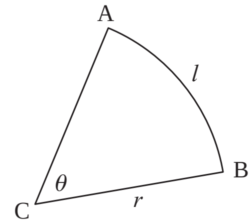
4 **a** Prove that for θ in radians:

i arc length AB, $l = r\theta$

ii sector area, $A = \frac{1}{2}r^2\theta$.

b Find the arc length and area of a sector of radius 5 cm and angle 2 radians.

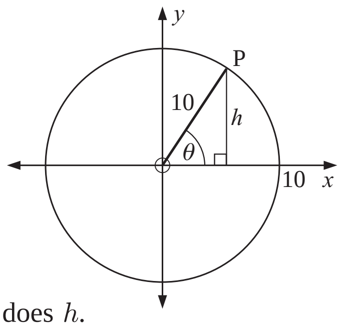
c If a sector has radius 10 cm and arc length 13 cm, find its area.



B PERIODIC FUNCTIONS FROM CIRCLES

Consider a Ferris wheel of radius 10 m revolving at constant speed. For convenience we place a set of axes through its centre of rotation.

The height of P, the point representing the person on the wheel relative to the principal axis at any given time, can be determined by using right angle triangle trigonometry.



As $\sin\theta = \dfrac{h}{10}$, then $h = 10\sin\theta$.

From this it is obvious that as time goes by θ changes and so does h.

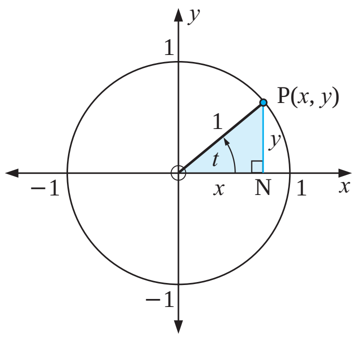
So, h is a function of θ, but more importantly h is a function of time t.

DEMO

SINE AND COSINE FROM THE UNIT CIRCLE

The **unit circle** is the circle, centre (0, 0) with radius 1 unit.

If P(x, y) moves around the unit circle such that OP makes an angle of t with the positive x-axis then:



> the x-coordinate of P is $\cos t$ and
> the y-coordinate of P is $\sin t$.

In the first quadrant, i.e., $0 < t < \frac{\pi}{2}$, we see the connection with right angled triangle trigonometry as:

$$\cos t = \frac{\text{ON}}{\text{OP}} = \frac{x}{1} = x$$

$$\text{and} \quad \sin t = \frac{\text{PN}}{\text{OP}} = \frac{y}{1} = y.$$

Notice also that in ΔONP, $x^2 + y^2 = 1$ {Pythagoras}

and so $[\cos t]^2 + [\sin t]^2 = 1$

or $\boxed{\cos^2 t + \sin^2 t = 1}$ if we use $\cos^2 t$ for $[\cos t]^2$, etc.

Notice also that as $-1 \leqslant x \leqslant 1$ and $-1 \leqslant y \leqslant 1$ for all points on the unit circle, then:

$$\boxed{-1 \leqslant \cos t \leqslant 1 \quad \text{and} \quad -1 \leqslant \sin t \leqslant 1 \quad \text{for all } t.}$$

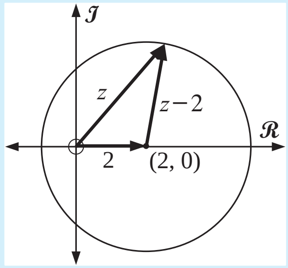
Example 30

a Illustrate $|z-2| = 3$ using geometric methods.

b Check **a** using algebraic argument.

a $|z-2| = 3$ means that the distance from z to 2 is always 3 units.

This means that z lies on a circle centre (2, 0), radius 3.
[2 is represented by the vector shown.]



b If $z = x + yi$ then $|z-2| = 3$ becomes

$$|x + yi - 2| = 3$$
$$|(x-2) + yi| = 3$$
$$\therefore \sqrt{(x-2)^2 + y^2} = 3$$
$$\therefore (x-2)^2 + y^2 = 9$$

which is a circle, centre (2, 0), radius $\sqrt{9} = 3$ units.

2 Illustrate the following using geometric methods and check your answer using an algebraic argument:

a $|z+i| = 2$ **b** $|z+1| \leqslant 2$ **c** $|z-1+i| > 2$

3 Illustrate:

a $\text{Re}(z) \geqslant 0$ **b** $\text{Re}(z) < 2$ **c** $-1 < \text{Re}(z) \leqslant 1$

d $\text{Im}(z) < 0$ **e** $\text{Im}(z) \geqslant -1$ **f** $0 \leqslant \text{Im}(z) < 1$

Example 31

a Illustrate $|z-2| = |z+2i|$. What Cartesian equation does its graph have?

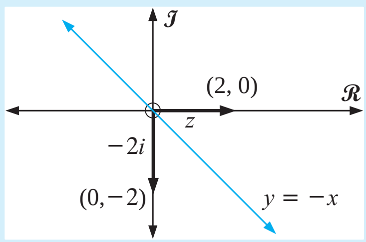
b Check **a** using algebraic argument.

a Since $|z-2| = |z-(-2i)|$, z's distance from 2 is equal to its distance from $-2i$.

This means that the end of z is equidistant from (2, 0) and (0, -2).

So, it lies on the perpendicular bisector of (2, 0) and (0, -2).

This is the line $y = -x$.



b If $z = x + iy$ then $|z-2| = |z+2i|$ becomes

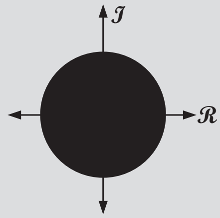
$$|x + iy - 2| = |x + iy + 2i|$$
$$\therefore |(x-2) + iy| = |x + (y+2)i|$$
$$\text{i.e., } \sqrt{(x-2)^2 + y^2} = \sqrt{x^2 + (y+2)^2}$$

The Julia set is a result of iterating $z \to z^2 + c$ for a fixed complex number c and varying z.

INVESTIGATION 4 $z \longrightarrow z^2 + c$ ITERATIONS WITH c FIXED AND REAL

$z \to z^2$ is a special case of $z \to z^2 + c$ with $c = 0$.

$z \to z^2$ generated the filled-in Julia set.



What filled-in Julia set results from $z \to z^2 - \frac{1}{2}$, say?

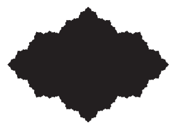
What to do:

1 Click on the icon to experiment with filled-in Julia sets resulting from:

a $c = -\frac{1}{2}$ **b** $c = i$ **c** $c = \frac{1}{2} + \frac{1}{2}i$ **d** $c = \frac{1}{2} - \frac{1}{2}i$

2 Choose points within, outside, and on the boundary and obtain their orbit plots.

You should have discovered that the filled-in Julia set for $z \to z^2 - \frac{1}{2}$ is:



$z \longrightarrow z^2 + c$ ITERATIONS WITH c FIXED AND REAL

In the investigation we observed the Julia set for the quadratic iteration $z \to z^2 - \frac{1}{2}$.

The question arises: "Does it have invariant points and cycles."

Consider the following example:

Example 34

Under $z \to z^2 - \frac{1}{2}$: **a** What are the invariant points? **b** Identify a 2-cycle.

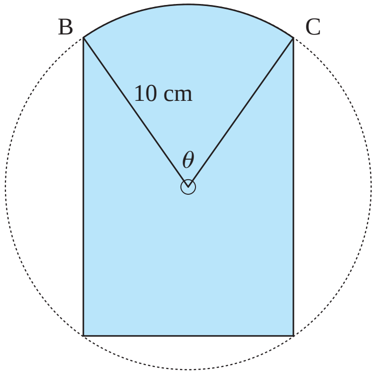
a For invariant points
$$z = z^2 - \tfrac{1}{2}$$
$$\therefore \quad z^2 - z - \tfrac{1}{2} = 0$$
$$\text{i.e.,} \quad 2z^2 - 2z - 1 = 0$$
$$\text{i.e.,} \quad z = \frac{2 \pm \sqrt{4 - 4(2)(-1)}}{4}$$
$$\text{i.e.,} \quad z = \frac{1 \pm \sqrt{3}}{2}$$

b $z \to z^2 - \frac{1}{2} \to (z^2 - \frac{1}{2})^2 - \frac{1}{2} = z^4 - z^2 - \frac{1}{4}$

So, solving $z^4 - z^2 - \frac{1}{4} = z$

i.e., $4z^4 - 4z^2 - 4z - 1 = 0$

EXERCISE 7B

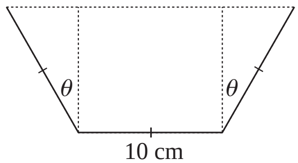


1 A circular piece of tinplate of radius 10 cm has 3 segments removed (as illustrated).

If θ is the measure of angle COB, show that the remaining area is given by

$$A = 50(\theta + 3\sin\theta).$$

Hence, find θ to the nearest $\frac{1}{10}$ of a degree when the area A is a maximum.



end view

2 A symmetrical gutter is made from a sheet of metal 30 cm wide by bending it twice (as shown).

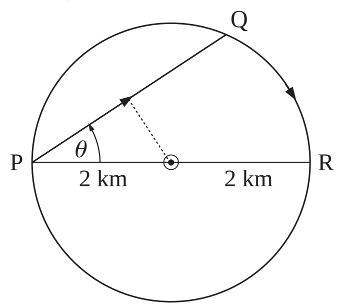
For θ as indicated:

a deduce that the cross-sectional area is given by

$A = 100\cos\theta(1 + \sin\theta)$.

b Hence, show that $\dfrac{dA}{d\theta} = 0$ when $\sin\theta = \frac{1}{2}$ or -1.

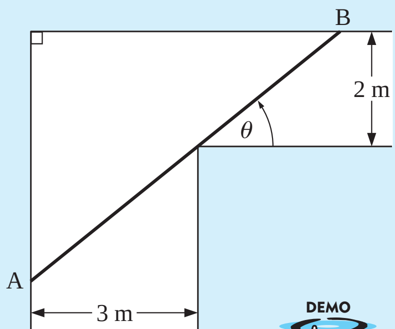
c What value is θ if the gutter has maximum carrying capacity?



3 Jack can row a boat across a circular lake of radius 2 km at 2 kmph. He can walk around the edge of the lake at 5 kmph.

What is Jack s shortest possible time to get from P to R by rowing from P to Q and walking from Q to R?

Example 7



Two corridors meet at right angles and are 2 m and 3 m wide respectively. θ is the angle marked on the given figure and AB is a thin metal tube which must be kept horizontal as it moves around the corner from one corridor to the other without bending it.

a Show that the length AB, is given by

$$L = \frac{3}{\cos\theta} + \frac{2}{\sin\theta}.$$

DEMO

b Show that $\dfrac{dL}{d\theta} = 0$ when $\theta = \tan^{-1}\left(\sqrt[3]{\frac{2}{3}}\right) \doteqdot 41.14^o$.

c Determine L when $\theta = \tan^{-1}\left(\sqrt[3]{\frac{2}{3}}\right)$ and comment on the significance of this value.

From **Investigation 5** you should have discovered that:

if $x' = -\alpha y,\ \ y' = \beta x,$ and $\alpha, \beta > 0,$

then $x(t) = A\cos(\omega t + \phi),\ \ y(t) = B\sin(\omega t + \phi)$

where $\omega = \sqrt{\alpha\beta}$ and $\dfrac{A}{B} = \sqrt{\dfrac{\alpha}{\beta}},\ \ x(0) = A\cos\phi,\ \ y(0) = B\sin\phi.$

Example 1

Solve $x' = -3y,\ \ y' = 2x$ with boundary conditions $x(0) = 0,\ \ y(0) = 2.$

$\alpha = 3$ and $\beta = 2 \quad \therefore \quad \omega = \sqrt{\alpha\beta} = \sqrt{6}$

$$\text{and} \quad \frac{A}{B} = \sqrt{\frac{\alpha}{\beta}} = \tfrac{\sqrt{3}}{\sqrt{2}}$$

But $x(0) = A\cos\phi = 0$ and $y(0) = B\sin\phi = 2$

$$\therefore \quad \frac{B\sin\phi}{A\cos\phi} = \frac{2}{0}$$

$$\tan\phi \quad \text{is undefined}$$

$$\therefore \quad \phi = \tfrac{\pi}{2}$$

$$\therefore \quad B\sin\tfrac{\pi}{2} = 2 \qquad \text{i.e.,} \quad B = 2$$

$$\text{and} \quad A = \tfrac{\sqrt{3}}{\sqrt{2}}B = \tfrac{\sqrt{3}}{\sqrt{2}} \times 2 = \sqrt{6}$$

$\therefore \quad x(t) = \sqrt{6}\cos(\sqrt{6}t + \tfrac{\pi}{2}),\ \ y(t) = 2\sin(\sqrt{6}t + \tfrac{\pi}{2})$ is the particular solution.

Note: Always check your solution by differentiating. Also recheck the initial conditions

EXERCISE 9A.3

1 Solve $x' = -2y,\ \ y' = 2x$ with boundary conditions $x(0) = -\sqrt{2},\ \ y(0) = \sqrt{2}.$

2 Solve $x' = -3y,\ \ y' = 3x$ with boundary conditions $x(0) = \sqrt{5},\ \ y(0) = -2.$

3 Solve $x' = -2y,\ \ y' = \dfrac{x}{2}$ with boundary conditions $x(0) = 1,\ \ y(0) = \tfrac{\sqrt{3}}{2}.$

4 Solve $x' = -\tfrac{2}{3}y,\ \ y' = \tfrac{3}{2}x$ with boundary conditions $x(0) = -1,\ \ y(0) = \tfrac{3\sqrt{3}}{2}.$

5 Solve $\dfrac{dI}{dt} = -\dfrac{1}{L}V$ and $\dfrac{dV}{dt} = \dfrac{1}{C}I$ for $L = 2$ henrys, $C = 0.5$ farads with $I(0) = 0$ and $V(0) = -\sqrt{\dfrac{L}{C}}I_p$. I_p is the maximum current.

3 Solve $\begin{cases} x' = -x + y \\ y' = x - y \end{cases}$ given that $x(0) = 2$, $y(0) = 8$.

4 Solve $\begin{cases} x' = x \\ y' = x - 2y \end{cases}$ given that $x(0) = -1$, $y(0) = -2$.

5 Solve $\begin{cases} x' = 2x + y \\ y' = x + 2y \end{cases}$ given that $x(0) = 4$, $y(0) = -6$.

6 Solve $\begin{cases} x' = -2x + 5y \\ y' = -2x \end{cases}$ given that $x(0) = 4$, $y(0) = 2$.

7 Solve $\begin{cases} x' = -2x + y \\ y' = -x - 4y \end{cases}$ given that $x(0) = 7$, $y(0) = 2$.

8 The interactive differential equations for an LC-circuit are $\dfrac{dI}{dt} = -\dfrac{1}{L}V$ and $\dfrac{dV}{dt} = \dfrac{1}{C}I$, L and C are positive constants.

Given that $I(0) = 0$, show that $I(t) = I_p \sin\left(\dfrac{t}{\sqrt{LC}}\right)$ and

$V(t) = -\sqrt{\dfrac{L}{C}} I_p \cos\left(\dfrac{t}{\sqrt{LC}}\right)$ $\{I_p$ is the peak or maximum current in the circuit$\}$.

9 For the chemical reaction example at the start of this chapter the reactant's system

$$\begin{cases} T' = k_1 D - k_2 T & \text{...... (1)} \\ D' = -k_1 D + k_2 T & \text{...... (2)} \end{cases} \quad \text{was examined.}$$

Given the initial conditions $T(0) = 0$ and $D(0) = D_0$, show that the actual solution is:

$$\begin{cases} D(t) = \dfrac{k_1 D_0}{k_1 + k_2}\left(e^{-(k_1+k_2)t} + \dfrac{k_2}{k_1}\right) \\[2ex] T(t) = \dfrac{k_1 D_0}{k_1 + k_2}\left(1 - e^{-(k_1+k_2)t}\right) \end{cases}$$

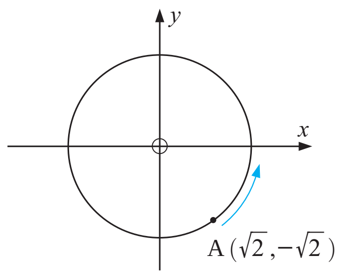
D PROBLEM SOLVING

EXERCISE 9D

1 The motion of a particle moving on a circle of radius 2 units is described by the interacting equations

$$\frac{dx}{dt} = -2y \quad \text{and} \quad \frac{dy}{dt} = 2x.$$

Find the solution to the system given that the particle starts at $A(\sqrt{2}, -\sqrt{2})$.



4 **a** If $x' = \alpha x + \beta y$ and $y' = \gamma x + \delta y$, show that $x'' - (\alpha + \delta)x' + (\alpha\delta - \beta\gamma) = 0$.

b In **a**, what form do the solutions take if $\begin{vmatrix} \alpha & \beta \\ \gamma & \delta \end{vmatrix} = \alpha\delta - \beta\gamma = 0$?

5 Solve $\begin{cases} x' = x - y \\ y' = -x + y \end{cases}$ given that $x(0) = 2$ and $y(0) = 3$.

6 The motion of a particle which is moving in a circle of radius 2 units is described by the interacting differential equations $x' = -3y$, $y' = 3x$.

Find the solution to the system given that the particle is initially at $(-2, 0)$.

7 A 3 kg mass is placed on the end of a spring with spring constant $k = 2.16$.

The extension s cm and velocity v cm/s are related by $s' = v$ and $v' = -\dfrac{k}{m}s - \dfrac{c}{m}v$

where $c = 5.4$ is the damping constant. Given that $s(0) = 5$ cm and $v(0) = -12$ cm/s find the particular solutions for $s(t)$ and $v(t)$.

8 In an LC circuit, $L = 30$ henrys and $C = 1.2$ farads and the voltage and current are related by $I' = -\dfrac{1}{L}V$ and $V' = \dfrac{1}{C}I$.

a Find $V(t)$ and $I(t)$ for boundary conditions $I(0) = 0$ amps and $V(0) = -54$ volts.

b What is the maximum current flow in the circuit?

REVIEW SET 9B

1 Consider the differential equations $x' = -\alpha y$, $y' = \beta x$ where the general solution is $x(t) = A\cos(wt + \phi)$, $y(t) = B\sin(wt + \phi)$.

a Show that $\omega = \sqrt{\alpha\beta}$ and $\dfrac{A}{B} = \sqrt{\dfrac{\alpha}{\beta}}$.

b Use **a** to solve $x' = -3y$, $y' = 2x$ where $x(0) = y(0) = 2$.

2 Find the solution of $y'' + 2y' + 3y = 0$ with boundary conditions $y(0) = 4$ and $y'(0) = 6$.

3 If $x' = 5x - y$ and $y' = -x - 2y$ find constants a and b such that $y'' + ay' + by = 0$.
What second order differential equation connects x'', x' and x?

4 **a** If $x' = \alpha x + \beta y$ and $y' = \gamma x + \delta y$, show that $y'' - (\alpha + \delta)y' + (\alpha\delta - \beta\gamma)y = 0$.

b If in **a** $\alpha + \delta = 0$, when will the solutions be:

i entirely exponential **ii** entirely trigonometric?

5 Solve $\begin{cases} x' = -x + y \\ y' = x - y \end{cases}$ given that $x(0) = 4$ and $y(0) = 3$.

MATHEMATICS FOR YR 12 - SPECIALIST MATHEMATICS
ERRATA (as at 7 September 2010)

back cover

ISBN should be 978-1-876543-79-2

QUESTIONS

page 21 **Exercise 1B.4**

4 b the difference between two odd functions is odd

page 28 **Exercise 1C.5**

5 in the diagram, θ should be the angle between BC and the horizontal

page 31 **Exercise 1D**

11 should start "Recall that if $f(x)$..."

page 32 **Exercise 1D**

15 e bottom line should be ".... and $\tan a = \frac{B}{A}$."

page 33 **Exercise 1D**

16 second line should end with ".... and $\tan a = \frac{B}{A}$."

page 37 **Review Set 1B**

3 should begin with "If $\cos\theta = -\frac{2}{3}$ and ...."

page 46 **Exercise 2A.3**

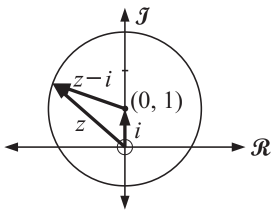
2 first line should be: "Find real numbers x and y if:"

page 50 **Exercise 2B.1**

first line of page should be:
"Notice that $\overline{z_1 + z_2 + z_3} = \overline{z_1 + z_2} + \overline{z_3}$..."

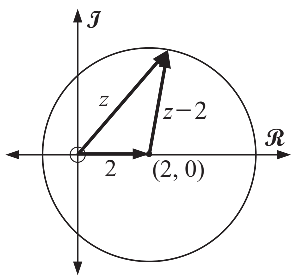
page 65 **Complex sets and their graphs**

first diagram should be:



page 66 **Example 30**

diagram should be:



page 66 **Example 31**

on the diagram, the equation of the line should be labelled as $y = -x$

page 67 **Exercise 2E**

8 b End of question should be "$x > 2$", not "$x \geqslant 2$"
9 θ is any angle in $0 \leqslant \theta \leqslant 2\pi$.

page 74 **Exercise 2G.2**

5 a the n roots of $z^n = 1$ are 1, w, w^2, w^3, ..., w^{n-1}

page 84 **Exercise 3B.1**

the blue box at the bottom of the page should read:
$\frac{P(x)}{ax^2+bx+c} = Q(x) + \frac{ex+f}{ax^2+bx+c}$

page 95 **Example 20**

the last line of the solution should read:
$= (x-2)(x+\sqrt{3})(x-\sqrt{3})$

page 96 **Example 20**

the last line of the solution should read:
$= (x-2)(x+\sqrt{3})(x-\sqrt{3})$

page 99

on the first diagram on the page the x-intercept should be at -1

page 103 **Example 25**

question should start "Find the quartic which..."

page 107 **Exercise 3G.2**

3 the last line of the question should be: "using $P(2-3i) = 0$"

page 109 **Exercise 3H**

3 b question should start:
"Explain why $(3-2w)(3-2w^2) = 19$..."

page 112 **Example 32**

a last line should read "$|z_4| = 0.4072$"

page 113 **Example 32**

b last line should read "$|z_4| = 9.866$"
c last line should read "$|z_4| = 25.9044$"

page 115 **Example 33**

b second line should read "$[2\,\text{cis}(-\frac{\pi}{6})]^2 = 4\,\text{cis}(-\frac{\pi}{3})$"

page 116 **Exercise 3J.2**

5 d last line of question should be "$n = 0, 1, 2, 3, 4, 5$ or 6"

page 120 **Example 36**

a last line should be:
"$\therefore\ c = \frac{1}{2} + \frac{1}{2}i$ is not a member of the Mandelbrot set."
b last line should be:
"$\therefore\ c = -1.940\,799\,807$ is a member of the Mandelbrot set..."

page 136 **Example 10**

the end of the first line of the question should be "Find **x** if:"

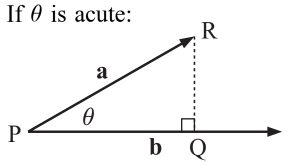
page 143

text in blue highlight box at bottom of page should be:
projection vector of **a** on **b** is $\left(\frac{\mathbf{a}\bullet\mathbf{b}}{|\mathbf{b}|}\right)\frac{\mathbf{b}}{|\mathbf{b}|}$
where $\frac{|\mathbf{a}\bullet\mathbf{b}|}{|\mathbf{b}|}$ is the length of the projection vector
and $\frac{\mathbf{b}}{|\mathbf{b}|}$ is the unit vector in the direction of **b**

page 144

the proof at the top of the page should be as follows:

If θ is acute:



In triangle PQR, $\cos\theta = \frac{\text{PQ}}{\text{PR}} = \frac{\text{PQ}}{|\mathbf{a}|}$

$\therefore\ \text{PQ} = |\mathbf{a}|\cos\theta$

$\therefore\ \text{PQ} = \frac{|\mathbf{a}||\mathbf{b}|\cos\theta}{|\mathbf{b}|}$

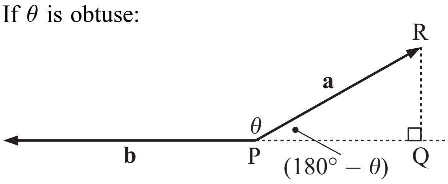
$\therefore\ \text{PQ} = \frac{\mathbf{a}\bullet\mathbf{b}}{|\mathbf{b}|}$

Now, the projection vector is in the same direction as **b**, so its unit vector is $\frac{\mathbf{b}}{|\mathbf{b}|}$.

$\therefore$ the projection vector of **a** on **b** is $\text{PQ} \times \frac{\mathbf{b}}{|\mathbf{b}|} = \left(\frac{\mathbf{a}\bullet\mathbf{b}}{|\mathbf{b}|}\right)\frac{\mathbf{b}}{|\mathbf{b}|}$.

(continued next page)

If θ is obtuse:



In triangle PQR, $\cos(180^o - \theta) = \frac{\text{PQ}}{\text{PR}} = \frac{\text{PQ}}{|\mathbf{a}|}$

$\therefore \quad \text{PQ} = |\mathbf{a}|\cos(180^o - \theta)$

$\therefore \quad \text{PQ} = -|\mathbf{a}|\cos\theta$

$\therefore \quad \text{PQ} = -\frac{|\mathbf{a}||\mathbf{b}|\cos\theta}{|\mathbf{b}|}$

$\therefore \quad \text{PQ} = -\frac{\mathbf{a}\bullet\mathbf{b}}{|\mathbf{b}|}$

Now, the projection vector is in the opposite direction to **b**, so its unit vector is $-\frac{\mathbf{b}}{|\mathbf{b}|}$.

$\therefore$ the projection vector of **a** on **b** is $\text{PQ} \times -\frac{\mathbf{b}}{|\mathbf{b}|} = \left(-\frac{\mathbf{a}\bullet\mathbf{b}}{|\mathbf{b}|}\right)\left(-\frac{\mathbf{b}}{|\mathbf{b}|}\right)$
$= \left(\frac{\mathbf{a}\bullet\mathbf{b}}{|\mathbf{b}|}\right)\frac{\mathbf{b}}{|\mathbf{b}|}$.

Note that in both cases $\text{PQ} = \frac{|\mathbf{a}\bullet\mathbf{b}|}{|\mathbf{b}|}$.

page 144 **Example 17**

c the projection vector of **b** on **a** $= \left(\frac{\mathbf{b}\bullet\mathbf{a}}{|\mathbf{a}|}\right)\frac{\mathbf{a}}{|\mathbf{a}|}$

page 153 **Exercise 4G.2**

5 c question should read:
"prove that $\mathbf{a} + \mathbf{b} = k\mathbf{c}$ for some scalar k"

page 167 **Exercise 4J**

14 Equation of plane in diagram and question should be:
"$Ax + By + Cz + D = 0$"

page 176

under the section dealing with midpoints the third line of the proof should be "$= \mathbf{a} + \frac{1}{2}(-\overrightarrow{\text{OA}} + \overrightarrow{\text{OB}})$"

page 182 **Exercise 5A.3**

1 In the diagram, vector **a** should go from O to A.

page 192 **Exercise 5B.1**

1 e XY is parallel to TZ

page 194 **Exercise 5B.1**

11 Last line of the question should read:
"that triangle XYC has constant perimeter."

page 203 **Review Set 5B**

4 On the diagram the point O should be labelled D.

page 214 **Example 3**

c second line of solution is missing. it should be:
$\therefore$ range $= x(5.954) - x(0) = 150.85 - 2 = 148.85$ m

page 215 **Exercise 6C.1**

1 c question should be: "How far from the line at $x = 0$ is the tip of the javelin when it is released?"

page 217

First paragraph after *step 3* should be:
"As t takes all values..."

The Bézier curves demo on the student CD produces incorrect results. A corrected version of this demo can be downloaded from our website (www.haeseandharris.com.au).

page 223 **Exercise 6D**

5 a should be "Find the initial position of P."
c should start "Explain why $|\mathbf{v}| = Rw$ ..."

page 227

first line of indented section of text should read:
"Likewise, $\frac{dy}{dt}$ is the rate at which B moves upwards."

page 232 **Exercise 6F**

10 third line of the problem should state:
"Michael starts walking westwards from B at 3 ms^{-1}"

page 232 **Review Set 6A**

1 b question should be:
"The speed of P is constant at $\sqrt{13}$ cm/s. Find a."

page 234 **Review Set 6C**

6 third line of problem should end:
"3 seconds after A passes through X."

page 237

third line from bottom of page should be:
"$= \sin x(0) + \cos x(1) \quad \{\text{as } h \to 0, \cos h \to 1, \frac{\sin h}{h} \to 1\}$"

page 238

Top of page, *Method 1:*, third line should be:
"$\therefore \quad y = \sin u \quad \text{where} \quad u = \frac{\pi}{2} - x$"

page 250 **Exercise 7C**

8 the flywheel rotates in a clockwise direction

page 258

the integral in both highlighted boxes should be $\int_a^b$

page 264 **Review Set 7A**

7 AM and BM are 1 km, not AP and BP.

page 267 **Review Set 7D**

7 a Equation should be:
$E(\theta) = \frac{dj_1}{\cos\theta} + (l - d\tan\theta)j_2, j_1 > j_2$

page 281

highlighted box should be:
"if $\frac{dy}{dx} = f(x)$ then $y = \int f(x)dx$"

page 293 **Exercise 8E.1**

7 the question should begin:
"A body moves to the right..."

page 294 **Example 18**

the worked solution should begin:

$q = -2\pi kr\frac{dT}{dr}$

$\therefore \quad 680 = -2\pi(0.2)r\frac{dT}{dr}$

$\therefore \quad \frac{1}{r} = \frac{-0.4\pi}{680}\frac{dT}{dr}$

$\therefore \quad \int \frac{1}{r}dr = \int \frac{-0.4}{680}\frac{dT}{dr}dr$

$\therefore \quad \int \frac{1}{r}dr = \int -\frac{0.4\pi}{680}dT$

$\therefore \quad \int \frac{1}{r}dr = -0.001\,848 \int 1dT$

page 297 **Example 20**

the second line onwards, from the top of the page should read:

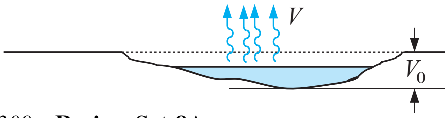
$\therefore \quad \frac{v - 29.4}{29.4} = -e^{-\frac{t}{3}} \quad (\text{since } v(0) = 0)$

$\therefore \quad v - 29.4 = -29.4e^{-\frac{t}{3}}$

page 297 **Exercise 8E.2**

6 the third line of the question should be:
"The equation for the motion is $\frac{dv}{dt} = g - 4v$..."

7 a question should read: "Explaining the symbols used, why is the differential equation $\frac{dV}{dt} = k(V_0 - V)$ appropriate?"



page 309 **Review Set 8A**

1 a question should end:

"... the differential equation $\frac{d^2y}{dt^2} = -4(y-3)$."

page 312 **Review Set 8D**

3 last line should end:
"...$(y-2)^2 = e^x(y-3)$."

page 316 **Opening Problem**

in the diagram, the $+$ and $-$ signs are the wrong way round and the second paragraph should begin:
"The extension s, will be positive if the spring is compressed."

page 317 **Investigation 1**

1 equation should be: "$y = \cos(4\pi x)$"
3 equation should be: "$y = \cos(4\pi x)\, e^{-x}$"

page 317 **Investigation 2**

in the diagram, the $+$ and $-$ signs are the wrong way round

page 333 **Investigation 6**

the first equation should be:
"$s(t) = s_0 e^{-\alpha t} \cos(\beta t)$"

page 337-338 **Investigation 7**

equation for S' should be: $S' = -\alpha S + D + 320$

What to do:

1 By differentiating (2) and using (1), show that a second order DE for the supply function is

$$S'' + \alpha S' + S = 320 \quad ...... (3)$$

2 Write down and solve the characteristic equation corresponding to the second order DE

$$S'' + \alpha S' + S = 0 \quad ...... (4)$$

3 Hence write down the general solution to (4) if:
i $\alpha = 1.8$ **ii** $\alpha = 2.0$ **iii** $\alpha = 2.2$

4 Show that if $y_0(t)$ satisfies $y'' + ay' + by = 0$ then $y_1(t) = y_0(t) + c$ satisfies $y'' + ay' + by = bc$.
Hence write down a general solution to (3) for each case:
i $\alpha = 1.8$ **ii** $\alpha = 2.0$ **iii** $\alpha = 2.2$
By considering the forms of the solutions obtained, predict which value of α is desired by Muhlack and Dangerfield.

5 Show that as $t \to \infty$, $S(t) \to 320$ for each of the general solutions obtained in **4**. Explain why this is not surprising.

6 For your chosen value of α:

a Use $S(0) = 0$ to evaluate one of the unknown constants.

b Use (2) evaluated at $t = 0$ to deduce the remaining coefficient.

c Write down a particular solution for $S(t)$.

d Use (2) to obtain a particular solution for $D(t)$.

e Graph $S(t)$ and $D(t)$ on the same set of axes, and hence check your prediction in **4**.

7 If the rate of change of gnome supply was $S' = -\alpha S + \beta D + k$, where $\beta > 0$, k a constant, what would the desired coefficient α be?
Show that β must equal $\alpha - 1$ for supply to match demand at equilibrium. **Hint:** When does $S' = D' = 0$?

page 341 **Investigation 9**

4 second equation should be:
"$y' = -\frac{1}{10} y \left(1 - \frac{3y}{10000} - \frac{x}{50000}\right)$"

page 343 **Review Set 9A**

7 spring constant should be: $k = 2.16$

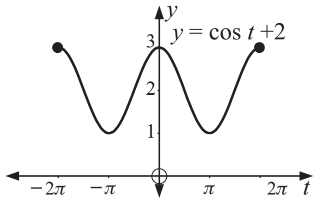
ANSWERS

page 346 **Exercise 1C.1**

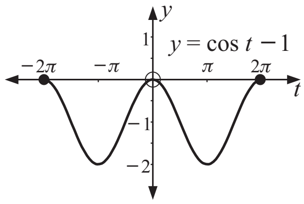
1 Graphs should have solid circles on their end points at $x = 0$ and $x = 4\pi$.

page 347 **Exercise 1C.3**

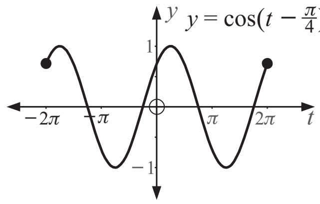
1 a



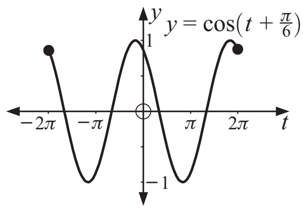
b



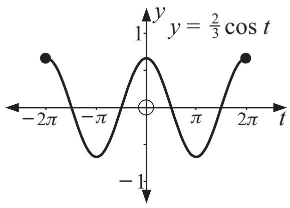
c



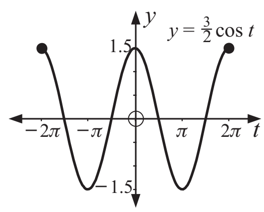
d



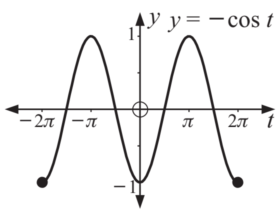
e



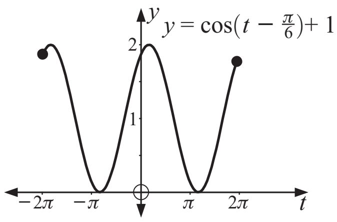
f



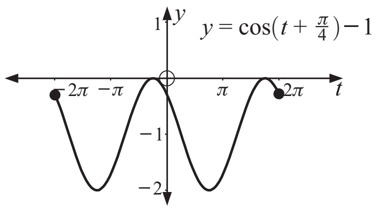
g



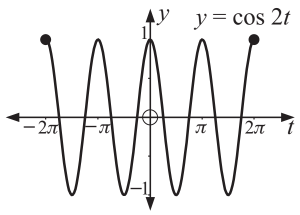
h



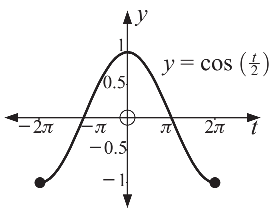
i



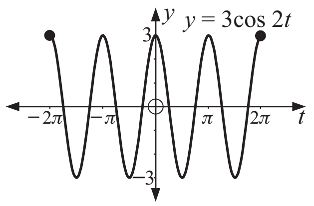
j



k

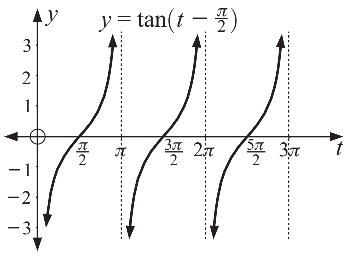


l

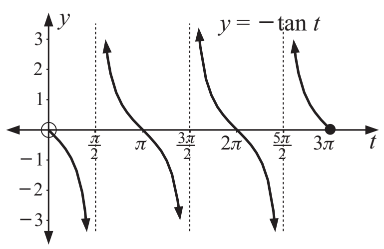


page 348 **Exercise 1C.4**

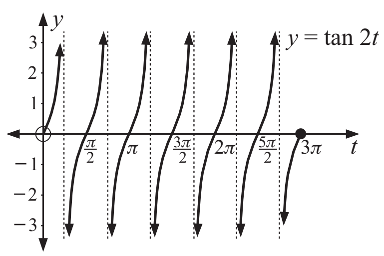
1 a i



ii



iii



page 348 **Exercise 1E.1**

1 d 5th number should be 6.3648.
h 0.2607, 1.8337, 6.5438

page 349 **Exercise 1E.2**

2 c $0, \frac{2\pi}{3}, \frac{4\pi}{3} \pm k2\pi$
d $\frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6} \pm k2\pi$
3 e 11:15 am

page 349 **Review Set 1A**

8 a all except $x = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$

page 352 **Exercise 2C.3**

11 b 0 if $a \neq 1$, undefined if $a = 1$

page 352 **Exercise 2D.3**

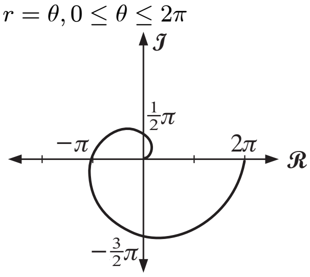
3 a $|z-1| = 2\sin\frac{\phi}{2}, \arg(z-1) = \frac{\phi}{2} + \frac{\pi}{2}$
b $z - 1 = \left(2\sin\frac{\phi}{2}\right) \text{cis}\left(\frac{\phi}{2} + \frac{\pi}{2}\right)$
c $\overline{z-1} = \left(2\sin\frac{\phi}{2}\right) \text{cis}\left(-\frac{\phi}{2} - \frac{\pi}{2}\right)$

page 352 **Exercise 2D.4**

4 a $a(x^2 + 2x + 4) = 0, a \neq 0$
b $a(x^2 - 2x + 2) = 0, a \neq 0$

page 354 **Exercise 2E**

9 $r = \theta, 0 \leq \theta \leq 2\pi$



page 355 **Exercise 2G.2**

1 a iii $z = \frac{1-w^n}{2}$, where $n = 0, 1, 2$ and $w = \text{cis}\,\frac{2\pi}{3}$
4 c $1 - w^5$

page 357 **Exercise 3C**

5 b $P(z) = a(z+2)(z^2+1) \quad a \neq 0$

page 359 **Exercise 3J.2**

5 b 3 cycle
c 3 cycle

page 359 **Exercise 3J.3**

1 a ii $0 \leftrightarrow -1$ is the 2-cycle
b i $\frac{1}{2} \pm \frac{\sqrt{33}}{6}$

page 359 **Exercise 3J.4**

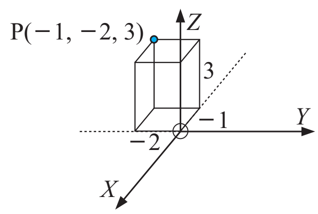
1 $(-1, 0)$

page 359 **Review Set 3A**

4 $a = 7, b = 0$ or $a = 4, b = \pm\sqrt{3}$

page 361 **Exercise 4A**

1 d



page 362 **Exercise 4B**

2 a $\overrightarrow{AB} = [4, -1, -3] \quad \overrightarrow{BA} = [-4, 1, 3]$

page 363 **Exercise 4F**

9 a -1
20 -7

page 363 **Exercise 4H**

4 a 4 units^3

page 364 **Exercise 4H**

5 c 9 units^3

page 364 **Exercise 4I**

9 a $\mathbf{a} = [2, 3, 6]$, $\mathbf{b} = [1, 1, 1]$

page 364 **Exercise 4K**

1 b i $a_1 = a_2, \quad b_1 = b_2, \quad c_1 = c_2$

page 365 **Review Set 4A**

1 c $-\frac{5}{14}[-1, 2, 3]$

page 365 **Review Set 4C**

2 a 3 units

page 366 **Exercise 5A.3**

5 $\overrightarrow{ON} = \frac{4}{5}\mathbf{a} - \frac{3}{5}\mathbf{b}$

page 366 **Exercise 5B.2**

1 a yes {opposite angles are supplementary}
b yes {one side subtends equal angles at the other two vertices}
c no {opposite angles are not supplementary}
d yes {opposite angles are supplementary}
e yes {one side subtends equal angles at the other two vertices}
f yes {opposite angles are supplementary}

page 367 **Exercise 6A.1**

1 c $v'(t) \doteqdot 5.6952t^{0.0865}$

page 367 **Exercise 6B**

2 d $t = 4.2$ hours

page 367 **Exercise 6C.1**

4 b $[3, 3]$, $3\sqrt{2}$ cms^{-1}
e $\sqrt{26}$ cms^{-1}
f maximum speed $9\sqrt{2}$ cms^{-1}, minimum speed 4.025 cms^{-1}

page 368 **Exercise 6C.2**

3 c left-most $(-2.552, 1.709)$, right-most $(1, 2)$

page 368 **Exercise 6C.3**

2 a $X(s) = (1-3s)x_0 + 3x_1 s, \quad Y(s) = (1-3s)y_0 + 3y_1 s$

page 368 **Exercise 6D**

3 c arc length l from $(R, 0) = R(\phi + t)$....
5 b $\mathbf{v} = k[-R\sin(wt+\phi), R\cos(wt+\phi)]$

page 368 **Exercise 6E.1**

1 g $(x-1)(2-y) = 6$

page 368 **Exercise 6E.2**

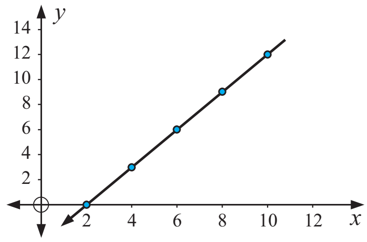
2 b $7x - 2y = 78$

page 368 **Exercise 6F**

2 a moving to the right at 1 unit per second
2 b moving to the right at 10000 units per second

page 369 **Review Set 6A**

1 c



2 a $y = \frac{1}{9}(x^2 - 9x + 18)$ for $x \geqslant 3$
b $(3, 0)$

page 369 **Review Set 6B**

2 b $(-15, 7)$ and $(-3, 1)$
6 a $V(r) = \frac{1}{3}\pi r^2\left(\frac{8r}{3}\right) = \frac{8\pi}{9}r^3$
b $\frac{dr}{dt} = -\frac{8}{375\pi}$ m per minute

page 369 **Review Set 6C**

1 a X23 $x_1 = 2 + t$, $y_1 = 4 - 3t$, $t \geqslant 0$
b Y18 $x_2 = 11 - (t-2)$, $y_2 = 3 + a(t-2)$, $t \geqslant 2$
c intercept occured at 2:22:30 pm
d true bearing of 192.7^o, 4.540 units per minute
4 a $X(t) = -1.125 - 1.25t$, $Y(t) = -2 + 5t$
b ii $k = -22.5$
6 3.601 ms^{-1}

page 369 **Exercise 7A.1**

7 a rising
b rising at 2.731 m per hour
8 a $-34\,000\pi$ units per second

page 371 **Exercise 7C**

8 b 100π radians per second

page 371 **Exercise 7D.2**

7 c $-\ln|\cos x| + c$

page 372 **Exercise 7D.5**

2 6.283 units^2

page 372 **Review Set 7C**

7 c increasing at 0.05 units per second

page 372 **Review Set 7D**

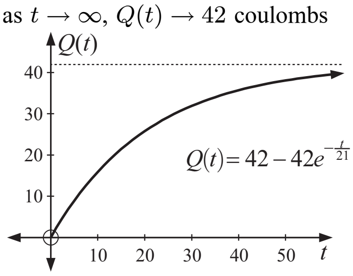
1 b $\frac{dy}{dx} = \frac{-y\cos x}{\sin x + 2y}$

page 373 **Exercise 8A.1**

1 a $P_0 = 10^6$
b i $2P_0 = 2 \times 10^6$
b ii $4P_0 = 4 \times 10^6$
b iii $64P_0 = 6.4 \times 10^7$

page 373 **Exercise 8A.2**

1 c as $t \to \infty$, $S \to 36.31$ gms
3 a $3\frac{dQ}{dt} + \frac{Q}{7} = 6$
b $k = -\frac{1}{21}, c = 42$
c $a = -42$
d as $t \to \infty$, $Q(t) \to 42$ coulombs
e



f 21.62 seconds

page 374 **Exercise 8D.1**

2 c $t = 6y^{\frac{1}{2}} + 26$
4 a $p = \frac{10}{e}$

page 374 **Exercise 8D.2**

3 $y = e^{-x^2}$
5 $y^2 = x^2 - 9, a = \pm 3\sqrt{2}$

page 374 **Exercise 8E.1**

3 \$1537.41, < 847 plates

page 374 **Exercise 8E.2**

1 a 399.8^oC
b 0.3867 m
c 0.1867 m

10 a $I = 2 + Ae^{-\frac{100}{3}t}$
b $I = 2\left(1 - e^{-\frac{100}{3}t}\right)$

page 374 **Exercise 8F**

1 d i 41 years
3 c $t \doteqdot 6.089 \times 10^{-5}$

page 375 **Review Set 8A**

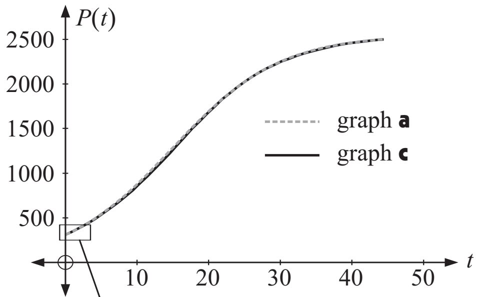
5 $y^2 = 20 - 4e^x$
7 c $t \doteqdot 2.057$ years

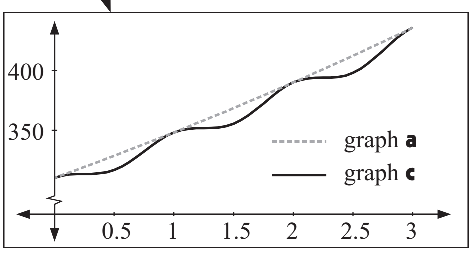
page 375 **Review Set 8B**

3 b 0.02479 m
4 a $\frac{dN}{dt} = kN$
6 a $y = 1 - \frac{2}{x^2+4x+1}$
b vertical asymptotes $x = -2 \pm \sqrt{3}$

page 375 **Review Set 8D**

5 a $P(t) = \dfrac{2550}{1 + 7.226e^{-0.1330t}}$
b $t = 14.87$ years
c $P = \dfrac{2550}{1 + 7.380e^{-0.1330t - 0.02116\cos 2\pi t}}$





d The student's initial model predicts the long-term population behaviour just as well as the modified model (seen from the graph with $0 \leqslant t \leqslant 50$). However, (as seen from the graph with $0 \leqslant t \leqslant 3$) the modified model accounts for the small-scale detail of seasonal fluctuations that the initial model lacks.

page 375 **Exercise 9A.3**

5 $I(t) = I_p \cos(t + \frac{3\pi}{2})$
$V(t) = 2I_p \sin(t + \frac{3\pi}{2})$

page 375 **Exercise 9B.1**

2 a line 2 should be "if $w \neq 0$, $y = Ae^{wt} + Be^{-wt}$"

page 375 **Exercise 9B.2**

1 a $y(t) = 3e^{2t} + 4e^{-t}$
2 b $y(t) = 4 - 3e^{3t}$

page 376 **Exercise 9C.2**

4 b i $\alpha = \beta = 0 : x(t) = A + Bt$
$\alpha = \beta \neq 0 : x(t) = A\cos\alpha t + B\sin\alpha t$
4 b ii $\alpha = 0$ or $\beta = 0 : x(t) = A + Bt$
$\alpha \neq 0$ and $\beta \neq 0 : x(t) = A\cos t\sqrt{\alpha\beta} + B\sin t\sqrt{\alpha\beta}$

page 376 **Exercise 9C.3**

6 second line should be:
$y(t) = e^{-t}(2\cos 3t - 2\sin 3t)$

page 376 **Review Set 9B**

4 b i when $\alpha\delta - \gamma\beta < 0$
ii when $\alpha\delta - \gamma\beta > 0$
7 second line of solution should be:
$v(t) = 4\cos 0.9t - 2.7\sin 0.9t$
8 a $A(t) = \frac{720}{17} - \frac{380}{17}e^{-1.7t}$
$B(t) = \frac{380}{17}e^{-1.7t} + \frac{640}{17}$