TOPIC 6: MENSURATION

UNITS

You should be familiar with the following units:

|	
Length:	mm, cm, m, km
Area:	mm^2, cm^2, m^2, ha, km^2
Volume:	mm^3, cm^3, m^3
Capacity:	ml, cl, l
Mass:	g, kg, t

To convert from **smaller** to **larger** units we **divide** by the conversion factor.

To convert from **larger** to **smaller** units we **multiply** by the conversion factor.
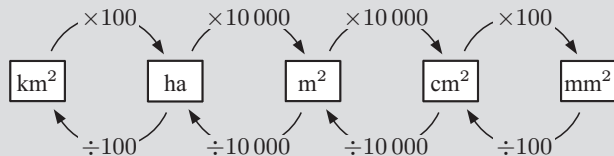
LENGTH

The **perimeter** of a figure is the measurement of the distance around its boundary.

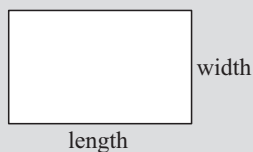For a **polygon** the perimeter is the sum of the lengths of all sides.
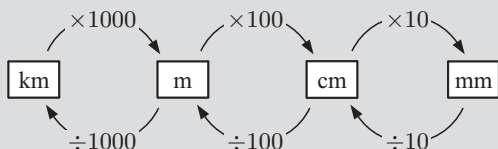
AREA

The **area** of a figure is the amount of surface within its boundaries.
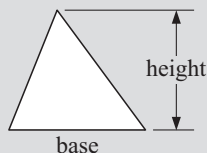
You should be able to use these formulae for area:
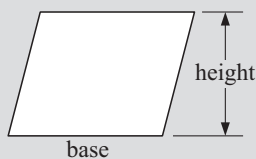
Rectangles

Area = length × width

Triangles

Area = $\frac{1}{2}$ (base × height)

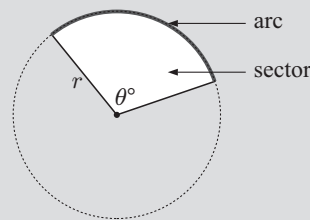**Parallelograms**

Area = base × height

Trapezia

Area = $\frac{1}{2}(a + b) \times h$
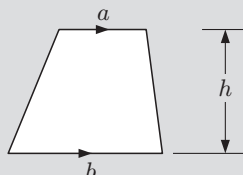
Circles and sectors

An **arc** is any continuous part of the circle. The length of an arc is called its **arclength**.

Every arc has a corresponding **sector**, which is the portion of the circle subtended by the same angle $\theta°$ as the arc.

For a circle: Circumference $C = \pi d = 2\pi r$
                Area $A = \pi r^2$

For a sector of angle $\theta°$:

   Arclength $s = \left(\frac{\theta}{360}\right) \times 2\pi r$
   Area $A = \left(\frac{\theta}{360}\right) \times \pi r^2$

SURFACE AREA
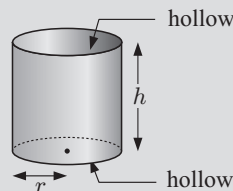
Solids with plane faces

The **surface area** of a three dimensional figure with plane faces is the sum of the areas of the faces.

To assist in your calculations, you can draw a **net** of the solid, correctly labelling the dimensions.
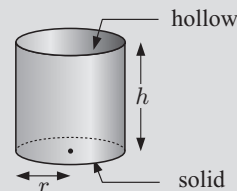
Solids with curved surfaces

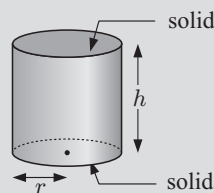You should be able to use these formulae for surface area:
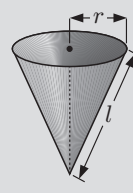
Hollow cylinder

$A = 2\pi rh$
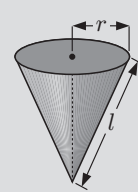
Hollow can

$A = 2\pi rh + \pi r^2$

Solid cylinder

$A = 2\pi rh + 2\pi r^2$
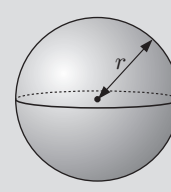
Hollow cone

$A = \pi rl$
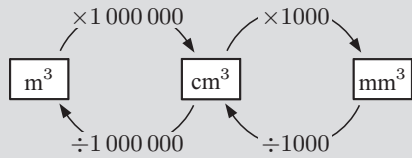
Solid cone

$A = \pi rl + \pi r^2$
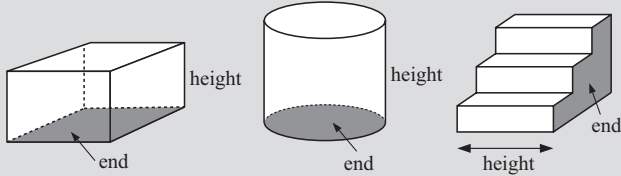
Sphere

$A = 4\pi r^2$

VOLUME

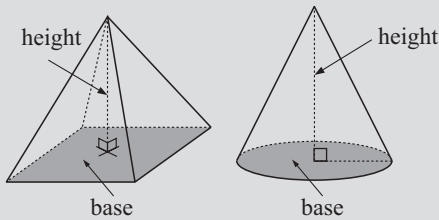The **volume** of a solid is the amount of space it occupies.

You should be able to use these formulae for volume:
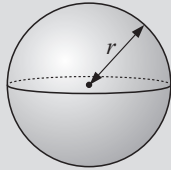
Solids of uniform cross-section

Volume of uniform solid = area of end × height

Pyramids and cones

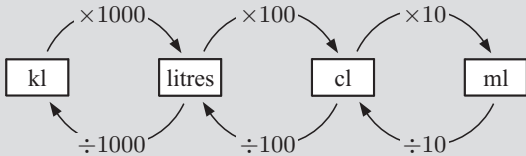Volume of a pyramid or cone = $\frac{1}{3}$(area of base × height)

Spheres

Volume of a sphere = $\frac{4}{3}\pi r^3$

You can find the volumes of compound solids by separating the solid into sections like those above.

CAPACITY

The **capacity** of a container is the quantity of fluid or gas required to fill it.
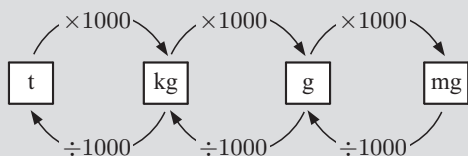

Connecting volume to capacity

$$1\,\text{ml} \equiv 1\,\text{cm}^3$$
$$1\,\text{litre} \equiv 1000\,\text{cm}^3$$
$$1\,\text{kl} = 1000\,\text{litres} \equiv 1\,\text{m}^3$$

MASS

The **mass** of an object is the amount of matter in it.

SKILL PRACTICE
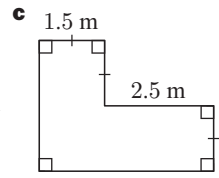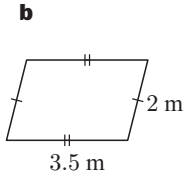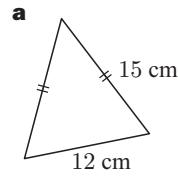
1 Convert:
 a 72 mm to cm
 b 5.8 m to mm
 c 9.75 km to m
 d 28 000 000 cm to km.

2 Kevin counts the light poles on the footpath as he walks to school. Kevin walks 2.4 km, and counts 80 light poles. How far is it between each light pole?

3 Find the perimeter of:

4 Convert:
 a 44 mm^2 to cm^2
 b 0.059 ha to cm^2
 c 21.85 ha to km^2
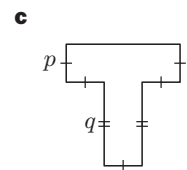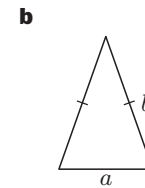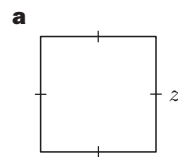 **d** 0.000 006 2 km^2 to mm^2
 e 360 m^2 to cm^2
 f 39 500 m^2 to ha.

5 A rectangle is 3.2 m by 2.4 m and has the same perimeter as a square. Find the length of the sides of the square.

6 The base area of a box of stickers is 85 cm^2. How many of these boxes will fit in one layer of a pallet of area 1.36 m^2?
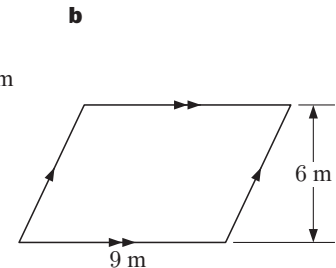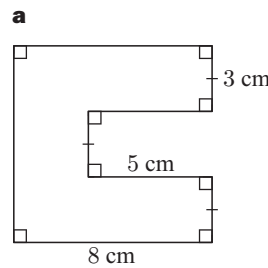
7 Find a formula for the perimeter P of:

8 A circle has area 36.4 m^2. Find:
 a its radius
 b its circumference.

9 Find the area of the following:

10 Find the surface area of:
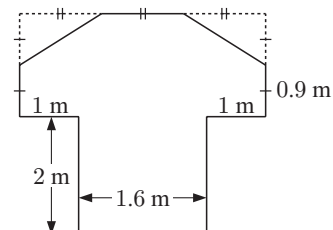 **a** a cube with sides 16 cm
 b a cuboid 36 mm × 48 mm × 21 mm.
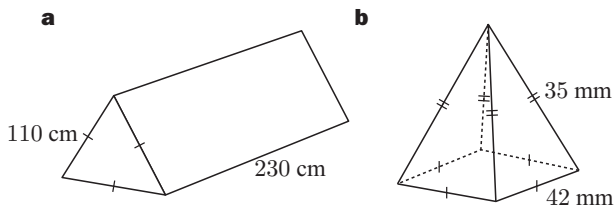
11 Convert:
 a 3.71 litres into cl
 b 58 215 ml into litres.

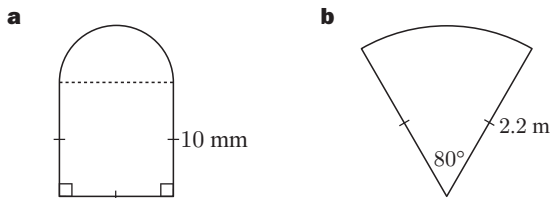**12** Calculate the length of guard rail needed to construct a safety fence for the following viewing platform:

13 Find the area of a kite whose diagonals have lengths 40 cm and 70 cm.

14 Find the surface area of:

a

110 cm
230 cm

b

35 mm
42 mm

15 Find the perimeter and area of the following figures:

a

10 mm

b

2.2 m
80°

16 Convert the following:

a $7.25 \, \text{m}^3$ to cm^3

b $2\,900\,000\,000 \, \text{mm}^3$ to m^3
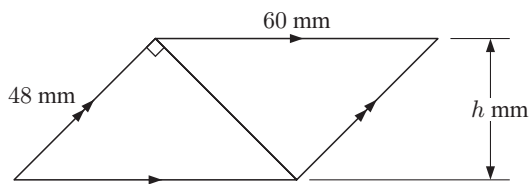
c $2500 \, \text{cm}^3$ to mm^3.

17 A chef uses 75 ml of milk in each serve of mashed potatoes. He makes an average of 235 serves each week. How many litres of milk does he use?

18 Adrian's new garage has the dimensions shown. Find the surface area of sheet metal required for the walls and roof.
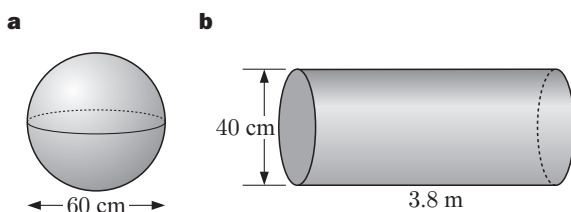
2.5 m
2 m
3.5 m
6 m
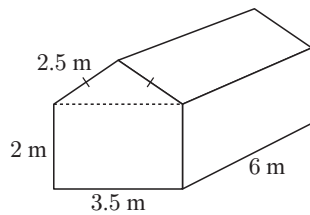
19 A sector has radius 4 cm and angle 250°. Find its area.

20

60 mm
48 mm
h mm

a Find the area of the parallelogram. **b** Find h.

21 Find the surface area of these solids:

a

60 cm

b

40 cm
3.8 m

22 A television cabinet has the dimensions shown. Find:
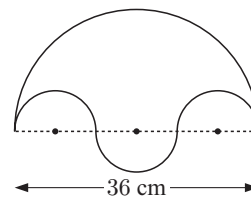
50 cm
42 cm
55 cm
110 cm
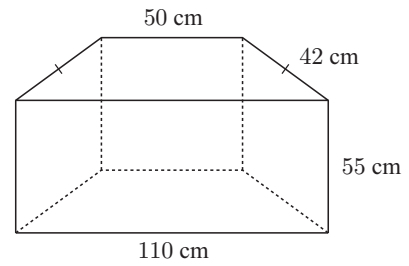
a the area of its top

b the total surface area of its four sides.

23 $40 \, \text{mm}^3$ of copper is required to make a single resistor. How many resistors can be made with $1000 \, \text{cm}^3$ of copper?
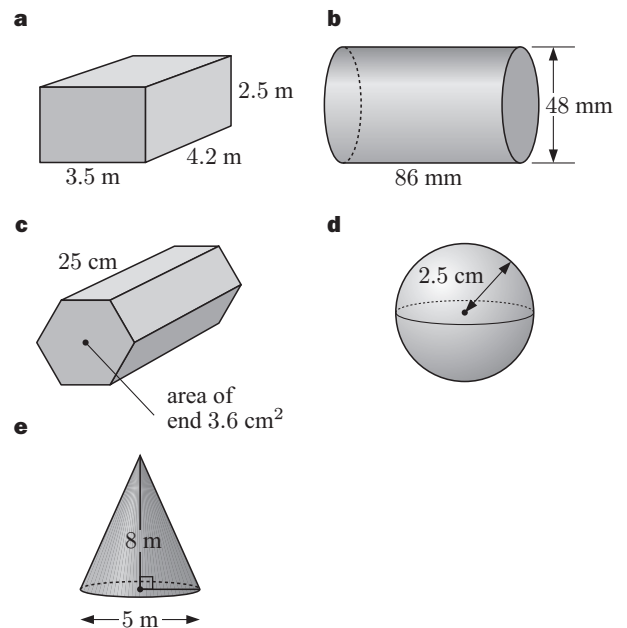
24 The engine of a 500 cc motorbike holds $500 \, \text{cm}^3$ of fuel-air mixture. Express this quantity in litres.

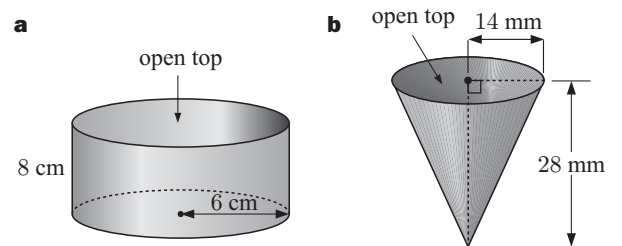**25** Find the perimeter and area of this figure:

36 cm

26 When full, a blow-up beach ball has diameter 36 cm. Find the surface area of rubber needed to make 200 of these balls.

27 Find the volume of the following:

a

2.5 m
3.5 m
4.2 m

b

48 mm
86 mm

c

25 cm
area of end 3.6 cm²

d

2.5 cm

e

8 m
5 m

28 Find the outer surface area of:

a

open top
8 cm
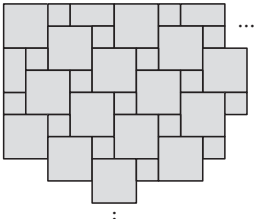6 cm

b

open top 14 mm
28 mm

29 A circular pie with radius 8.5 cm is served on a square plate with sides 21 cm long. What proportion of the area of the plate does the pie cover?

30 A cylindrical drinking flask has radius 3.42 cm and height 16.33 cm. Find its capacity.

31 How many cylindrical cookies with diameter 5 cm and thickness 1 cm could be made from a rectangular block of dough 20 cm × 15 cm × 8 cm?

**32**

Three sizes of tile are used to form the 3.25 m × 2.25 m floor of a bathroom using the pattern shown. The large tiles are 10 cm × 10 cm. What proportion of the area is covered by the smallest tiles?

33 Pauline has a wooden block with the dimensions shown. She paints a 1 cm wide border around the edge of every face. Find:
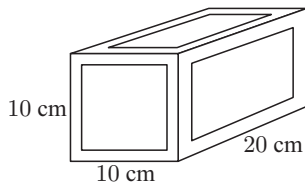
a the total surface area of the block

b the painted area   **c** the unpainted area.

34 Find a formula for the area A of the following regions:

a

b   **c**

d
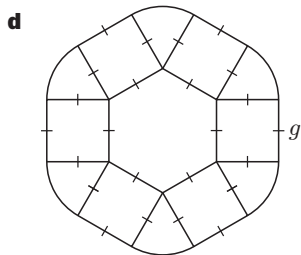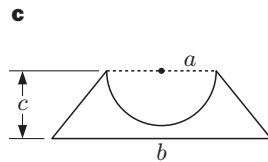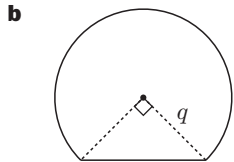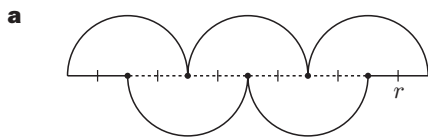
35 A solid cone has diameter 15 mm and slant height 34 mm. Find its surface area.

**36** Emma has just bought 60 timber posts to help build a fence. Each post is a cylinder 1.8 m long with diameter 16 cm. The total mass of Emma's posts is 1.08 tonnes.

Find: **a** the mass of each post in kilograms

b the volume of each post in m^3

**37** Find a formula for the surface area A of the following solids:

a   **b**
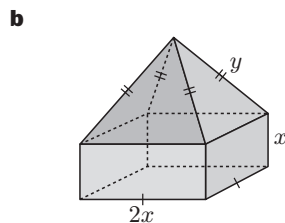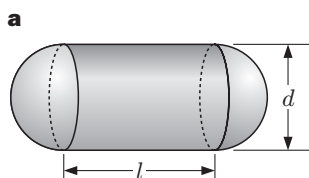
38 Eliza has a bucket with the dimensions shown. She fills it with water, but there is a hole in the bucket, so the water drips out at a rate of 1.2 ml/min. How much water remains in the bucket when Eliza returns 3 hours later?
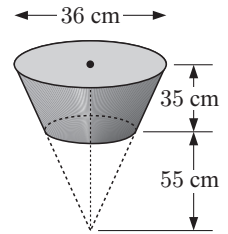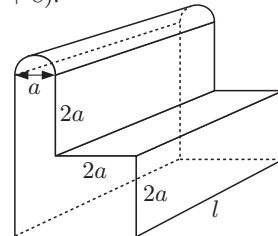
**39** Find formulae for the volume V of the following objects:

a   **b**

c
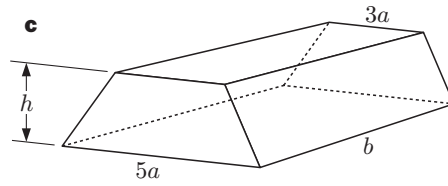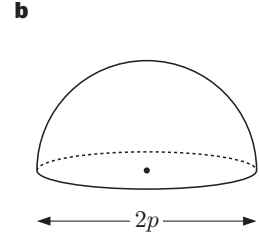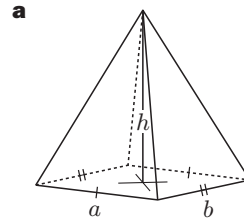
40 Des buys a 500 g wedge of his favourite cheese. The wedge is a right angle and is 6.1 cm high. Its volume is 460 cm^3.
Find the radius of the wedge.
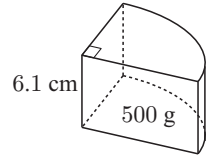
41 A concrete bench for a bus stop is made with the dimensions shown. Show that the volume of concrete used is given by the formula $V = a^2 l(\frac{\pi}{8} + 8)$.
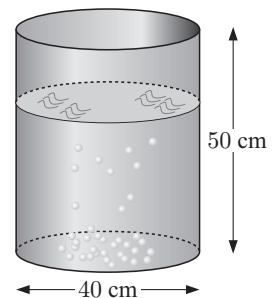
42 A metal door handle is formed from three cylindrical pieces. The handles are 4 cm deep and have radius 3 cm. The shaft in the middle has length 12 cm and radius 1.5 cm.
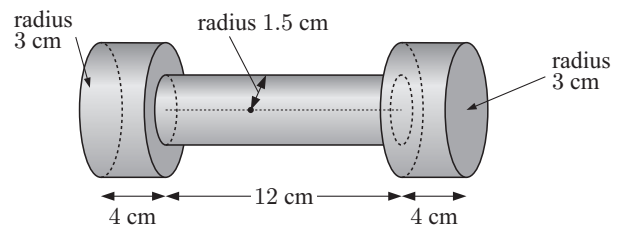
Find the total volume of the door handle.

43 **a** 55 litres of water is added to the cylindrical aquarium shown.
How far from the top does the water rise?

b Glass marbles of diameter 12 mm are carefully added to the aquarium. How many marbles can be added without causing the water to overflow?