



# HAESE MATHEMATICS

This document provides brief notes for the use of the Haese Mathematics books for the International Baccalaureate Further Mathematics HL course.

## Preparation during the Middle Years Program

The following chapters will appear in the online versions of the MYP texts. They are not essential in order to do FM, but may be useful in encouraging students to take on the course, and also then in providing some basic background knowledge.

- End of MYP 3: Introduction to networks (graphs) – Chapter 23  
They are referred to here as networks, to avoid confusion with the graph of a function.
- End of MYP 4: Logic – Chapter 27  
I believe mathematical logic is important for understanding what constitutes a rigorous proof.
- End of MYP 5 Extended: Circle and ellipses – Chapter 27  
Provides some background work for the Conic Sections in the Geometry topic.
- Matrices – Chapter 28  
Provides some background work for the Linear Algebra topic.

In addition there are some other sections of our MYP books with work necessary for FM students. This is generally marked Extension in the MYP 5 Extended book, and is more specifically described below.

## Order of study for Further Mathematics topics

We recommend the following topic order, since it fits with our presentation of the HL Core course without running into difficulties with assumed knowledge.

### 1. Linear Algebra

- Systems of linear equations (without geometric interpretation, which is covered in HL Core (vectors)).
- In determinants of  $3 \times 3$  matrices, there is a question involving function notation and factors of a polynomial.
- Vector spaces are done from the point of view of  $n \times 1$  matrices, so knowledge of HL vectors is not required.
- Spanning sets is self-contained, so there is no need to have already done the Sets, Relations, and Groups Option.
- Geometric transformations extends directly from MYP 5 Extended, so there is no need to have already covered HL transforming functions.

- In preparation for the rotations section, the students should have done Exercise 21I from the MYP 5 Extended book, marked Extension (compound angle formula).

## 2. Sets, Relations, and Groups

- Sets, subsets, algebra of sets, and properties of sets are all covered in MYP.
- Relations and functions covered in HL Core Chapter 2. Students should have completed this by now.
- Some questions later (for example, Section G) use  $i = \sqrt{-1}$ , which students should have seen by now in HL Core Chapter 6.
- In Exercise J, there is a question involving  $f(x) = \ln x$ , which students should have seen already in HL Core Chapter 4.

## 3. Discrete Mathematics

- Need to have studied induction from HL Core Chapter 9.
- The recurrence relations section covers sequences which students should have seen in HL Core Chapter 7.
- Under 2<sup>nd</sup> degree linear homogenous recurrence relations with constant coefficients, the solution to the characteristic equation may be complex, so we need  $i$  and complex conjugates.  $\text{cis}$  is mentioned, and this is not covered in HL Core until chapter 16, so students may not be that far advanced, but the use of  $\text{cis}$  and polar form here is not essential.
- In Section F.1, there is a reference to the Sets, Relations, and Groups Option.

## 4. Geometry

- With earlier deductive geometry, there are no issues through to equations of tangents and normals of conic sections. This section could be done using quadratic theory, but in the following section, the syllabus explicitly refers to parametric differentiation. So, this needs to be done after the HL Core calculus anyway. We therefore choose to do tangents and normal using implicit differentiation, with dual purpose to practice this skill learnt in HL Core.
- The last section relies on eigenvalues and eigenvectors from Linear Algebra.

## 5. Calculus

- Relies extensively on functions, including trigonometric functions, exponentials, and logarithms. The work requires HL Core calculus, so it should be started after completing HL Core Chapter 22.

## 6. Statistics and Probability

- This Option must be chosen for the HL course. It flows directly from the last chapter of the HL Core book.