-dimensional. Students completing the full A level course will encounter three-dimensional vectors in our A2 book.

The final section of the chapter covers vector proofs, and allows students to practise some of the proof techniques from Chapter 13.

CHAPTER 19: MECHANICS: FORCES AND NEWTON'S LAWS

A	Newton's First Law of Motion	R1
B	Newton's Second Law of Motion	R2
C	Weight	R3
D	Newton's Third Law of Motion	R4
E	Driving and resistance forces	R4
F	Tension	R4

The material in this chapter may be unfamiliar to many students, so it is important that classes allocate sufficient time for this chapter.

The different examination bodies require different levels of accuracy for the acceleration due to gravity g. We have used $g = 9.8\ \text{m s}^{-2}$.

This chapter contains several discussions and historical notes, allowing students to explore the relevance of this content to their daily lives, and to discuss the reasonableness of the assumptions used in mechanics modelling.

CHAPTER 20: SINGLE-VARIABLE STATISTICS

A	Sampling	K1
B	Numerical data	L1, L4
C	Displaying numerical data	L1
D	Measuring the centre of data	L3
E	Variance and standard deviation	L3

We begin our study of statistics in this book with single-variable statistics.

Section A will be unfamiliar to most students as it introduces many new ideas such as sampling methods and sources of error.

Sections B to D should mostly be revision, covering methods for organising and displaying (numerical) data seen in GCSE. We decided not to include categorical data, quartiles, box plots, and cumulative frequency plots as the students should have done these in GCSE, and the chapter is already large.

We deal with discrete and continuous data separately in Section C as we felt this was the most sensible way to structure the sections. We have omitted "frequency histograms" to avoid confusion with the syllabus' more rigid definition of a histogram which requires that area = frequency.

In Section E we introduce variance and standard deviation. We have included both population and sample formulae where the sample formulae use a denominator of $(n-1)$. You should be aware that some examination boards may use a denominator of n instead.

The activity at the end of the chapter helps provide motivation for using the sample estimators over the population ones. Classes who are struggling with this can use this as a directed investigation or as an introduction to the concept of "sample estimators".

CHAPTER 21: BIVARIATE STATISTICS

A	Association between numerical variables	L2, L4
B	Measuring correlation	L2
C	Linear models	L2
D	Exponential and power models	F4
E	Problem solving	F4

Students should already be familiar with the concepts in Section A such as scatter diagrams and informal assessments of linear correlation from GCSE. Section B continues in a similar vein, introducing the product moment correlation coefficient formally and the use of technology to calculate this value. We have also included a subsection on the "coefficient of determination". Although not strictly a part of the course, we felt it was appropriate to include considering the emphasis on fitting models later in the chapter.

Section C introduces linear models and linear regression (via technology). Some students may only fit a "line of best fit" to data by eye. However, while the syllabus states that "calculations involving regression lines are excluded", we have interpreted this as "regression coefficients should not be calculated by hand". As such, we have included examples and exercises where calculation of the equation of the regression line using technology is needed. This allows us to use the equation to *predict* values. Manipulation of the regression line should be straightforward given earlier coverage of equations of lines in Chapter 1.

One aspect that students may struggle with is using exact values in their calculations. They should be aware that their calculators store regression coefficients and should be confident in accessing these values. We have provided various graphical calculator instructions on this topic to assist students.

In Sections D and E we use linear regression to fit exponential and power models. We have included this here instead of the exponentials and logarithms chapters because they fit better thematically with linear regression. There are some basic exercises to help students get re-accustomed with using exponentials and logarithms before delving into questions with data.

There are also several activities placed throughout the chapter which can serve as extension material. The activity on Anscombe's quartet is particularly important as it illustrates the importance of visualising data in exploratory analysis, an overarching theme in the Large Data Set component of the course.

CHAPTER 22: PROBABILITY

A	Outcomes and sample spaces	
B	Theoretical probability	
C	Compound events	M1
D	Tree diagrams	
E	Sets and Venn diagrams	
F	The addition law of probability	M1

This chapter on probability will mostly serve as revision of probability from GCSE and preparation for Chapter 23: Statistical distributions.

The concepts of counting and probability generally go hand in hand (especially with the enumeration of sample spaces), so the counting material in Chapter 9 is put to good use. This link between counting and probability will be more formally established in Chapter 23.

Although tree diagrams, sets, and Venn diagrams are only included in the full A level course, we have covered them here because:

- they are generally useful for aiding probability calculations, even outside of the study of conditional probability
- motivating the probability formula for mutually exclusive events is easiest done with the aid of set notation and Venn diagrams.

For students doing the full A level course, Sections D and E will prepare the students for their use in the context of conditional probability. For students only doing the AS course, these sections may be skipped if time is short.

CHAPTER 23: STATISTICAL DISTRIBUTIONS

A	Random variables	
B	Discrete probability distributions	N1
C	The binomial distribution	N1
D	Statistical hypothesis testing	O1, O2

In this chapter we introduce the concept of random variables and probability distributions. Up until this point, students have only seen statistics in the context of data which has already been observed, rather than a random variable whose value has not yet been determined. We believe that this is best approached in the context of probability, and as such leads on directly from the work on probability in Chapter 22.

Section C introduces the binomial distribution as a special case of some of the discrete random variables encountered in the previous section. The introduction of this section links directly to the use of the binomial coefficient. The use of technology in the latter half of the section prepares students for some of the more complex scenarios in Section D.

Section D provides a basic introduction to hypothesis testing. The syllabus is rather open-ended when it comes to this topic, as are the specifications provided by the various awarding organisations. We have included a full testing procedure in the context of a binomial distribution including the calculation of p-values, and critical regions and values. Because of the complexity that discrete distributions add, various results are stated without full explanation, such as the significance level in a hypothesis test being the probability of incorrectly rejecting the null. (This is explored in detail in the investigation on randomised tests at the end of the chapter.) Consequently, we place a greater emphasis on identifying terminology in context and discussion, rather than using the testing procedure itself.

It is unusual for hypothesis testing to be formally introduced without the normal distribution as:

- it is generally more intuitive
- certain aspects of hypothesis testing can be more easily explored with a continuous distribution.

Students taking the full A level course, will therefore study hypothesis testing much more completely in our A2 book.