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### YEAR 12 MATHEMATICS METHODS

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The main purpose of this chapter is for students to familiarise themselves with exponential, logarithmic, and trigonometric functions, before studying the calculus of these functions in later chapters.

Section B provides a brief recap of logarithms, on the basis that some students may have already encountered logarithms in Year 11. However, students following the WACE syllabus may not have studied logarithms in Year 11. For this reason, an introduction to logarithms, based on the work in the Year 11 book, is provided as an online link.

Dealing with logarithms at this early stage allows us to include the derivatives and integrals of logarithmic functions in the calculus chapters, rather than having to address logarithmic functions separately later.

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In this chapter we recap first principles and simple rules of differentiation from Year 11. Students are then introduced to the chain, product, and quotient rules, before differentiating the exponential, logarithmic, and trigonometric functions studied in Chapter 1.

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In this chapter we look at the applications of differential calculus. The chapter gives students the opportunity to practise differentiating a wide variety of functions.

Many of the concepts explained in this chapter will be familiar to students, as they were studied in Year 11. The focus this year is in applying the concepts to more complicated functions.

Students also have the chance to explore the surge function and the logistic function in real-world contexts.

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We begin our study of integration by calculating the area under a curve, using the ideas of limits. We feel this approach is consistent with how integral calculus was developed historically. We then move on to consider how the area under a curve relates to antiderivatives of functions.

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We now explore some applications of integration, including the area under and between curves, kinematics, and problem solving. We now formally establish that for functions  $f(x) \leq 0$ , we must negate the integral to find the area between  $f(x)$  and the  $x$ -axis.

It is at this point that the Methods and Specialist classes will need to be well coordinated. Chapter 7 of the Specialist textbook follows directly on from this chapter, so this chapter must be completed before the Specialist classes reach Chapter 7.

## CHAPTER 6: STATISTICS

- A Key statistical concepts
- B Measuring the centre of data
- C Variance and standard deviation

This chapter gives students the opportunity to revise some important statistical concepts.

Students following the WACE syllabus did not do any statistics in Year 11, and it is possible that they have never encountered variance and standard deviation. It is important that these students understand the basic ideas behind variance and standard deviation, before they encounter more advanced treatments of these measures of spread in later chapters.

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We now explore the properties of discrete random variables. In Section D, we study the variance and standard deviation of discrete random variables, which is why it is important that students gain some familiarity with these concepts in Chapter 6.

Students should recognise that the Bernoulli random variable is a special case of the binomial random variable, where the trial is performed only once. The relationship between these variables will be further developed in Chapter 9.

## CHAPTER 8: CONTINUOUS RANDOM VARIABLES

- A Continuous random variables 4.2.1
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- D Probabilities using a calculator 4.2.7
- E The standard normal distribution 4.2.6  
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When defining the mean and standard deviation of a continuous random variable, the syllabus describes an integral over the infinite domain from  $-\infty$  to  $\infty$ . However, in the overwhelming majority of cases the students will encounter, continuous random variables are defined over a finite domain  $[a, b]$ , and we have defined the mean and standard deviation accordingly. It seems inappropriate to provide these definitions in a form that the students do not have the tools to evaluate.

## CHAPTER 9: SAMPLING AND CONFIDENCE INTERVALS

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D	Confidence intervals for means		
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F	Confidence intervals for proportions		4.3.7, 4.3.8, 4.3.9, 4.3.10

When presenting the Central Limit Theorem, we state that  $n = 30$  is used as a “rule of thumb” to decide whether a sample size is large enough for the distribution of sample means to be approximately normal. However, students should understand that this is a guide only, and there is no “magical threshold” at which distributions change from being non-normal to normal; it is a gradual process through which the approximation to normality improves as the sample size increases.

In the SACE syllabus, the work on statistical inference for both sample means and sample proportions is in the Mathematical Methods course, which is why this material all appears in the Methods textbook.

For students following the WACE syllabus, the work on statistical inference for sample means is actually part of the Mathematics Specialist course. This should not be a problem, as students studying Mathematics Specialist will also be studying Mathematics Methods, and so they should have both textbooks. If possible, it would be best if these students complete the material on sample means and sample proportions together rather than separately, as the ideas involved are very similar, and the results from both components each stem from the Central Limit Theorem in Section C.

Students studying only Mathematics Methods need only complete Sections E and F. That being said, their understanding of the work would be enhanced by studying some relevant aspects of the earlier sections, especially the Central Limit Theorem in Section C.