b $V \approx 28.9x + 127$

c There is a strong, positive association between the variables.
The linear model is not appropriate as the data does not lie in a straight line.

d We could draw a smooth curve through the points and extend it to $x = 49$.

SOLUTIONS TO PRACTICE EXAM 1

Paper 2

1
$$z = \frac{2}{3y-4}$$
$$\therefore\ z(3y-4) = \frac{2}{3y-4}(3y-4)$$
$$\therefore\ z(3y-4) = 2$$
$$\therefore\ \frac{z(3y-4)}{z} = \frac{2}{z}$$
$$\therefore\ 3y-4 = \frac{2}{z}$$
$$\therefore\ 3y-4+4 = \frac{2}{z}+4$$
$$\therefore\ 3y = \frac{2}{z}+4$$
$$\therefore\ \frac{3y}{3} = \frac{\frac{2}{z}+4}{3}$$
$$\therefore\ y = \frac{2}{3z}+\frac{4}{3}$$

2 **a** $3^6 = 729 \Leftrightarrow \log_3 729 = 6$

b $5\sqrt{5} = 5^{1.5} \Leftrightarrow \log_5 5\sqrt{5} = 1.5$

3 **a**
$$\sin 60^\circ = \frac{5}{x}$$
$$\therefore\ \frac{\sqrt{3}}{2} = \frac{5}{x}$$
$$\therefore\ \frac{\sqrt{3}}{2}\times 2x = \frac{5}{x}\times 2x$$
$$\therefore\ \sqrt{3}x = 10$$
$$\therefore\ \frac{\sqrt{3}x}{\sqrt{3}} = \frac{10}{\sqrt{3}}$$
$$\therefore\ x = \frac{10}{\sqrt{3}}$$

b
$$\cos x^\circ = \frac{3\sqrt{2}}{6}$$
$$= \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$$
$$\therefore\ x^\circ = \cos^{-1}(\tfrac{1}{\sqrt{2}})$$
$$\therefore\ x = 45$$

4 **a** 81, 80, 78, 75, 71, 66, 60
differences: -1, -2, -3, -4, -5, -6

b The difference table is:

n	1	2	3	4	5	6	7
u_n	81	80	78	75	71	66	60
$\Delta 1$	-1	-2	-3	-4	-5	-6	
$\Delta 2$	-1	-1	-1	-1	-1		

$\Delta 2$ values are constant, so the sequence is quadratic with general term $u_n = an^2 + bn + c$.

$2a = -1$, so $a = -\frac{1}{2}$

$3a + b = -1$, so $-\frac{3}{2} + b = -1$ $\quad\therefore\ b = \frac{1}{2}$

$a + b + c = 81$, so $-\frac{1}{2} + \frac{1}{2} + c = 81$ $\quad\therefore\ c = 81$

$\therefore$ the general term is $u_n = -\frac{1}{2}n^2 + \frac{1}{2}n + 81$

c ...., 60, 53, 45, 36, 26, 15, 3, -10
differences: -7, -8, -9, -10, -11, -12, -13

The first negative term is -10.
It is the 14th term.

5 Interest payable for 1 year $= 6\%$ of £1800
$= 0.06 \times$ £1800

$\therefore$ interest payable for 40 months which is $\frac{40}{12}$ years
$= 0.06 \times$ £1800 $\times \frac{40}{12}$
$=$ £360

6 $g(x) = \dfrac{2-x}{x^3}$

a $g(-1) = \dfrac{2-(-1)}{(-1)^3} = \dfrac{2+1}{-1} = -3$

b $g(3) = \dfrac{2-3}{3^3} = -\frac{1}{27}$

c $g(-5) = \dfrac{2-(-5)}{(-5)^3} = \dfrac{2+5}{-125} = -\frac{7}{125}$

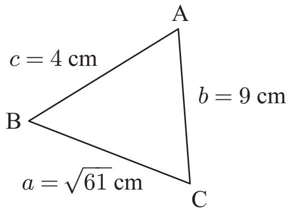
7 Let $x = 0.\overline{12}$
$= 0.121\,212\ldots$
$\therefore\ 100x = 12.1212\ldots$
$= 12 + x$
$\therefore\ 99x = 12$
$\therefore\ x = \frac{12}{99}$
$\therefore\ x = \frac{4}{33}$, so x is rational.

8 **a** $\{x \mid 0 \leqslant x \leqslant 8,\ x \in \mathbb{Z}\}$

b $\{x \mid x < -1$ or $x \geqslant 2,\ x \in \mathbb{R}\}$

9 **a** Area $= \frac{1}{2} \times 12 \times 18 \times \sin 30^\circ$
$= \frac{1}{2} \times 12 \times 18 \times \frac{1}{2}$
$= 54$ cm^2

b **i**



$$\cos A = \frac{b^2+c^2-a^2}{2bc}$$
$$= \frac{9^2+4^2-(\sqrt{61})^2}{2\times 9\times 4}$$
$$= \frac{81+16-61}{72}$$
$$= \tfrac{36}{72} = \tfrac{1}{2}$$
$$\therefore\ A = 60^\circ$$

ii Area $= \frac{1}{2} \times 9 \times 4 \times \sin 60^\circ$
$= 18 \times \frac{\sqrt{3}}{2}$
$= 9\sqrt{3}$ cm^2

10 **a** **B** **b** **C** **c** **A**

11 **a** $\overrightarrow{XZ} = \begin{pmatrix} -6-4 \\ -2-3 \end{pmatrix} = \begin{pmatrix} -10 \\ -5 \end{pmatrix}$

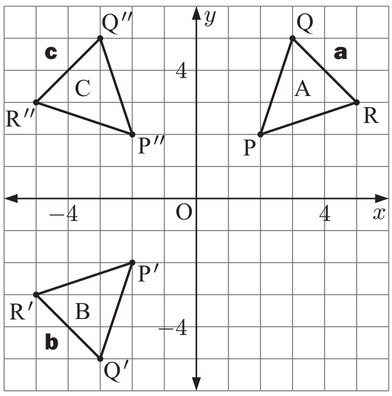
b $\overrightarrow{WY} = \begin{pmatrix} 2--2 \\ -1-2 \end{pmatrix} = \begin{pmatrix} 4 \\ -3 \end{pmatrix}$

c $\overrightarrow{VY} = \begin{pmatrix} 2-0 \\ -1--4 \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$

d $\overrightarrow{WZ} = \begin{pmatrix} -6--2 \\ -2-2 \end{pmatrix} = \begin{pmatrix} -4 \\ -4 \end{pmatrix}$

Paper 4

1



d a reflection in the y-axis

2 $f(x) = 3x^2 - 6x + 1$

a $f(3) = 3(3)^2 - 6(3) + 1$
$= 27 - 18 + 1$
$= 10$

b Cuts y-axis when $x = 0$
$f(0) = 1$, so y-intercept is 1.
Cuts x-axis when $y = 0$

$$\therefore\ 0 = 3x^2 - 6x + 1$$
$$\therefore\ x = \frac{6 \pm \sqrt{36 - 4 \times 3}}{2 \times 3} = \frac{6 \pm \sqrt{24}}{6} = \frac{6 \pm 2\sqrt{6}}{6} = 1 \pm \tfrac{\sqrt{6}}{3}$$

$\therefore$ the x-intercepts are $1 - \frac{\sqrt{6}}{3}$ and $1 + \frac{\sqrt{6}}{3}$.

c The quadratic has a line of symmetry halfway between the x-intercepts.
The average of $1 - \frac{\sqrt{6}}{3}$ and $1 + \frac{\sqrt{6}}{3}$ is 1, so the line of symmetry is $x = 1$.
The line of symmetry passes through the vertex of the quadratic.
$\therefore$ the x-coordinate of the vertex is 1.
Now $f(1) = 3(1)^2 - 6(1) + 1$
$= -2$
$\therefore$ the vertex is $(1, -2)$

d $f(x) = 3(x - 1)^2 - 2$

e $f(x)$ has vertex $(1, -2)$
Under $T\begin{pmatrix} 1 \\ 2 \end{pmatrix}$, $g(x)$ has vertex $(2, 0)$.

$\therefore\ g(x) = 3(x - 2)^2$
$= 3(x^2 - 4x + 4)$
$= 3x^2 - 12x + 12$

3 $h = 3r$

a $V = \pi r^2 h$
$= \pi r^2(3r)$
$= 3\pi r^3$

b $3\pi r^3 = 192\pi$
$\therefore\ 3r^3 = 192$
$\therefore\ r^3 = 64$
$\therefore\ r = 4$

c Surface area
$= \text{area of curved surface} + \text{area of 2 ends}$
$= 2\pi rh + 2\pi r^2$
$= 2\pi r(3r) + 2\pi r^2$
$= 6\pi r^2 + 2\pi r^2$
$= 8\pi r^2$
$= 8\pi \times 4^2$
$\approx 402\ \text{cm}^2$

d Each cm^3 of lead weighs 11.37 g.
$\therefore$ using **b**, mass $= 11.37 \times 192\pi$
≈ 6860 g

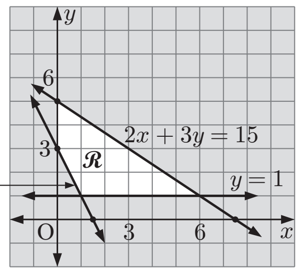
e $V = \frac{4}{3}\pi r^3$
$\therefore\ 192\pi = \frac{4}{3}\pi r^3$
$\therefore\ r^3 = 192 \times \frac{3}{4}$
$\therefore\ r = \sqrt[3]{144}$
≈ 5.24
$\therefore$ the radius is approximately 5.24 cm.

4 **a** $2x + y = 3$

x	0	1.5
y	3	0

$2x + 3y = 15$

x	0	7.5
y	5	0



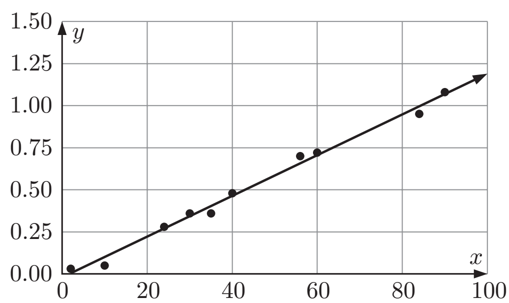
b

x	1	2	3	1	2	3	0
y	1	1	1	2	2	2	3
$x + 2y$	3	4	5	5	6	7	6

all other y values are too large

(2, 1) is the only point with integer values such that $x + 2y = 4$.

5 **a**, **d**



b very strong, positive correlation

c There is very strong, positive, linear correlation between the two variables, so as x increases, so does y. This indicates that the two variables are directly proportional.

d $y \approx 0.0121x - 0.0201$ (see above for graph)

e gradient ≈ 0.0121
For every extra female teacher on staff, a student will get an extra 0.0121 detentions a year.

f **i** 0.440 detentions **ii** 1.37 detentions

The prediction for 38 teachers is reasonable, since the prediction is within the domain of the data and the variables are very strongly correlated.

The prediction for 115 teachers is not as reliable, as it is outside the domain of the data.

6 **a** 0.45 km = 450 m

b **i** between 20 s and 50 s after starting

ii $\text{average speed} = \frac{\text{distance}}{\text{time}} = \frac{0.15 \text{ km}}{30 \text{ s}} = \frac{150 \text{ m}}{30 \text{ s}} = 5 \text{ m/s}$

c It happens 80 seconds after starting. He is stuck for 10 seconds.

d **i** $\text{average speed for race} = \frac{0.45 \text{ km}}{170 \text{ s}} = \frac{0.45 \text{ km}}{\frac{170}{60\times 60} \text{ h}} = \frac{0.45 \times 60 \times 60}{170} \text{ km/h} \approx 9.53 \text{ km/h}$

ii $\text{average speed} = \frac{0.45 \text{ km}}{170 \text{ s}} = \frac{450 \text{ m}}{170 \text{ s}} \approx 2.65 \text{ m/s}$

e To complete the course Michel travels 0.45 km at 10.1 km/h,

so time taken $= \frac{0.45}{10.1} \times 3600 + 10 \text{ s} \approx 160.4 + 10 \text{ s} \approx 170.4 \text{ s}$

So, Clinton finished in the quicker time.

7 **a** $\triangle$s ACE and BCD are congruent (SAS) because

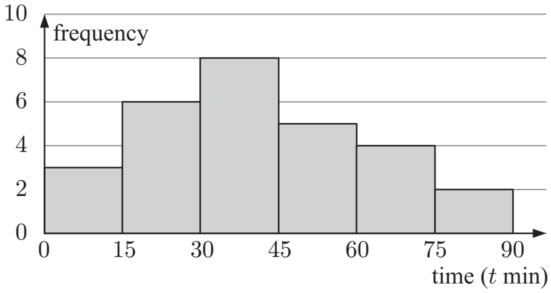
AC = BC {given}
CE = CD {given}
$\widehat{ACE} = \widehat{BCD}$ {vertically opposite}
$\therefore$ AE = BD {corresponding sides}

b In $\triangle$s ABE, ADB,
AB is common
AE = BD {part **a**}
AC + CD = BC + CE {AC = BC, CD = CE}
$\therefore$ AD = BE
$\therefore$ $\triangle$s ABE, ADB are congruent (SSS)

c From **b**, $\widehat{AEB} = \widehat{ADB}$
But these are angles subtended by AB.
$\therefore$ ABDE is a cyclic quadrilateral.

8 **a** Data was recorded on $3+6+8+5+4+2 = 28$ days

b **Histogram of Surfing times**



c The modal class is $30 \leqslant t < 45$ min.

d

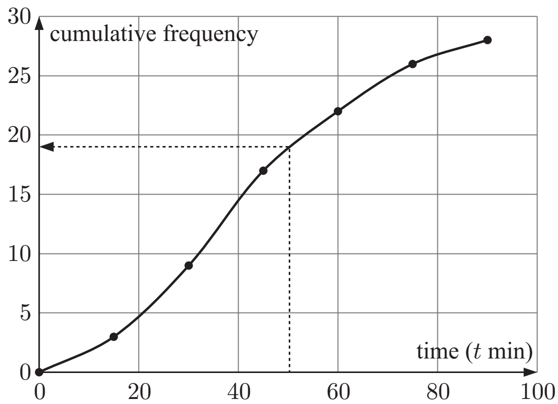
Middle value (x)	*Frequency* (f)
7.5	3
22.5	6
37.5	8
52.5	5
67.5	4
82.5	2

The mean length of time $\approx$ 41.3 min.

e

Surfing time (mins)	*Freq.*	*Cumul. Freq.*
$0 \leqslant t < 15$	3	3
$15 \leqslant t < 30$	6	9
$30 \leqslant t < 45$	8	17
$45 \leqslant t < 60$	5	22
$60 \leqslant t < 75$	4	26
$75 \leqslant t < 90$	2	28

Cumulative frequency graph



f Toby spends up to 50 minutes on approximately 19 days, so he spends more than 50 minutes on $28 - 19 = 9$ days.

$\therefore$ the probability that Toby 'fines' himself is $\frac{9}{28}$.

In a 30 day month he fines himself

$\frac{9}{28} \times 30 \approx 10$ days (nearest day).

$\therefore$ Toby will pay approximately £5 × 10 = £50 in 'fines'.

9 $f(x) = 5 \times 4^{-x}$

a **i** $f(-\frac{3}{2}) = 5 \times 4^{-(-\frac{3}{2})} = 5 \times \left(2^2\right)^{\frac{3}{2}} = 5 \times 2^3 = 40$

ii $f(x-1) = 5 \times 4^{-(x-1)}$
$= 5 \times 4^{-x} \times 4$
$= 20 \times 4^{-x}$

$4f(x) = 4 \times 5 \times 4^{-x}$
$= 20 \times 4^{-x}$

$\therefore\ f(x-1) = 4f(x)$

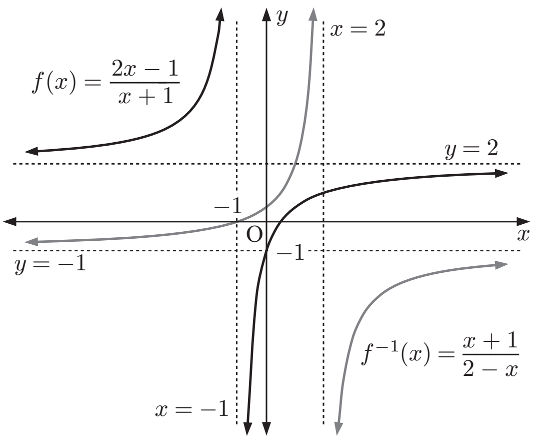
iii Domain $= \{x \mid x \in \mathbb{R}\}$
Range $= \{y \mid y > 0,\ y \in \mathbb{R}\}$

iv Asymptote is $y = 0$.

b By interchanging x and y, the inverse of $y = \dfrac{2x-1}{x+1}$

is $x = \dfrac{2y-1}{y+1}$

$\therefore\ x(y+1) = 2y-1$
$\therefore\ xy + x = 2y - 1$
$\therefore\ xy - 2y = -x - 1$
$\therefore\ y(x-2) = -x-1$
$\therefore\ y = \dfrac{-x-1}{x-2} = f^{-1}(x)$ or $f^{-1}(x) = \dfrac{x+1}{2-x}$



10 **a** $m_{PQ} = \dfrac{-1-(-5)}{6-(-1)} = \dfrac{-1+5}{6+1} = \frac{4}{7}$

$m_{PR} = \dfrac{1-(-5)}{2-(-1)} = \frac{6}{3} = 2$

$m_{QR} = \dfrac{1-(-1)}{2-6} = \frac{2}{-4} = -\frac{1}{2}$

$m_{QR} \times m_{PR} = -\frac{1}{2} \times 2 = -1$

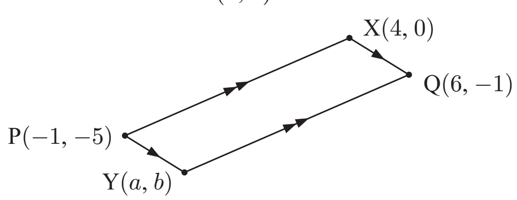
$\therefore$ QR is perpendicular to PR.
$\therefore$ $\triangle$PQR is right angled at R.

b midpoint X of QR is $\left(\dfrac{6+2}{2}, \dfrac{-1+1}{2}\right)$

$\therefore$ X is $(4, 0)$.

distance PX $= \sqrt{(4-(-1))^2 + (0-(-5))^2}$
$= \sqrt{5^2+5^2}$
$= \sqrt{50}$
$= 5\sqrt{2}$ units

c Let Y have coordinates (a, b).



For a parallelogram, the midpoints of the diagonals are the same point.

Midpoint of PQ is $\left(\dfrac{-1+6}{2}, \dfrac{-5+-1}{2}\right) = (\frac{5}{2}, -3)$

Midpoint of XY is $\left(\dfrac{a+4}{2}, \dfrac{b+0}{2}\right) = \left(\dfrac{a+4}{2}, \dfrac{b}{2}\right)$

$\therefore\ \dfrac{a+4}{2} = \frac{5}{2}$ and $\dfrac{b}{2} = -3$

$\therefore\ a+4 = 5$ and $b = -6$
$\therefore\ a = 1$

So, Y is $(1, -6)$.

11 **a** We are given: $x + 2y = 12$ .... (1)
$x - 4y = -12$ .... (2)

$\therefore\ 2x + 4y = 24$ $\{(1) \times 2\}$
$x - 4y = -12$
$3x = 12$ {adding}
$\therefore\ x = 4$

Substituting in (1) gives
$4 + 2y = 12$
$\therefore\ 2y = 8$
$\therefore\ y = 4$
So, $x = 4,\ y = 4$.

b Using technology,
$x = 4$ or $x = -12$
$\therefore\ x = 4,\ y = 4$ or $x = -12,\ y = 0$

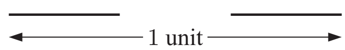
12 Let Jo have €x.
$\therefore$ Gemma has €$3x$.
Together they have €x + €$3x$ = €$4x$
and $4x = 28.60$
$\therefore\ x = 7.15$
and $3x = 21.45$
So, Jo has €7.15 and Gemma has €21.45.

Paper 6

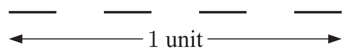
A. Investigation: The Hilbert curve

Part I

1 Using the base of the first pattern, the square is 1 unit wide. The second pattern has the same width. So 3 equal lengths make up 1 unit, and the length of each line segment is $\frac{1}{3}$ unit.



In the 3rd pattern, 7 lengths make up 1 unit, and so on.



Hilbert curve order	*Length of line segments*
1	1
2	$\frac{1}{3}$
3	$\frac{1}{7}$
4	$\frac{1}{15}$
5	$\frac{1}{31}$

2 The denominator is 2 raised to the power of the order number, then 1 is subtracted from it.

3 $u_n = \frac{1}{2^n - 1}$

For $n = 6$,
$$u_6 = \frac{1}{2^6 - 1} = \frac{1}{64 - 1} = \tfrac{1}{63}$$

For $n = 7$,
$$u_7 = \frac{1}{2^7 - 1} = \frac{1}{128 - 1} = \tfrac{1}{127}$$

4 As n becomes very large, the length becomes very small.

Part II

1 3 **2** 15 **3** $u_{n+1} = 4u_n + 3$

4

Hilbert curve order	*Number of line segments*
1	3 $(= 2^2 - 1)$
2	15 $(= 2^4 - 1)$
3	63 $(= 2^6 - 1)$
4	255 $(= 2^8 - 1)$
5	1023 $(= 2^{10} - 1)$

5 **a** $u_n = 2^{2n} - 1$ or $4^n - 1$

b **i** $u_6 = 4^6 - 1 = 4095$

ii $u_{10} = 4^{10} - 1 = 1\,048\,575$

iii $u_{15} = 4^{15} - 1 = 1\,073\,741\,823$

6
$$\begin{aligned} u_{n+1} &= 4^{n+1} - 1 \\ &= 4 \times 4^n - 1 \\ &= 4 \times (4^n - 1) + 3 \\ &= 4u_n + 3 \end{aligned}$$

7 As n becomes very large, the number of line segments becomes very large.

Part III

1

Hilbert curve order	*Length of each line segment*	*Number of line segments*	*Total length*
1	1	3	3
2	$\frac{1}{3}$	15	5
3	$\frac{1}{7}$	63	9
4	$\frac{1}{15}$	255	17
5	$\frac{1}{31}$	1023	33

2 $L_n = \frac{1}{2^n - 1} \times (2^{2n} - 1)$

from **Part I**, **3** from **Part II**, **5**

$$\therefore \quad L_n = \frac{1}{2^n - 1} \times (2^n + 1)(2^n - 1) = 2^n + 1$$

a $L_6 = 2^6 + 1 = 65$ **b** $L_7 = 2^7 + 1 = 129$

c $L_{20} = 2^{20} + 1 = 1\,048\,577$

3 **a** As n gets larger, L_n becomes very large.

b As n gets very large, the number of line segments gets extremely large, but the length of each line segment gets very small.

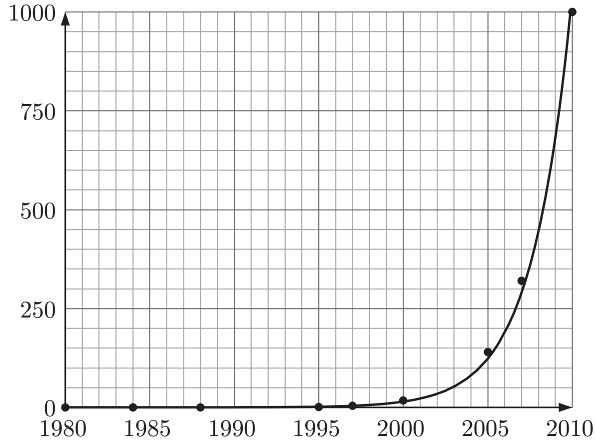
However, in $L_n = \frac{1}{2^n - 1} \times (2^{2n} - 1)$,

2^{2n} becomes larger much faster than 2^n.

So, L_n becomes larger as n becomes larger.

B. Modelling: Hard Disk Storage

1



2 **a** **i** $S \approx 0.002\,76 \times 1.534^7 \approx 0.0552$ GB

ii $S \approx 0.002\,76 \times 1.534^{12} \approx 0.469$ GB

b Using answers to **2 a i**, **ii**:

$$\left(\frac{0.469 - 0.0552}{0.0552}\right) \times 100\% \approx 750\% \text{ increase}$$

c If $S_t = 0.002\,76 \times 1.534^t$, then in 5 years,

$$\begin{aligned} S_{t+5} &= 0.002\,76 \times 1.534^{t+5} \\ &= \underbrace{0.002\,76 \times 1.534^t}_{\text{this is } S_t} \times 1.534^5 \\ &\approx 8.49 \times S_t \end{aligned}$$

This corresponds to a 749% increase, so it will increase by approximately the same percentage every 5 years.

3 **a** $S = 6$ when $t \approx 17.96$ {using solver}

The hard drive was made at the end of 1997 or start of 1998.

b Our prediction should be fairly reliable, since it is within the domain of the data, and the variables have strong correlation.

4 **a** When $t = 40$, $S \approx 74\,804$ GB

b Our prediction is outside the domain of the data, so it is not really reliable.

SOLUTIONS TO PRACTICE EXAM 2

Paper 2

1 **a**
$$\begin{aligned} &-(-1)^3 \\ &= -(-1) \\ &= 1 \end{aligned}$$

b
$$\begin{aligned} &\sqrt{28} - \sqrt{7} \\ &= \sqrt{4 \times 7} - \sqrt{7} \\ &= \sqrt{4} \times \sqrt{7} - \sqrt{7} \\ &= 2\sqrt{7} - \sqrt{7} \\ &= \sqrt{7} \end{aligned}$$