HAESE MATHEMATICS
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TEACHER NOTES FOR YEAR 11 SPECIALIST MATHEMATICS

In the Mathematical Methods book: **NUMBER SEQUENCES**

		SACE
A	Number sequences	Topic 7 Sub-topic 7.1, 7.2
B	Arithmetic sequences	
C	Geometric sequences	
D	Series	
E	Arithmetic series	
F	Geometric series	

Number sequences are in the SACE "Specialist" syllabus and in the ACARA Methods syllabus. The chapter that includes this material is in the Methods textbook, since SA Specialist students will also have this.

The chapter provides a fairly standard treatment of sequences and series, including both the explicit and recursive rules for generating arithmetic and geometric sequences. We explore applications of sequences, such as growth and decay, and compound interest.

CHAPTER 1: COUNTING

		SACE	ACARA
A	The product principle		Unit 1 Topic 1
B	Counting paths		
C	Factorial notation		
D	Permutations		
E	Combinations		
F	The Inclusion-Exclusion principle		
G	The Pigeonhole principle		

Counting is in the ACARA Specialist Mathematics syllabus, but the counting context in the SACE syllabus (Sub-topic 4.1) is covered in the Methods textbook. South Australian students may therefore skip this chapter. However, SACE students may like to do sections F and G as extension work.

In the ACARA syllabus, combinations are listed last in Topic 1. We have instead placed them directly after permutations, since they are a logical progression.

Permutations introduces the idea of dividing by a factorial. For example, suppose we have 10 letters, and take 4 of them to form a word. The order of the 4 selected letters is important, so the number of ways to do this is $10\times9\times8\times7=\frac{10!}{6!}$. In other words, there are 10! ways to order all of the letters, but we divide by 6! because the order of the 6 letters not in the word is not important.

This leads to permutations where certain objects are indistinguishable. Here we have the same discussion: since the objects are indistinguishable, the order in which they are selected is not important.

This leads naturally to our work on combinations: if a team of 4 people is selected from a group of 10, the order of the 4 that are selected is not important, and the order of the 6 that are not selected is not important, so the total number of ways to do this is $\frac{10!}{4!6!}$.

The inclusion-exclusion principle is then presented. Here we assume knowledge of sets and Venn diagrams from previous years, including Venn diagrams with 3 sets. This section is quite short, as there are not many sensible questions which can be written.

The chapter finishes with a basic treatment of the pigeonhole principle. This only considers the case where there must be at least one box with more than *one* pigeon.

CHAPTER 2: CIRCLE GEOMETRY

		SACE	ACARA
A	The angle in a semi-circle theorem	Topic 8 Sub-topic 8.1, 8.2	Unit 1 Topic 3
B	The chord of a circle theorem		
C	The radius-tangent theorem		
D	The angle at the centre theorem		
E	Angles subtended by the same arc		
F	Tangents from an external point		
G	Angle between tangent and chord		
H	Cyclic quadrilaterals		
I	Intersecting chords and secants theorems		

In this chapter we present a variety of theorems associated with circle geometry.

This chapter contains a lot of formal proofs, both as proofs of theorems, and also asking students to provide proofs for other theorems and geometric properties. There is a useful appendix on proof at the end of the book, which gives a more general treatment of the nature of proof, and outlines what a formal proof entails.

The 10A chapter "Geometry of circles" is a very good preparation for this chapter. It introduces the theorems and their application, allowing students in Year 11 to concentrate more on the rigour and proof.

The more of this work the students can complete at the 10A level, the faster they will be able to advance. Knowing that this material is not directly examinable in Year 12, students should not get bogged down in this chapter.

Cyclic quadrilaterals and the intersecting chords and secant theorems were not covered in 10A. We have included the chord of a circle theorem, even though it was not listed in the syllabus, because we needed it in a proof.

It is important that these theorems are presented in the correct order, otherwise you may find that, in trying to prove one theorem, you require another theorem which you have not yet proved. In proof we *must* avoid circular references.

A lot of what is in this chapter will push the students in terms of rigorous proof from what they have done before. It is unfortunate that we were not forewarned that words such as contrapositive, implication, converse, equivalence, and negation would be in this course, otherwise we would have included a chapter on Mathematical Logic in previous years.

CHAPTER 3: VECTORS

		SACE	ACARA
A	Vectors and scalars	Topic 9 Sub-topic 9.1	
B	Geometric operations with vectors		
C	Vectors in the plane	Topic 9 Sub-topic 9.2	Unit 1 Topic 2
D	The magnitude of a vector		
E	Operations with plane vectors		
F	The vector between two points		
G	Parallelism		
H	Problems involving vector operations		
I	The scalar product of two vectors	Topic 9 Sub-topic 9.3	
J	The angle between two vectors		
K	Vector projection		
L	Proof using vector geometry	Topic 3, Sub-topic 3.4	

This chapter gives students their first look at vectors. In accordance with the syllabus, this chapter deals with vectors in 2 dimensions, leaving 3-dimensional vectors for Year 12.

In the SACE syllabus, there are several statements in the right hand column that we feel are inappropriate or misleading:

1) "Calculating the projection algebraically leads to the definition of the dot product and the formula for the cosine of the angle between the two vectors." This is like saying that the vector cross product should lead to the definition of a 3×3 matrix determinant. In practice we define 3×3 determinants first, then find that they have an application in the vector cross product. In the same way, we should define the scalar product first, then explore its application in projection. Otherwise we need to introduce the concept of projection, backtrack to a definition of dot product, and then return to the original context. This way, when the angle between two vectors is investigated, we find the scalar product embedded within it, and the implications of a positive, negative, or zero scalar product are clear.

2) The syllabus also cites calculating the assistance a plane receives from wind in a particular direction as an example of an application of projections. However, care must be taken here, because we cannot simply take the projection of the wind vector onto the direction of travel to analyse the change in course. For example, if I want to fly due north, and there is a wind from the south-west, the projection of the wind onto my flight does *NOT* give the assistance of the wind. This is because the wind is actually blowing the plane off course. Therefore, we need to turn the plane to face slightly north-west, and let the wind blow it back on course. We have included a discussion to clarify this issue.

As an application of scalar multiplication, we have included an investigation on linear combinations.

The chapter concludes with a section on proof using vector geometry. Teachers should be aware that students may be encountering these types of proofs for the first time.

CHAPTER 4: TRIGONOMETRIC FUNCTIONS

		SACE	ACARA
A	The general tangent function		
B	General trigonometric functions		
C	Reciprocal trigonometric functions		
D	Solving trigonometric equations	Topic 10	
E	Modelling using trigonometric functions	Sub-topic 10.1, 10.2	Unit 2 Topic 1
F	Trigonometric relationships		
G	Double angle identities		
H	Angle sum and difference identities		
I	Trigonometric equations in quadratic form (Extension)		

This chapter follows directly from the chapter "Trigonometric functions" in the Methods textbook. It should not be started until students have completed the Methods chapter. If necessary, Specialist students may move ahead to Chapter 5 "Matrices" while the Methods trigonometry content is completed, and then return to this chapter after.

In this chapter we revise and extend the work done on general trigonometric functions and their graphs. We introduce reciprocal trigonometric functions, and solve more complicated trigonometric equations. We are then in a position to form detailed trigonometric models of real-world situations. Finally, we move on to study and apply important trigonometric identities.

CHAPTER 5: MATRICES

		SACE	ACARA
A	Matrix structure		
B	Matrix operations and definitions	Topic 11	
C	Matrix multiplication	Sub-topic 11.1	
D	The inverse of a 2×2 matrix		
E	Simultaneous linear equations		
F	Translations and lines in 2-D		Unit 2
G	Linear transformations		Topic 2
H	Rotations about the origin	Topic 11	
I	Reflections	Sub-topic 11.2	
J	Dilations		
K	Compositions of transformations		
L	The inverse of a linear transformation		

The chapter starts with a fairly traditional treatment of matrices. We look at matrix structure and operations, and we use inverse matrices to solve simultaneous linear equations. We note that only 2×2 inverses are covered, which means that the intersection of planes in Year 12 will need to be done by row operations (as per IB HL Mathematics).

The remainder of the chapter is more advanced, looking at transformations in the plane. We know that this material will be unfamiliar to many teachers as well as students, and we are looking to run some PD workshops to help you.

It is unclear exactly what is required for “translations and their representation as column vectors”. We took the opportunity to present the vector work on the parametric equation of a line and its real-world applications. The solution to these problems corresponds to the translation of a point through a scalar multiple of a vector matrix.

We finish with the composition of transformations and inverse of transformations, which provides interesting contrast with what we did with functions.

CHAPTER 6: MATHEMATICAL INDUCTION

		SACE	ACARA
A	The process of induction		
B	The principle of mathematical induction		
C	Proof of divisibility	Topic 12	Unit 2
D	Proofs for sequences and series	Sub-topic 12.2	Topic 3
E	Proofs for products		
F	Induction with matrices		
G	Proof of inequalities		

The Principle of Mathematical Induction is a sub-topic of ‘Real and complex numbers’, but it can be applied to many other areas, such as sequences and series, matrices, and trigonometry.

We therefore took the opportunity to present a more complete picture of what students can do with induction. It ties together material from earlier in the course, providing the opportunity for revision. Teachers should feel free to skip through the induction applied to these other areas if they feel it is not appropriate for their students.

This chapter will also appear in the Year 12 Specialist textbook, to address Topic 1 in the SACE Stage 2 Specialist Mathematics syllabus. Classes should cover as much of the chapter as they feel is appropriate in Year 11, and finish the rest in Year 12.

CHAPTER 7: REAL AND COMPLEX NUMBERS

		SACE	ACARA
A	Interval notation		
B	Rational numbers	Topic 12	
C	Division by surds	Sub-topic 12.1	
D	Irrational numbers		Unit 2
E	The ‘invention’ of imaginary numbers		Topic 3
F	Complex numbers	Topic 12	
G	Complex conjugates	Sub-topic 12.3,	
H	Complex numbers as 2-D vectors	12.4, 12.5	
I	Modulus		

Sections A to D focus on the real number line, looking at interval notation, and rational and irrational numbers. We have skipped the basic revision of surds, as this is covered in the Methods textbook. However, division by surds is included, as this will be important later in the chapter.

The remainder of the chapter provides an introduction to complex numbers which will be taken up and extended in Year 12.

We find it inconsistent that the right hand column of the SACE syllabus presents division of complex numbers as a rearrangement of the product of complex numbers, for example $(2 - i)(1 + i) = (3 + i)$, when we are already asked to draw parallels with the arithmetic surds. In fact, the division of surds and the use of radical conjugates is included solely for the purpose of comparing with complex conjugates.