b **i** $\overrightarrow{PR} = \overrightarrow{PB} + \overrightarrow{BC} + \overrightarrow{CR}$
$= \frac{1}{2}\mathbf{a} + \mathbf{b} - \frac{1}{2}\mathbf{a}$
$= \mathbf{b}$
$\therefore\ |\overrightarrow{PR}| = |\mathbf{b}|$
$= |\mathbf{a}|$ {ABCD rhombus $\therefore\ |\mathbf{a}| = |\mathbf{b}|$ }

ii $\overrightarrow{SQ} = \overrightarrow{SD} + \overrightarrow{DC} + \overrightarrow{CQ}$
$= \frac{1}{2}\mathbf{b} + \mathbf{a} - \frac{1}{2}\mathbf{b}$
$= \mathbf{a}$
$\therefore\ |\overrightarrow{SQ}| = |\mathbf{a}|$

c From **a** we have $\overrightarrow{PS} = \overrightarrow{QR}$ and $\overrightarrow{PQ} = \overrightarrow{SR}$
$\therefore$ we deduce PQRS is a parallelogram.
Also, from **b**, the diagonals are equal in length.
$\therefore$ PQRS is a rectangle.

44 **a** T_5 maps triangle 0 to triangle 5
T_7 maps triangle 5 to triangle 2
$\therefore$ T_5 then T_7 is equivalent to T_2.

b T_1 maps triangle 0 to triangle 1
T_4 maps triangle 1 to triangle 5
$\therefore$ T_1 then T_4 is equivalent to T_5.

c T_6 maps triangle 0 to triangle 6
T_6 maps triangle 6 to triangle 4
$\therefore$ T_6 then T_6 is equivalent to T_4.

d T_3 maps triangle 0 to triangle 3
T_2 maps triangle 3 to triangle 5
$\therefore$ T_3 then T_2 is equivalent to T_5.

SOLUTIONS TO TOPIC 6: MENSURATION

1 **a** 72 mm
$= (72 \div 10)$ cm
$= 7.2$ cm

b 5.8 m
$= (5.8 \times 100)$ cm
$= (5.8 \times 100 \times 10)$ mm
$= 5800$ mm

c 9.75 km
$= (9.75 \times 1000)$ m
$= 9750$ m

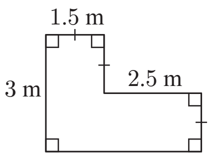
d 28 000 000 cm
$= (28\,000\,000 \div 100)$ m
$= (28\,000\,000 \div 100 \div 1000)$ km
$= 280$ km

2 Distance between light poles $= \dfrac{2.4\text{ km}}{80} = \dfrac{2400\text{ m}}{80} = 30$ m

3 **a** Perimeter
$= 2 \times 15 + 12$
$= 42$ cm

b Perimeter
$= 2 \times 3.5 + 2 \times 2$
$= 11$ m

c Perimeter
$= 3 \times 1.5 + 2.5 + 4 + 3$
$= 14$ m



4 **a** 44 mm^2
$= (44 \div 100)\text{ cm}^2$
$= 0.44\text{ cm}^2$

b 0.059 ha
$= (0.059 \times 10\,000)\text{ m}^2$
$= (0.059 \times 10\,000 \times 10\,000)\text{ cm}^2$
$= 5\,900\,000\text{ cm}^2$

c 21.85 ha
$= (21.85 \div 100)\text{ km}^2$
$= 0.2185\text{ km}^2$

d $0.000\,006\,2\text{ km}^2$
$= (0.000\,006\,2 \times 1\,000\,000)\text{ m}^2$
$= (0.000\,006\,2 \times 1\,000\,000 \times 1\,000\,000)\text{ mm}^2$
$= 6\,200\,000\text{ mm}^2$

e 360 m^2
$= (360 \times 10\,000)\text{ cm}^2$
$= 3\,600\,000\text{ cm}^2$

f $39\,500\text{ m}^2$
$= (39\,500 \div 10\,000)$ ha
$= 3.95$ ha

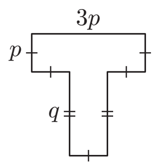
5 The rectangle has perimeter $= 2 \times 3.2 + 2 \times 2.4$
$= 11.2$ m
$\therefore$ the perimeter of the square is also 11.2 m, and hence the length of its sides is $\dfrac{11.2\text{ m}}{4} = 2.8$ m.

6 $1.36\text{ m}^2 = 13\,600\text{ cm}^2$
$\therefore$ the number of boxes on the pallet $= \dfrac{13\,600}{85} = 160$

7 **a** $P = 4z$

b $P = a + 2b$

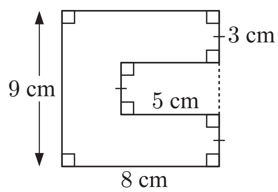
c $P = 3p + 5 \times p + 2 \times q$
$= 8p + 2q$



8 **a** $\pi r^2 = 36.4$
$\therefore\ r^2 = \dfrac{36.4}{\pi}$
$\therefore\ r = \sqrt{\dfrac{36.4}{\pi}}$ $\{r > 0\}$
$\therefore\ r \approx 3.4039$
The radius of the circle is 3.40 m.

b $C = 2\pi r$
$\approx 2\pi \times 3.4039$
≈ 21.4
The circumference of the circle is 21.4 m.

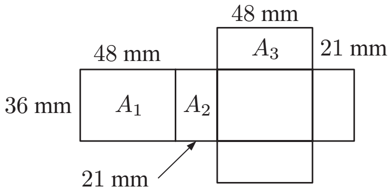
9 **a** Area $= 9 \times 8 - 3 \times 5$
$= 57\text{ cm}^2$



b Area = base × height
$= 9 \times 6$
$= 54\text{ m}^2$

10 **a** The cube has 6 identical faces, each with area $= 16 \times 16 = 256\text{ cm}^2$.
$\therefore$ the surface area of the cube $= 6 \times 256 = 1536\text{ cm}^2$.

b The net of the prism is:



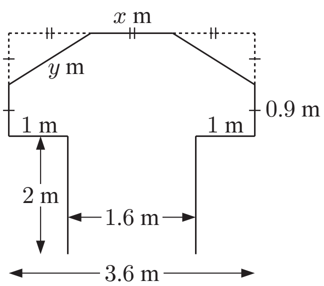
$A_1 = 36 \times 48 = 1728\,\text{mm}^2$ {bottom and top}
$A_2 = 36 \times 21 = 756\,\text{mm}^2$ {sides}
$A_3 = 48 \times 21 = 1008\,\text{mm}^2$ {front and back}

$\therefore$ total surface area $= 2 \times A_1 + 2 \times A_2 + 2 \times A_3$
$= 2 \times 1728 + 2 \times 756 + 2 \times 1008$
$= 6984\,\text{mm}^2$

11 **a** 3.71 litres
$= (3.71 \times 100)$ cl
$= 371$ cl

b 58 215 ml
$= (58\,215 \div 1000)$ litres
$= 58.215$ litres

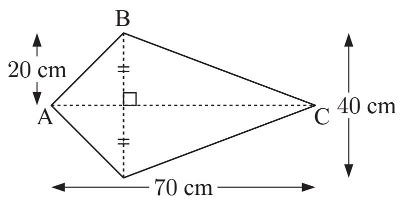
12



$x = \frac{3.6}{3} = 1.2$
$\therefore\ y^2 = 1.2^2 + 0.9^2$ {Pythagoras}
$\therefore\ y^2 = 2.25$
$\therefore\ y = 1.5$ $\{y > 0\}$

$\therefore$ guard rail length
$= 1.2 + 2 \times 1.5 + 2 \times 0.9 + 2 \times 1 + 2 \times 2$
$= 12$ m

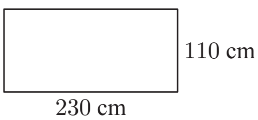
13



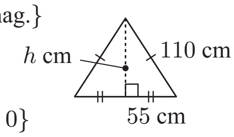
area of kite $= 2 \times$ (area of $\triangle$ABC)
$= 2\left(\frac{1}{2} \times 70 \times 20\right)$
$= 1400\,\text{cm}^2$

14 **a** The figure has:

- 3 rectangular faces



- 2 triangular faces

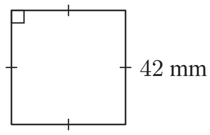


$h^2 + 55^2 = 110^2$ {Pythag.}
$\therefore\ h^2 + 3025 = 12\,100$
$\therefore\ h^2 = 9075$
$\therefore\ h \approx 95.3$ $\{h > 0\}$

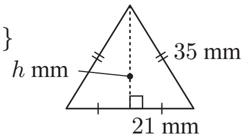
$\therefore$ total surface area
$\approx 3 \times (230 \times 110) + 2 \times \left(\frac{1}{2} \times 110 \times 95.3\right)$
$\approx 86\,400\,\text{cm}^2$

b The figure has:

- 1 square base



- 4 triangular faces



$h^2 + 21^2 = 35^2$ {Pythag.}
$\therefore\ h^2 + 441 = 1225$
$\therefore\ h^2 = 784$
$\therefore\ h = 28$ $\{h > 0\}$

$\therefore$ total surface area
$= 42 \times 42 + 4 \times \left(\frac{1}{2} \times 42 \times 28\right)$
$= 4116\,\text{mm}^2$

15 **a** Distance around semi-circle $= \frac{1}{2} \times \pi \times 10 \approx 15.7$ mm
$\therefore$ total perimeter $\approx 15.7 + 3 \times 10 \approx 45.7$ mm
Area $= 10 \times 10 + \frac{1}{2} \times \pi \times 5^2 \approx 139\,\text{mm}^2$

b Perimeter $= 2.2 + 2.2 +$ length of arc
$= 4.4 + \left(\frac{80}{360}\right) \times 2 \times \pi \times 2.2$
≈ 7.47 m

Area $= \left(\frac{80}{360}\right) \times \pi \times 2.2^2 \approx 3.38\,\text{m}^2$

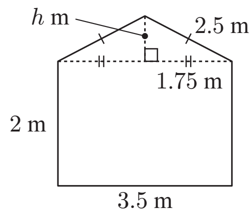
16 **a** $7.25\,\text{m}^3$
$= (7.25 \times 1\,000\,000)\,\text{cm}^3$
$= 7\,250\,000\,\text{cm}^3$

b $2\,900\,000\,000\,\text{mm}^3$
$= (2\,900\,000\,000 \div 1000)\,\text{cm}^3$
$= (2\,900\,000\,000 \div 1000 \div 1\,000\,000)\,\text{m}^3$
$= 2.9\,\text{m}^3$

c $2500\,\text{cm}^3$
$= (2500 \times 1000)\,\text{mm}^3$
$= 2\,500\,000\,\text{mm}^3$

17 Volume of milk used each week
$= 235 \times 75$ ml
$= 17\,625$ ml
$= 17.625$ l

18



$h^2 + 1.75^2 = 2.5^2$ {Pythagoras}
$\therefore\ h^2 + 3.0625 = 6.25$
$\therefore\ h^2 = 3.1875$
$\therefore\ h \approx 1.785$ $\{h > 0\}$

$\therefore$ area of end $\approx 2 \times 3.5 + \frac{1}{2} \times 3.5 \times 1.785$
$\approx 10.124\,\text{m}^2$

$\therefore$ total surface area
$\approx 2 \times 10.124 + 2 \times (6 \times 2) + 2 \times (6 \times 2.5)$
$\approx 74.2\,\text{m}^2$

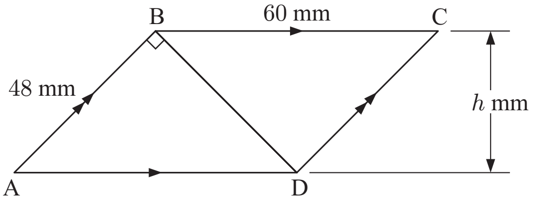
So, $74.2\,\text{m}^2$ of sheet metal is required.

19 Area $= \left(\frac{\theta}{360}\right) \times \pi r^2$

$= \frac{250}{360} \times \pi \times 4^2$

$\approx 34.9\,\text{cm}^2$

20



a ABCD is a parallelogram

$\therefore$ AD = 60 mm, DC = 48 mm.

$\therefore$ in $\triangle$ABD, $BD^2 + 48^2 = 60^2$ {Pythagoras}

$\therefore$ $BD = \sqrt{60^2 - 48^2}$ {BD > 0}

$= 36\,\text{mm}$

Now $B\widehat{D}C = A\widehat{B}D = 90°$ {alternate angles}

$\therefore$ area of parallelogram = area $\triangle$ABD + area $\triangle$BCD

$= \frac{1}{2} \times 48 \times 36 + \frac{1}{2} \times 48 \times 36$

$= 1728\,\text{mm}^2$

b area of parallelogram $= 1728\,\text{mm}^2$

$\therefore$ $60 \times h = 1728$

$h = 28.8$

21 **a** The sphere has radius 30 cm.

$\therefore$ surface area $= 4 \times \pi \times 30^2$

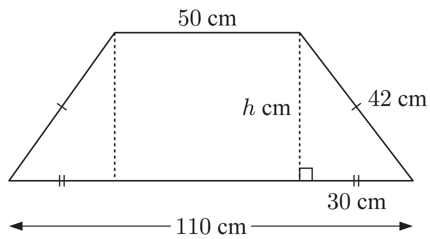
$\approx 11\,300\,\text{cm}^2$

b surface area $= 2\pi r^2 + 2\pi rh$

$= 2 \times \pi \times 20^2 + 2 \times \pi \times 20 \times 380$ {3.8 m = 380 cm}

$\approx 50\,300\,\text{cm}^2$

22 **a**



$h^2 + 30^2 = 42^2$ {Pythagoras}

$\therefore$ $h = \sqrt{42^2 - 30^2}$ {h > 0}

$\therefore$ $h \approx 29.39$

$\therefore$ area of top $\approx \left(\frac{50 + 110}{2}\right) \times 29.39$

$\approx 2350\,\text{cm}^2$

b area of four sides

$= (110 \times 55) + 2 \times (42 \times 55) + (50 \times 55)$

$= 13\,420\,\text{cm}^2$

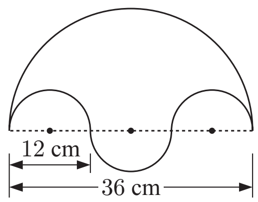
23 $1000\,\text{cm}^3 = 1\,000\,000\,\text{mm}^3$

$\therefore$ number of resistors $= \frac{1\,000\,000}{40} = 25\,000$

24 $500\,\text{cm}^3 \equiv 500\,\text{ml}$

$\equiv 0.5\,\text{l}$

25



Perimeter of figure

= (perimeter of large semi-circle) + 3 × (perimeter of small semi-circle)

$= \frac{1}{2} \times \pi \times 36 + 3 \times \left(\frac{1}{2} \times \pi \times 12\right)$

$\approx 113\,\text{cm}$

Area of figure

= (area of large semi-circle) − (area of 1 small semi-circle)

$= \frac{1}{2} \times \pi \times 18^2 - \frac{1}{2} \times \pi \times 6^2$

$\approx 452\,\text{cm}^2$

26 The beach balls have radius 18 cm.

$\therefore$ surface area of ball $= 4 \times \pi \times 18^2$

$= 1296\,\pi\,\text{cm}^2$

$\therefore$ surface area of 200 balls $= 200 \times 1296\pi$

$\approx 814\,000\,\text{cm}^2$

$\approx 81.4\,\text{m}^2$

So, $81.4\,\text{m}^2$ of rubber is needed.

27 **a** Volume

= length × width × depth

$= 3.5 \times 4.2 \times 2.5$

$\approx 36.8\,\text{m}^3$

b Volume

$= \pi r^2 \times h$

$= \pi \times 24^2 \times 86$

$\approx 156\,000\,\text{mm}^3$

c Volume

= area of end × height

$= 3.6 \times 25$

$= 90\,\text{cm}^3$

d Volume

$= \frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \pi \times 2.5^3$

$\approx 65.4\,\text{cm}^3$

e Volume

$= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \pi \times 2.5^2 \times 8$

$\approx 52.4\,\text{m}^3$

28 **a** Surface area $= 2\pi rh + \pi r^2$

$= 2 \times \pi \times 6 \times 8 + \pi \times 6^2$

$\approx 415\,\text{cm}^2$

b $l^2 = 14^2 + 28^2$ {Pythagoras}

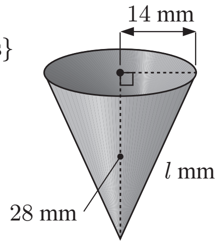
$\therefore$ $l = \sqrt{14^2 + 28^2}$ {l > 0}

$\therefore$ $l \approx 31.3$

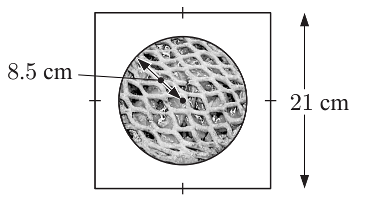
$\therefore$ surface area $= \pi rl$

$\approx \pi \times 14 \times 31.3$

$\approx 1380\,\text{mm}^2$



29



area of pie $= \pi \times 8.5^2 = 72.25\,\pi\,\text{cm}^2$

area of plate $= 21 \times 21 = 441\,\text{cm}^2$

Now $\frac{72.25\pi}{441} \approx 0.515 \approx 51.5\%$

So, the pie covers 51.5% of the plate.

30 Volume of flask $= \pi r^2 h$

$= \pi \times 3.42^2 \times 16.33$

$\approx 600\,\text{cm}^3$

$\therefore$ capacity of flask ≈ 600 ml

31 Volume of cookies $= \pi r^2 h$

$= \pi \times 2.5^2 \times 1$

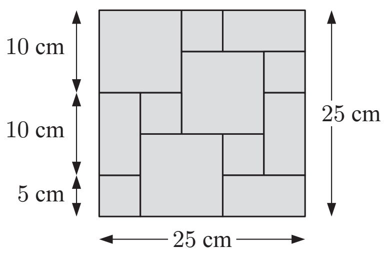
$= 6.25\,\pi\,\text{cm}^3$

Volume of dough $= 20 \times 15 \times 8$

$= 2400\,\text{cm}^3$

$\frac{2400}{6.25\pi} \approx 122$, so 122 cookies can be made from the block.

32 The pattern can be divided into 25 cm by 25 cm squares as shown:



The square has area $25 \times 25 = 625\,\text{cm}^2$.

The small tiles are 5 cm by 5 cm, and there are 5 of them in the square.

So, the small tiles occupy a total of $5 \times (5 \times 5) = 125\,\text{cm}^2$.

$\therefore$ the proportion of the area covered by the smallest tile is $\frac{125}{625} = 20\%$.

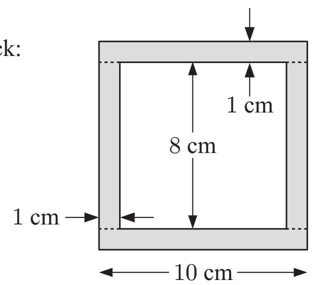
33 **a** Total surface area

$= 2 \times (10 \times 10) + 4 \times (10 \times 20)$

(front and back) (top, bottom and sides)

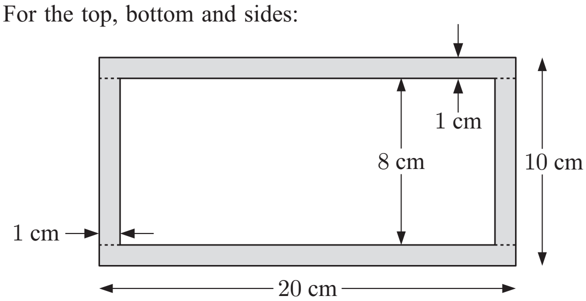
$= 1000\,\text{cm}^2$

b For the front and back:



painted area $= 2 \times (10 \times 1) + 2 \times (8 \times 1) = 36\ \text{cm}^2$

For the top, bottom and sides:



painted area $= 2 \times (20 \times 1) + 2 \times (8 \times 1) = 56\ \text{cm}^2$

$\therefore$ total painted area $= 2 \times 36 + 4 \times 56 = 296\ \text{cm}^2$

c Unpainted area $= 1000 - 296 = 704\ \text{cm}^2$

34 **a** $A =$ area of 5 semi-circles

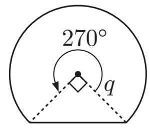
$= 5 \times \left(\frac{1}{2}\pi r^2\right)$

$= \frac{5}{2}\pi r^2$

b $A =$ sector area $+$ area of $\triangle$

$= \frac{270}{360} \times \pi q^2 + \frac{1}{2} \times q \times q$

$= q^2\left(\frac{3}{4}\pi + \frac{1}{2}\right)$

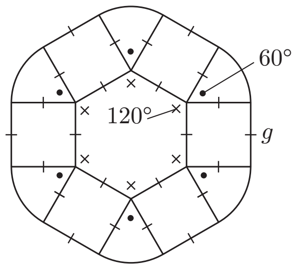


c $A =$ area of trapezium $-$ area of semi-circle

$= \left(\dfrac{2a+b}{2}\right) \times c - \frac{1}{2}\pi a^2$

$= \dfrac{(2a+b)c - \pi a^2}{2}$

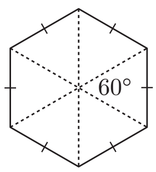
d



Area of each square $= g^2$

Area of each circle sector $= \frac{60}{360} \times \pi g^2$

The inner hexagon is made up of 6 equilateral triangles of length g.



$\therefore$ area of hexagon $= 6 \times \left(\frac{1}{2} \times g \times g \times \sin 60^\circ\right)$

$= 3g^2 \times \frac{\sqrt{3}}{2}$

$= \frac{3\sqrt{3}}{2}g^2$

$\therefore\ A = 6 \times g^2 + 6 \times \left(\frac{60}{360}\right)\pi g^2 + \frac{3\sqrt{3}}{2}g^2$

$= \left(6 + \pi + \frac{3\sqrt{3}}{2}\right)g^2$

35 Surface area $= \pi r l + \pi r^2$

$= \pi \times 7.5 \times 34 + \pi \times 7.5^2$

$\approx 978\ \text{mm}^2$

36 **a** Total mass $= 1.08$ tonnes $= 1080$ kg

$\therefore$ mass of each post $= \frac{1080}{60}$ kg $= 18$ kg

b Volume $= \pi r^2 h$

$= \pi \times 0.08^2 \times 1.8$ {8 cm = 0.08 m}

$\approx 0.0362\ \text{m}^3$

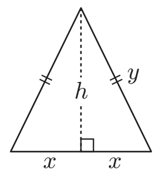
37 **a** $A =$ area of hollow cylinder $+$ area of sphere

$= 2\pi\left(\dfrac{d}{2}\right)l + 4\pi\left(\dfrac{d}{2}\right)^2$

$= \pi d l + \pi d^2$

$= \pi d(l + d)$

b In the triangular faces,



$h^2 + x^2 = y^2$

$\therefore\ h = \sqrt{y^2 - x^2}$ $\{h > 0\}$

$\therefore$ area of each triangular face

$= \frac{1}{2} \times 2x \times \sqrt{y^2 - x^2}$

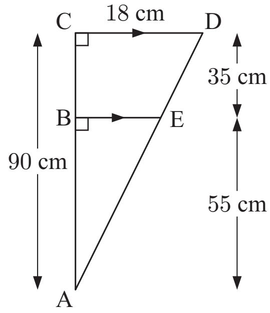
$= x\sqrt{y^2 - x^2}$

$\therefore\ A = \underbrace{4 \times x\sqrt{y^2 - x^2}}_{\text{triangular faces}} + \underbrace{4(2x \times x)}_{\text{sides}} + \underbrace{(2x \times 2x)}_{\text{base}}$

$= 4x\sqrt{y^2 - x^2} + 8x^2 + 4x^2$

$= 4x\left(\sqrt{y^2 - x^2} + 3x\right)$

38



$\widehat{AEB} = \widehat{ADC}$ {corresponding angles}

$\widehat{ABE} = \widehat{ACD} = 90°$

$\therefore$ △s ABE and ACD are similar, and

$\frac{BE}{CD} = \frac{AB}{AC}$ {same ratio}

$\therefore \frac{BE}{18} = \frac{55}{90}$

$\therefore$ BE = 11

$\therefore$ volume of bucket

= volume of large cone − volume of small cone

$= \frac{1}{3}\pi(18)^2 \times 90 - \frac{1}{3}\pi(11)^2 \times 55$

$\approx 23\,567\text{ cm}^3$

$\therefore$ the bucket has capacity 23 567 ml.

In 3 hours or 180 minutes, the bucket loses $180 \times 1.2 = 216$ ml of water.

$\therefore$ the amount of water remaining $\approx (23\,567 - 216)$ ml

$\approx 23\,351$ ml

≈ 23.4 litres

39 **a** $V = \frac{1}{3} \times$ area of base $\times$ height

$= \frac{1}{3}abh$

b V = volume of hemisphere

$= \frac{1}{2} \times \left(\frac{4}{3}\pi p^3\right)$

$= \frac{2}{3}\pi p^3$

c V = area of trapezium $\times$ length

$= \left(\frac{3a+5a}{2} \times h\right) \times b$

$= 4abh$

40 Let the radius of the wedge be r cm.

Now volume = 460 cm^3

$\therefore \frac{1}{4}\pi r^2 \times 6.1 = 460$

$\therefore r^2 = \frac{1840}{6.1\pi}$

$\therefore r = \sqrt{\frac{1840}{6.1\pi}}$ $\{r > 0\}$

$\therefore r \approx 9.80$

$\therefore$ the radius of the wedge is 9.80 cm.

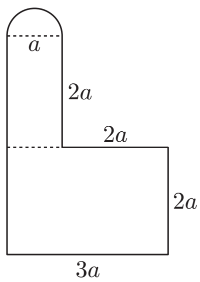
41 Area of end $= \frac{1}{2} \times \pi \left(\frac{a}{2}\right)^2 + 2a \times a + 2a \times 3a$

$= \frac{\pi a^2}{8} + 2a^2 + 6a^2$

$= a^2\left(\frac{\pi}{8} + 8\right)$

$\therefore V = a^2\left(\frac{\pi}{8} + 8\right) \times l$

$= a^2 l\left(\frac{\pi}{8} + 8\right)$



42 Volume of each handle $= \pi \times 3^2 \times 4$

$= 36\pi\text{ cm}^3$

Volume of shaft $= \pi \times 1.5^2 \times 12$

$= 27\pi\text{ cm}^3$

$\therefore$ total volume of door handle $= 2 \times 36\pi \times 27\pi$

$= 99\pi$

$\approx 311\text{ cm}^3$

43 **a** 55 litres = 55 000 ml

$\therefore$ the water has volume 55 000 cm^3.

Suppose the water rises to a height of h cm.

$\therefore \pi \times 20^2 \times h = 55\,000$

$\therefore h = \frac{55\,000}{400\pi}$

$\therefore h \approx 43.77$

$\therefore$ the water is $50 - 43.77 = 6.23$ cm from the top.

b Volume of space remaining in aquarium

$\approx \pi \times 20^2 \times 6.23$

$\approx 7831.853\text{ cm}^3$

$\approx 7\,831\,853\text{ mm}^3$

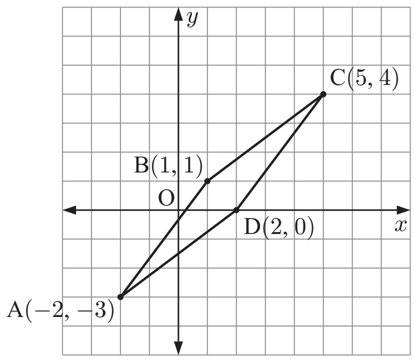
Volume of each marble $= \frac{4}{3}\pi \times 6^3$

$\approx 904.8\text{ mm}^3$

So, $\frac{7\,831\,853}{904.8} \approx 8660$ marbles can be added before the aquarium overflows.

SOLUTIONS TO TOPIC 7: COORDINATE GEOMETRY

1 **a**



b $\overrightarrow{AB} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$, and $AB = \sqrt{3^2 + 4^2} = 5$ units

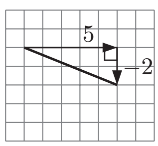
$\overrightarrow{BC} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$, and $BC = \sqrt{4^2 + 3^2} = 5$ units

$\overrightarrow{CD} = \begin{pmatrix} -3 \\ -4 \end{pmatrix}$, and $CD = \sqrt{(-3)^2 + (-4)^2} = 5$ units

$\overrightarrow{DA} = \begin{pmatrix} -4 \\ -3 \end{pmatrix}$, and $DA = \sqrt{(-4)^2 + (-3)^2} = 5$ units

All four sides are equal in length, so the points form a rhombus.

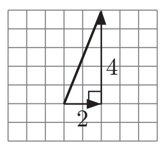
2 **a**



gradient $= \frac{-2}{5}$

$= -\frac{2}{5}$

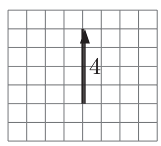
b



gradient $= \frac{4}{2}$

$= 2$

c



gradient $= \frac{4}{0}$ which is undefined