

- 37** The yield of apples depends on the number of cold days experienced by the tree. The following table shows the total yield of apples from an orchard over several years with different numbers of cold days.

Cold days (n)	32	55	67	28	43	50
Yield (Y tonnes)	41.1	26.8	16.2	45.3	38.6	32.5

- Name the independent and dependent variables.
- Draw a scatter diagram for the data.
- Use technology to find the equation of the linear regression line.
- Interpret the gradient and vertical intercept of this line.
- Use the regression line to predict the yield from the orchard if the number of cold days was:
 - 40
 - 100.
- Comment on the reliability of your answers in e.

- 38** The times taken for a group of students to complete a logic puzzle were recorded and the results tabled.

Time (t min)	Frequency
$0 \leq t < 1$	3
$1 \leq t < 2$	11
$2 \leq t < 3$	19
$3 \leq t < 4$	15
$4 \leq t < 5$	7
$5 \leq t < 6$	5

- Construct a cumulative frequency curve for the data.
- Estimate the IQR and explain what it means.
- Estimate the 80th percentile and explain what it represents.
- The fastest 10 students will be selected for an inter-school competition. Estimate the cut-off time for selection.

- 39** The average value of a rare bottle of wine sold at auction over the period from 1971 to 2005, is shown in dollars in the table below.

Year	1971	1980	1990	1995	2000	2003	2005
Value (\$)	221	350	587	723	976	1062	1224

- Let x be the number of years since 1971 and V be the value of the bottle of wine in dollars. Draw a scatter diagram of V against x .
- Find a linear model for V in terms of x .
- Comment on the association between the variables. Is the linear model appropriate? Why or why not?
- How could you estimate the value of the bottle of wine in 2020?

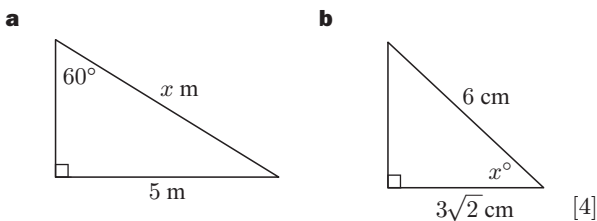
PRACTICE EXAM 1

Paper 2: 45 minutes / 40 marks

1 Make y the subject of the formula: $z = \frac{2}{3y - 4}$. [3]

2 Write an equivalent logarithmic statement for:
a $3^6 = 729$ **b** $5\sqrt{5} = 5^{1.5}$ [4]

- 3** Find the exact value of the unknown in:



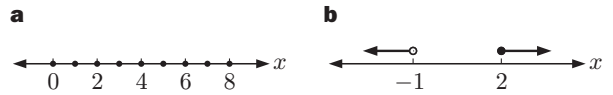
- Write down the next three terms of the sequence 81, 80, 78, 75, ... [1]
- Write down a formula for the n th term of the sequence. [2]
- What is the first negative term of the sequence, and where does it occur? [2]

- 5** Calculate the simple interest on a £1800 loan at 6% for 40 months. [2]

6 Given $g(x) = \frac{2-x}{x^3}$, find:
a $g(-1)$ **b** $g(3)$ **c** $g(-5)$ [3]

- 7** Show that $0.\overline{12}$ is a rational number. [2]

- 8** Write these sets in interval notation:

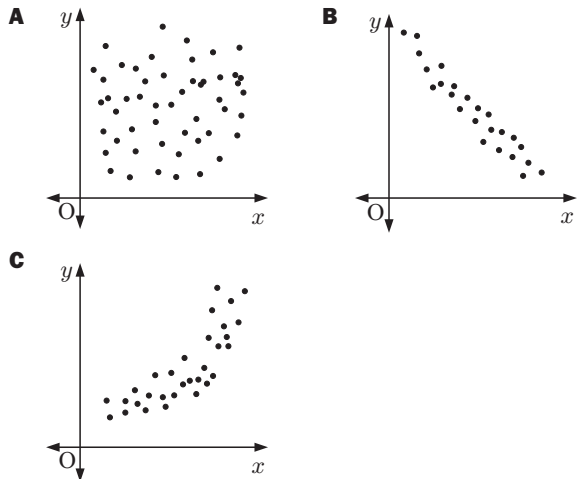


- 9** Find:

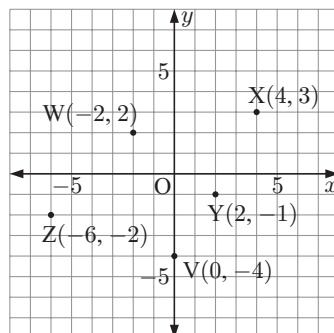
- the area of:
- the measure of angle A in $\triangle ABC$ if $a = \sqrt{61}$ cm, $b = 9$ cm and $c = 4$ cm.
 - Hence find the exact area of the triangle. [8]

- 10** Match each scatter diagram to its description:

- strong, negative, linear correlation [1]
- moderate, non-linear correlation [1]
- no correlation [1]



- 11** Write in component form:



- \vec{XZ} [1]
- \vec{WY} [1]
- \vec{VY} [1]
- \vec{WZ} [1]

Paper 4: 2 hours 15 minutes / 120 marks

- 1** Triangle PQR has vertices P(2, 2), Q(3, 5), and R(5, 3).
- Draw triangle PQR on a set of axes. Label this triangle A. [2]
 - Rotate triangle A 180° about O. Label this image B. [2]
 - Reflect image B in the x -axis. Label this image C. [2]
 - What single transformation maps A onto C? [1]

- 2** Consider the function $f(x) = 3x^2 - 6x + 1$.
- Find $f(3)$. [2]
 - Find the axes intercepts of the function. [3]
 - Find the axis of symmetry of the function, and hence find its vertex. [3]
 - Use **c** to write $f(x)$ in the form $a(x - h)^2 + k$. [1]
 - Suppose $g(x)$ is obtained by translating $f(x)$ through $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$. Find $g(x)$, giving your answer in the form $g(x) = ax^2 + bx + c$. [3]

- 3** The height of a solid cylinder is three times its radius r .
- Write down an expression for the volume of the cylinder, in terms of r . [2]
 - Find r given that the cylinder has volume $192\pi \text{ cm}^3$. [3]
 - Find the surface area of the cylinder correct to the nearest cm^2 . [4]
 - Suppose this cylinder was made of pure lead. Each cubic centimetre of lead weighs 11.37 g. Find the mass of the cylinder. [2]
 - The cylinder is melted down and moulded into a sphere. Find the radius of the sphere. [3]

- 4**
- Graph the region \mathcal{R} for which $x \geq 0$, $y \geq 1$, $2x + y \geq 3$, and $2x + 3y \leq 15$. [4]
 - Find all points in \mathcal{R} with integer coordinates, such that $x + 2y = 4$. [3]

- 5** A school superintendent has a theory that the number of detentions given in a school per student is proportional to the percentage of female teachers on its staff. He has collected the following data from the schools in his district.

No. of female teachers (x)	60	30	2	10	56
No. of detentions per student per year (y)	0.72	0.36	0.03	0.05	0.70
No. of female teachers (x)	90	35	40	84	24
No. of detentions per student per year (y)	1.08	0.36	0.48	0.95	0.28

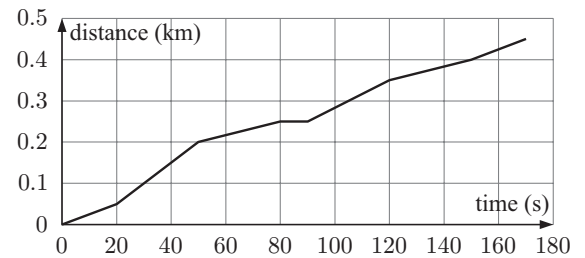
- Draw a scatter diagram for the data. [3]
- Comment on the correlation between the two variables. [2]
- Is the superintendent's theory correct? Explain your answer. [2]
- Use technology to determine the equation of the line of best fit. Plot this on the graph. [2]
- Interpret the gradient of the line of best fit. [1]

- f** Estimate the number of detentions per student given in a year in a school with:

- 38 female teachers
- 115 female teachers.

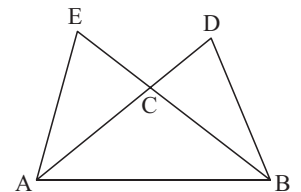
Comment on whether these predictions are reasonable. [4]

- 6** Clinton is in a kayak race. His progress through the course is shown on the travel graph below:



- How long is the course? [1]
- When is Clinton travelling fastest? [3]
 - What is his speed at this time? [3]
- During the race, Clinton's kayak becomes stuck on a rock. When does this happen, and for how long is he stuck? [2]
- What is Clinton's average speed in:
 - km/h
 - m/s? [2]
- Clinton's main rival Michel paddles at an average speed of 10.1 km/h, but gets stuck at the same rock for 10 seconds. Who finished in the quicker time? [2]

- 7** Triangle ABC is isosceles with $AC = BC$. BC is produced to E and AC is produced to D such that $CE = CD$. Prove that:



- $AE = BD$ [4]
- triangles ABE and ADB are congruent [4]
- ABDE is a cyclic quadrilateral. [3]

- 8** Toby recorded the length of time he spent each day surfing the internet. He summarised the results in the table below:

Surfing time (mins)	Frequency
$0 \leq t < 15$	3
$15 \leq t < 30$	6
$30 \leq t < 45$	8
$45 \leq t < 60$	5
$60 \leq t < 75$	4
$75 \leq t < 90$	2

- For how many days did Toby record data? [1]
- Construct a histogram to display the data. [4]
- State the modal class for the data. [1]
- Use technology to estimate the mean length of time Toby spends surfing the internet each day. [1]
- Construct a cumulative frequency graph for the data. [4]
- Toby decides to cut back on his internet surfing. He decides that next month he will 'fine' himself £5 for every day he spends more than 50 minutes online. If there are 30 days in the month, estimate how much money Toby will pay in 'fines'. [3]

- 9 a** Suppose $f(x) = 5 \times 4^{-x}$.
- Evaluate $f(-\frac{3}{2})$.
 - Show that $f(x - 1) = 4f(x)$.
 - State the domain and range of $f(x)$.
 - Write down the equation of the asymptote to $f(x)$. [5]
- b** Given $f(x) = \frac{2x - 1}{x + 1}$, find $f^{-1}(x)$. Hence graph $f(x)$ and $f^{-1}(x)$ on the same set of axes. [5]
- 10** Triangle PQR has vertices P(-1, -5), Q(6, -1), and R(2, 1).
- Show that triangle PQR is right angled, and state the vertex where the right angle occurs. [5]
 - Find the distance from P to X, the midpoint of QR. [3]
 - Point Y is located such that PXQY is a parallelogram. Find the coordinates of Y. [3]
- 11 a** Use the method of elimination to solve simultaneously:
- $$\begin{aligned} x - 4y &= -12 \\ x + 2y &= 12 \end{aligned}$$
- [4]
- b** Use your graphics calculator to solve simultaneously:
- $$\begin{aligned} x - 4y &= -12 \\ |x| + 2y &= 12 \end{aligned}$$
- [2]
- 12** Gemma and Jo have a total of €28.60 between them. Gemma has three times as much money as Jo. How much money does each have? [4]

Paper 6: 1 hour 30 minutes / 40 marks

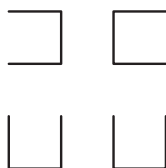
A. Investigation: The Hilbert curve 25 marks

The Hilbert curve was first described in 1891. It is used in compressing and processing digital images.

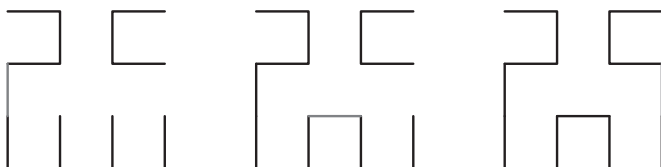
The starting image of the Hilbert curve (called the *first order* Hilbert curve) is a 'cup', which is three sides of a square as shown.



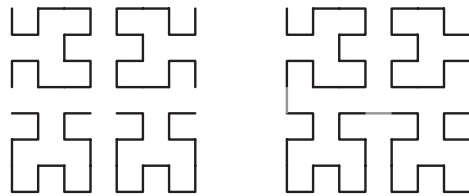
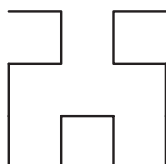
To get the second order Hilbert curve, we start with a square of the original size, and place a smaller 'cup' in each corner:



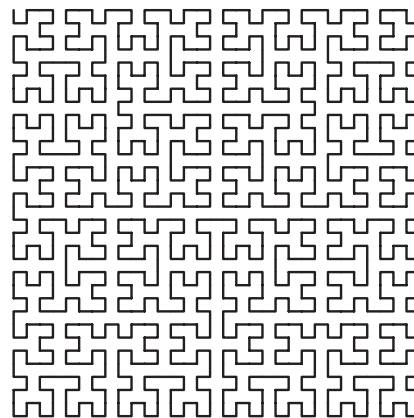
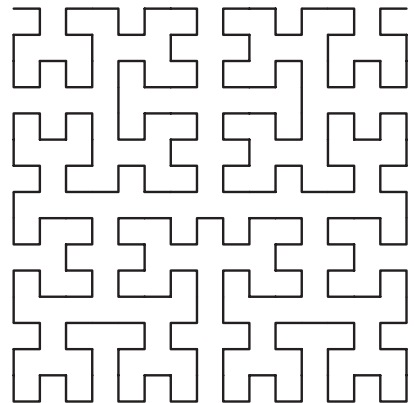
We then draw connecting line segments as shown:



To get to the third order Hilbert curve, we use a similar process using the new shape:



The fourth and fifth order Hilbert curves are shown below, enlarged so you can see the detail:



Rules

The next Hilbert curve is obtained from the current one by:

- shrinking the current curve
- putting a 'copy' in each corner of the square, and rotating it by some multiple of 90 degrees, and
- connecting them up with line segments.

Part I

In this part, we will consider the effect of successive iterations on the line segments of the curve.

- Use the diagrams given to fill in the following table:

Hilbert curve order	Length of line segments
1	1
2	$\frac{1}{3}$
3	
4	
5	

- Consider the denominator of each of the fractions in **1**. Describe what is happening to the denominator in words.
Hint: Consider powers of 2.

- 3** Suggest a formula for the length of line segments in the n th order Hilbert curve. Hence find the length of the line segments in the:
- a** sixth **b** seventh iteration.
- 4** As n gets very large, what happens to the length of the line segments in the curve?

Part II

In this part we consider the *number* of line segments needed to make each iteration of the Hilbert curve. Remember that all of the line segments in the curve have the same length.

- 1** Consider the first order Hilbert curve. How many line segments are needed to make it?
- 2** Now consider the second order Hilbert curve in **Part I** above. How many line segments are needed to make it?
- 3** Let u_n be the number of line segments in the n th Hilbert curve. Using the list of rules for generating the next Hilbert curve, write a formula for u_{n+1} in terms of u_n .
- 4** Complete the table below:

Hilbert curve order	Number of line segments
1	
2	
3	
4	
5	

- 5 a** Using the table above, write a formula for u_n in terms of n only.
- b** Hence, find the number of line segments needed for:
- i** u_6 **ii** u_{10} **iii** u_{15} .
- 6** Consider your answer to **5 a**. By replacing n with the expression $n + 1$, show that this formula is equivalent to the one you found in **3**.
- 7** As n gets very large, what happens to the number of line segments needed?

Part III

Now, we will consider the effect that successive iterations have on the *total* length of the Hilbert curve.

- 1** Copy your answers from **Part I** and **Part II** into the table below, and hence fill out the *Total length* column.

Hilbert curve order	Length of each line segment	Number of line segments	Total length
1	1	3	$1 \times 3 = 3$
2	$\frac{1}{3}$		
3			
4			
5			

- 2** Suggest a formula for L_n , the total length of the n th order Hilbert curve. Hence find:
- a** L_6 **b** L_7 **c** L_{20}
- 3 a** As n gets larger, what happens to L_n ?
- b** Use your answers to **Part I**, question **4** and **Part II**, question **7** to explain your answer to **a**.

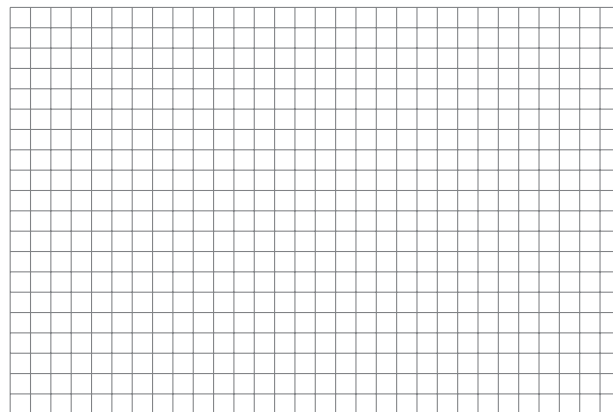
B. Modelling: Hard disk storage

15 marks

Ada, a computer science student, is investigating how the storage capacity of computer hard drives has increased over the past thirty years. She has collected the following data:

Year	1980	1984	1988	1995	1997
Storage (S GB)	0.005	0.015	0.04	1.2	4.3
Year	2000	2005	2007	2010	
Storage (S GB)	17.3	140	320	1000	

- 1** Let t be the number of years since 1980. Draw a scatter diagram of S against t .



- 2** The storage capacity S fits a model of the form $S \approx 0.00276 \times 1.534^t$, where t is the number of years after 1980.
- a** Find the approximate size of a hard drive in:
- i** 1987 **ii** 1992.
- b** What is the percentage increase in storage capacity between 1987 and 1992?
- c** Will hard drive capacity increase by the same percentage every five years? Explain your answer.
- 3** During a major cleanout, Ada finds an old 6 GB hard drive without a date of manufacture.
- a** Use the model to estimate when the hard drive was made.
- b** Comment on the reliability of your prediction.
- 4** A study has predicted that in 2020, hard drives will reach 14 terabytes (14 000 GB) in size.
- a** Use the model to predict the storage capacity of a hard drive in 2020.
- b** Explain why your answer might be different from the study's prediction.

PRACTICE EXAM 2

Paper 2: 45 minutes / 40 marks

- 1** Simplify:
- a** $-(-1)^3$ [1]
- b** $\sqrt{28} - \sqrt{7}$ [2]
- c** $\log 9 + \log 15 - \log 5$ [2]
- 2** Consider the sequence $-1, 2, -4, 8, -16, \dots$
- a** Find the next two terms of the sequence. [2]
- b** Write down a formula for the n th term of the sequence. [3]