Chapter 8

Coordinate geometry

Syllabus reference: 5.1, 5.2

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OPENING PROBLEM

A city has two hospitals:
Ridgehaven located at $R(6, -9)$,
and Sunport located at $S(-8, 5)$.

Things to think about:

- **a** Trish lives at $T(4, 4)$. Which hospital is Trish closest to?
- **b** Can you find the point midway between the hospitals?
- **c** The city’s planning council wants to define a ‘boundary line’ so that people will go to the hospital closest to them. Can you find the equation of this boundary line?

HISTORICAL NOTE

History now shows that the two Frenchmen René Descartes and Pierre de Fermat arrived at the idea of analytical geometry at about the same time. Descartes’ work *La Geometrie* was published first, in 1637, while Fermat’s *Introduction to Loci* was not published until after his death.

Today, they are considered the co-founders of this important branch of mathematics, which links algebra and geometry.

The initial approaches used by these mathematicians were quite opposite. Descartes began with a line or curve and then found the equation which described it. Fermat, to a large extent, started with an equation and investigated the shape of the curve it described. This interaction between algebra and geometry shows the power of analytical geometry as a branch of mathematics.

Analytical geometry and its use of coordinates provided the mathematical tools which enabled Isaac Newton to later develop another important branch of mathematics called calculus. Newton humbly stated: "If I have seen further than Descartes, it is because I have stood on the shoulders of giants."
THE NUMBER PLANE

The position of any point in the number plane can be specified in terms of an ordered pair of numbers \((x, y)\), where:

- \(x\) is the horizontal step from a fixed point or origin \(O\), and
- \(y\) is the vertical step from \(O\).

Once the origin \(O\) has been given, two perpendicular axes are drawn. The \(x\)-axis is horizontal and the \(y\)-axis is vertical. The axes divide the number plane into four quadrants.

The number plane is also known as either:

- the 2-dimensional plane, or
- the Cartesian plane, named after René Descartes.

In the diagram, the point \(P\) is at \((a, b)\).

- \(a\) and \(b\) are referred to as the coordinates of \(P\).
- \(a\) is called the \(x\)-coordinate.
- \(b\) is called the \(y\)-coordinate.

Examples: The coordinates of the given points are

- \(A(4, 2)\)
- \(B(0, 2)\)
- \(C(-3, 1)\)
- \(D(-1, 0)\)
- \(E(1, -2)\).

DISTANCE BETWEEN TWO POINTS

Suppose we have the points \(A(1, 2)\) and \(B(5, 4)\), and we want to find the distance \(d\) between \(A\) and \(B\).

By drawing line segments \(AC\) and \(BC\) along the grid lines, we form a right angled triangle with hypotenuse \(AB\).

Now, using Pythagoras’ theorem,

\[
d^2 = 4^2 + 2^2
\]

\[
\therefore d^2 = 20
\]

\[
\therefore d = \sqrt{20} \quad \{\text{as } d > 0\}
\]

So, the distance between \(A\) and \(B\) is \(\sqrt{20}\) units.
EXERCISE 8A.1

1 Use Pythagoras’ theorem where appropriate to find the distance between:
   a) A and B  
   b) D and E  
   c) A and E  
   d) F and H  
   e) D and F  
   f) B and F  
   g) C and D  
   h) H and B.

2 Jack lives at J(2, 1). His friends Kevin, Ling, and Martin live at K(5, −7), L(8, 7), and M(−5, −4) respectively. Which of Jack’s friends lives closest to him?

THE DISTANCE FORMULA

You will have noticed it takes quite a lot of time to draw a diagram and then apply Pythagoras’ theorem.

To make the process quicker, we can develop a formula.

To go from A$(x_1, y_1)$ to B$(x_2, y_2)$, we find the

\[ x\text{-step} = x_2 - x_1 \]

and \[ y\text{-step} = y_2 - y_1. \]

As before,

\[ (AB)^2 = (x\text{-step})^2 + (y\text{-step})^2 \quad \text{Pythagoras} \]

\[ AB = \sqrt{(x\text{-step})^2 + (y\text{-step})^2} \]

\[ d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}. \]

The distance between two points A$(x_1, y_1)$ and B$(x_2, y_2)$ is given by

\[ AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}. \]
The distance formula saves us having to graph the points each time we want to find a distance.

Find the distance between A(6, 3) and B(8, -2).

\[
A(6, 3) \quad B(8, -2) \quad AB = \sqrt{(8 - 6)^2 + (-2 - 3)^2} \\
= \sqrt{2^2 + (-5)^2} \\
= \sqrt{4 + 25} \\
= \sqrt{29} \text{ units}
\]

**EXERCISE 8A.2**

1 Find the distance from:
   - a A(2, 6) to B(3, 3)
   - b C(-2, 3) to D(1, 5)
   - c M(2, 4) to N(-1, -3)
   - d O(0, 0) to P(-2, 4)
   - e R(3, -2) to S(5, -2)
   - f T(0, 3) to U(2, -1)
   - g W(-4, 0) to X(0, 3)
   - h Y(-1, -4) to Z(-3, 3).

2 In the map alongside, each grid unit represents 1 km.
Find the distance between:
   - a the lighthouse and the tree
   - b the jetty and the lighthouse
   - c the well and the tree
   - d the lighthouse and the well.

**Example 2**

The points A(2, -1), B(5, 1) and C(0, 2) form a triangle ABC.

a Use the distance formula to classify the triangle as equilateral, isosceles, or scalene.

\[
AB = \sqrt{(5 - 2)^2 + (1 - 1)^2} \\
= \sqrt{3^2 + 0^2} \\
= \sqrt{9} \text{ units}
\]

\[
AC = \sqrt{(0 - 2)^2 + (2 - (-1))^2} \\
= \sqrt{(-2)^2 + 3^2} \\
= \sqrt{13} \text{ units}
\]

\[
BC = \sqrt{(0 - 5)^2 + (2 - 1)^2} \\
= \sqrt{(-5)^2 + 1^2} \\
= \sqrt{26} \text{ units}
\]

Since \(AB = AC\) but not \(BC\), triangle ABC is isosceles.
\[ AB^2 + AC^2 = 13 + 13 = 26 = BC^2 \]

So, using the converse of Pythagoras’ theorem, triangle ABC is right angled. The right angle is opposite the longest side, so the right angle is at A.

3 Use the distance formula to classify triangle ABC as either equilateral, isosceles or scalene.

\[ a \quad A(-1, 0), B(-2, 3), C(-5, 4) \]
\[ b \quad A(-2, -4), B(1, 4), C(2, -3) \]
\[ c \quad A(0, 1), B(0, -1), C(\sqrt{3}, 0) \]
\[ d \quad A(0, -4), B(\sqrt{3}, 1), C(3\sqrt{3}, -5) \]

4 Use the distance formula to see if the following triangles are right angled. If they are, state the vertex where the right angle is.

\[ a \quad A(1, -1), B(-1, 2), C(7, 3) \]
\[ b \quad A(-1, 2), B(3, 4), C(5, 0) \]
\[ c \quad A(-2, 3), B(-5, 4), C(1, 2) \]
\[ d \quad A(5, 4), B(-4, 6), C(-3, 2) \]

5 Fully classify the triangles formed by the following points:

\[ a \quad A(-4, 5), B(3, 4), C(8, -1) \]
\[ b \quad A(-2, -5), B(-2, 2), C(-4, -1) \]
\[ c \quad A(-2, 1), B(-3, 4), C(1, 2) \]
\[ d \quad A(\sqrt{3}, -1), B(0, 2), C(-\sqrt{3}, -1) \]

### Example 3

Find \( q \) given that \( P(-2, 4) \) and \( Q(-1, q) \) are \( \sqrt{10} \) units apart.

From \( P \) to \( Q \), the \( x \)-step is \(-1 - (-2) = 1 \)
and the \( y \)-step is \( q - 4 \)

\[ \therefore \sqrt{1^2 + (q - 4)^2} = \sqrt{10} \]
\[ \therefore 1 + (q - 4)^2 = 10 \quad \{\text{squaring both sides}\} \]
\[ \therefore (q - 4)^2 = 9 \quad \{\text{subtracting 1 from both sides}\} \]
\[ \therefore q - 4 = \pm 3 \quad \{\text{if} \ X^2 = k \ \text{then} \ X = \pm \sqrt{k}\} \]
\[ \therefore q = 4 \pm 3 \]
So, \( q = 1 \) or \( 7 \).

6 Find \( q \) given that:

\[ a \quad P(2, 1) \text{ and } Q(q, -3) \text{ are 5 units apart} \]
\[ b \quad P(q, 6) \text{ and } Q(-2, 1) \text{ are } \sqrt{29} \text{ units apart} \]
\[ c \quad P(q, q) \text{ is } \sqrt{3} \text{ units from the origin} \]
\[ d \quad Q(3, q) \text{ is equidistant from } A(-1, 5) \text{ and } B(6, 4) \]
Classify the triangle formed by the points \(A(a, b)\), \(B(a, -b)\), and \(C(1, 0)\) as scalene, isosceles, or equilateral.

**The point M halfway between points A and B is called the midpoint of AB.**

Consider the points \(A(3, 1)\) and \(B(5, 5)\). M is at \((4, 3)\) on the line segment connecting A and B.

Using the distance formula, we can see that

\[
AM = \sqrt{(4 - 3)^2 + (3 - 1)^2} = \sqrt{5} \text{ units, and} \]
\[
MB = \sqrt{(5 - 4)^2 + (5 - 3)^2} = \sqrt{5} \text{ units.}
\]

So, M is the midpoint of AB.

The \(x\)-coordinate of M is the **average** of the \(x\)-coordinates of A and B.

The \(y\)-coordinate of M is the **average** of the \(y\)-coordinates of A and B.

\[
\frac{x\text{-coordinate of A}}{2} = \frac{3 + 5}{2} = 4 \quad \text{and} \quad \frac{y\text{-coordinate of A}}{2} = \frac{1 + 5}{2} = 3
\]

\[
\frac{x\text{-coordinate of M}}{2} = 2 \quad \text{and} \quad \frac{y\text{-coordinate of M}}{2} = \frac{1}{2}
\]

**THE MIDPOINT FORMULA**

If A is the point \((x_1, y_1)\) and B is \((x_2, y_2)\), then the midpoint \(M\) of AB has coordinates

\[
\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right).
\]

**Example 4**

Find the coordinates of M, the midpoint of AB, given \(A(-1, 3)\) and \(B(5, -2)\).

The \(x\)-coordinate of M

\[
= \frac{-1 + 5}{2} = \frac{4}{2} = 2
\]

The \(y\)-coordinate of M

\[
= \frac{3 + (-2)}{2} = \frac{1}{2}
\]

\(\therefore\) the midpoint of AB is \(M(2, \frac{1}{2})\).
EXERCISE 8B

1. a Use the distance formula to check that:
   i. M is the midpoint of AB
   ii. N is the midpoint of CD.

   b Use the midpoint formula to check your answers to part a.

Using the diagram only, find the coordinates of the midpoint of the line segment:
   a ST
   b UV
   c WX
   d YZ
   e SV
   f UT
   g YT
   h TV

2. Find the coordinates of the midpoint of the line segment that joins:
   a (2, 5) and (4, 7)
   b (1, 6) and (4, 2)
   c (0, 3) and (2, 0)
   d (3, −2) and (3, 2)
   e (−1, 4) and (2, 2)
   f (0, −3) and (−2, 5)
   g (−4, −1) and (3, −2)
   h (1, 0) and (−6, 8).

Example 5

M is the midpoint of AB. If A is (−1, 4) and M is (2, 3), find the coordinates of B.

Let B have coordinates (a, b).

\[
\frac{a + (-1)}{2} = 2 \quad \text{and} \quad \frac{b + 4}{2} = 3
\]

\[
\therefore a - 1 = 4 \quad \text{and} \quad b + 4 = 6
\]

\[
\therefore a = 5 \quad \text{and} \quad b = 2
\]

So, B is the point (5, 2).

4 M is the midpoint of AB. Find the coordinates of B if:
   a A is (1, 3) and M is (2, −1)
   b A is (2, 1) and M is (0, 2)
   c A is (−2, 1) and M is (−1, 2, 3)
   d A is (3, −2) and M is (3, −2)
   e A is (0, 0) and M is (2, −2)
   f A is (−3, 1) and M is (0, 0).
M is the midpoint of AB. Use equal steps to find the coordinates of B, given A is \((-4, 3)\) and M is \((-1, 2)\).

\[
\begin{align*}
A(-4, 3) & \quad +3 \\
M(-1, 2) & \quad +1 \\
B(a, b) & \quad -1
\end{align*}
\]

\[
\begin{align*}
x\text{-step:} & \quad -4 \quad +3 \quad -1 \quad +3 \quad 2 \\
y\text{-step:} & \quad 3 \quad -1 \quad 2 \quad -1 \quad 1
\end{align*}
\]

\[\therefore \text{B is (2, 1)}\]

5 Check your answers to questions 4a and 4b using equal steps.

6 If P is the midpoint of IJ, find the coordinates of I for:

a P(2, -6) and J(4, -3)

b P(0, -2) and J(-5, 1).

7 PQ is the diameter of a circle, centre C. If P is (4, -7) and Q is (-2, -3), find the coordinates of C.

8 AB is a diameter of a circle, centre \((3\frac{1}{2}, -1)\). Find the coordinates of A given that B is (2, 0).

9 Torvald gets into a rowboat at A(1, 2) on one side of a circular lake. He rows in a straight line towards the other side. He stops in the middle of the lake for a rest, at M(-2, 3).

a What are the coordinates of the point Torvald is aiming for?

b If distances are in km, how much further does he have to row?

10 A flagpole at F is held by four wires pegged into the ground at A, B, C and D. Opposite pegs are the same distance away from the pole. What are the coordinates of D?

11 Molly the cat stands at A\((-1, -2)\), watching in fear as Susan and Sandra throw water balloons at each other. Susan is at B(2, 3), and Sandra is at C(0, 4). The two girls throw at the same time, and their balloons collide and explode midway between them. Units are given in metres.

a Find the coordinates of the explosion point.

b How far is Molly from the explosion?
Example 7

Use midpoints to find the fourth vertex of the given parallelogram:

Since ABCD is a parallelogram, the diagonals bisect each other.

\[ \text{the midpoint of DB is the same as the midpoint of AC.} \]

If D is \((a, b)\), then

\[ \frac{a + 1}{2} = \frac{-3 + 0}{2} \quad \text{and} \quad \frac{b + 3}{2} = \frac{4 + (-2)}{2} \]

\[ \therefore a + 1 = -3 \quad \text{and} \quad b + 3 = 2 \]

\[ \therefore a = -4 \quad \text{and} \quad b = -1 \]

So, D is \((-4, -1)\).

12 Use midpoints to find the fourth vertex of the given parallelograms:

13 An inaccurate sketch of quadrilateral ABCD is given. P, Q, R and S are the midpoints of AB, BC, CD and DA respectively.

- a Find the coordinates of:
  - i P
  - ii Q
  - iii R
  - iv S

- b Find the length of:
  - i PQ
  - ii QR
  - iii RS
  - iv SP

- c What can be deduced about quadrilateral PQRS from b?

C

GRADIENT

Consider the following lines. Which do you think is the steepest?
We can see that line 2 rises much faster than the other two lines, so line 2 is the steepest. However, most people would find it hard to tell which of lines 1 and 3 is steeper just by looking at them. We therefore need a more precise way to measure the steepness of a line.

The gradient of a line is a measure of its steepness.

To calculate the gradient of a line, we first choose any two distinct points on the line. We can move from one point to the other by making a positive horizontal step followed by a vertical step.

If the line is sloping upwards, the vertical step will be positive.

If the line is sloping downwards, the vertical step will be negative.

The gradient is calculated by dividing the vertical step by the horizontal step.

\[ \text{The gradient of a line} = \frac{\text{vertical step}}{\text{horizontal step}} \text{ or } \frac{y\text{-step}}{x\text{-step}} \]

For lines with the same horizontal step, as the lines get steeper the vertical step increases. This results in a higher gradient.

**Example 8**

Find the gradient of each line segment:
From the previous example, you should have found that:

- the gradient of horizontal lines is 0.
- the gradient of vertical lines is undefined.

Lines like \[ \frac{\text{rise}}{\text{run}} = \frac{2}{1} \] are upwards sloping and have **positive gradients**.

Lines like \[ \frac{\text{rise}}{\text{run}} = \frac{-1}{3} \] are downwards sloping and have **negative gradients**.

**EXERCISE 8C.1**

1. Find the gradient of each line segment:

   \[
   \begin{array}{cccc}
   \text{a} & \text{b} & \text{c} & \text{d} \\
   \frac{\text{rise}}{\text{run}} &= &\frac{-1}{3} & \frac{0}{3} \\
   \end{array}
   \]

   which is undefined

2. On grid paper draw a line segment with gradient:

   \[
   \begin{array}{cccc}
   \text{a} & \text{b} & \text{c} & \text{d} \\
   \frac{\text{rise}}{\text{run}} &= &-2 & -\frac{1}{3} \\
   \end{array}
   \]

3. Consider these line segments:

   a. Which two lines have the same gradient?
   b. Which line is the steepest?
Draw a line through \((1, 3)\) with gradient \(-\frac{1}{2}\).

Plot the point \((1, 3)\).
The gradient \(= \frac{y\text{-step}}{x\text{-step}} = -\frac{1}{2}\)
\(\therefore\) let \(y\text{-step} = -1, \ x\text{-step} = 2\).
We use these steps to find another point and draw the line through these points.

4 On the same set of axes, draw lines through \((2, 3)\) with gradients \(\frac{1}{3}, \frac{3}{4}, 2\) and 4.

5 On the same set of axes, draw lines through \((-1, 2)\) with gradients 0, \(-\frac{2}{5}, -2\) and -5.

**THE GRADIENT FORMULA**

If a line passes through \(A(x_1, y_1)\) and \(B(x_2, y_2)\), then the horizontal or \(x\text{-step}\) is \(x_2 - x_1\), and the vertical or \(y\text{-step}\) is \(y_2 - y_1\).

The gradient of the line passing through \(A(x_1, y_1)\) and \(B(x_2, y_2)\) is \(\frac{y_2 - y_1}{x_2 - x_1}\).

**Example 10**

Find the gradient of the line through \((-2, 1)\) and \((2, 9)\).

\[\text{gradient} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{9 - 1}{2 - (-2)} = \frac{8}{4} = 2\]
EXERCISE 8C.2

1 Find the gradients of the line segments joining the following pairs of points:
   a \((1, 3)\) and \((6, 8)\)  
   b \((-4, 5)\) and \((4, 3)\)  
   c \((0, 0)\) and \((3, 5)\)  
   d \((5, 2)\) and \((2, 9)\)  
   e \((1, -4)\) and \((-5, -2)\)  
   f \((-3, 4)\) and \((2, 4)\)  
   g \((-6, 0)\) and \((0, -4)\)  
   h \((-3, 5)\) and \((-3, 1)\)  
   i \((-5, -8)\) and \((3, 4)\).

2 Find the gradient of line:
   a \(1\)  
   b \(2\)  
   c \(3\)  
   d \(4\)  
   e \(5\)  
   f \(6\)

Example 11

Find \(a\) given that the line joining \(P(a, -4)\) to \(Q(1, 8)\) has gradient 3.

The gradient of \(PQ = 3\), so 
\[
rac{8 - (-4)}{1 - a} = 3 \quad \{\text{gradient formula}\}
\]
\[
12 = 3(1 - a) \quad \therefore 12 = 3 - 3a
\]
\[
\therefore 3a = -9 \quad \therefore a = -3
\]

3 Find \(a\) given that the line joining:
   a \(P(1, 5)\) to \(Q(4, a)\) has gradient 2  
   b \(M(-2, a)\) to \(N(0, -2)\) has gradient -4  
   c \(A(a, 8)\) to \(B(-3, -4)\) has gradient \(\frac{2}{5}\).

4 A line with gradient \(-2\) passes through the point \((-1, 10)\). Determine where this line cuts the \(x\)-axis.
   Hint: A point on the \(x\)-axis has coordinates \((a, 0)\).

PARALLEL LINES

The figure ABCD alongside is a trapezium, with \(AB\) parallel to \(DC\).

\(DC\) has a gradient of \(\frac{1}{2}\), and \(AB\) has a gradient of \(\frac{3}{9} = \frac{1}{3}\).

Thus, \(AB\) and \(DC\) have the same gradient.
If two lines are parallel, they have equal gradient.
If two lines have equal gradient, they are parallel.

PERPENDICULAR LINES

The figure alongside is a square, so AB is perpendicular to BC.

AB has a gradient of $\frac{2}{3}$, and BC has a gradient of $\frac{2}{3} = -\frac{2}{3}$.

The gradients are negative reciprocals of each other and their product is $\frac{3}{2} \times -\frac{2}{3} = -1$.

For lines which are not horizontal or vertical:

- if the lines are perpendicular, their gradients are negative reciprocals.
- if the gradients are negative reciprocals, the lines are perpendicular.

**Example 12**

If a line has gradient $\frac{2}{3}$, find the gradient of all lines:

a. parallel to the given line
b. perpendicular to the given line.

a. The original line has gradient $\frac{2}{3}$, so the gradient of all parallel lines is also $\frac{2}{3}$.
b. The gradient of all perpendicular lines is $-\frac{5}{2}$. {the negative reciprocal}

**EXERCISE 8C.3**

1. Find the gradient of all lines  
   i. parallel  
   ii. perpendicular to a line with gradient:
      a. $\frac{3}{4}$  
      b. $\frac{1}{2}$  
      c. 4  
      d. $-3$  
      e. $-\frac{3}{7}$  
      f. $-4\frac{1}{2}$  
      g. 0  
      h. $-1$

2. The gradients of several pairs of lines are listed below. Which of the line pairs are perpendicular?
   a. $2, \frac{1}{2}$  
   b. $\frac{3}{5}, -\frac{3}{5}$  
   c. 4, $-\frac{1}{4}$  
   d. $\frac{3}{4}, 1\frac{1}{3}$  
   e. $-\frac{2}{5}, 2\frac{1}{7}$  
   f. $-\frac{3}{8}, 2\frac{1}{8}$

3. Consider the line segments below.

Identify any pairs of lines which are:

a. parallel  
   b. perpendicular.
Find \( t \) given that the line joining \( A(1, 4) \) to \( B(5, t) \) is perpendicular to a line with gradient \( \frac{2}{3} \).

The gradient of \( AB = \frac{-3}{2} \) \{perpendicular to line with gradient \( \frac{2}{3} \)\}.

\[
\therefore \frac{t - 4}{5 - 1} = \frac{-3}{2} \{\text{gradient formula}\}
\]

\[
\therefore \frac{t - 4}{4} = \frac{-6}{4} \{\text{writing fractions with equal denominators}\}
\]

\[
\therefore t - 4 = -6 \{\text{equating numerators}\}
\]

\[
\therefore t = -2
\]

4 Find \( t \) given that the line joining:

\( a \) \( C(2, 5) \) and \( D(4, t) \) is perpendicular to a line with gradient \( \frac{1}{2} \).

\( b \) \( X(-3, -1) \) and \( Y(t, 1) \) is perpendicular to a line with gradient \( 3\frac{1}{2} \).

5 Given the points \( A(1, 2), \ B(-3, 0), \ C(5, 3), \) and \( D(3, k), \) find \( k \) if:

\( a \) \( AB \) is parallel to \( CD \)

\( b \) \( AC \) is parallel to \( DB \)

\( c \) \( AB \) is perpendicular to \( CD \)

\( d \) \( AD \) is perpendicular to \( BC \).

6 Consider the triangle \( ABC \) alongside.

\( a \) Find the length of each side. Hence, show that the triangle is right angled at \( B \).

\( b \) Find the gradients of \( AB \) and \( BC \). Hence verify that \( AB \) is perpendicular to \( BC \).

### COLLINEAR POINTS

Three or more points are **collinear** if they lie on the same straight line.

Consider the three collinear points \( A, B, \) and \( C, \) which all lie on the line \( L \).

gradient of \( AB = \) gradient of \( BC = \) gradient of \( L \).

Three points \( A, B, \) and \( C \) are **collinear** if:

\[
\text{gradient of } AB = \text{gradient of } BC = (\text{gradient of } AC)
\]
Show that the points \( A(-5, -3) \), \( B(-1, -1) \), and \( C(7, 3) \) are collinear.

The gradient of \( AB = \frac{-1 - (-3)}{-1 - (-5)} = \frac{2}{4} = \frac{1}{2} \).

The gradient of \( BC = \frac{3 - (-1)}{7 - (-1)} = \frac{4}{8} = \frac{1}{2} \).

\( AB \) and \( BC \) have equal gradients, and so \( A \), \( B \), and \( C \) are collinear.

**EXERCISE 8C.4**

1. Determine whether the following sets of three points are collinear:
   
   a. \( A(-1, 7) \), \( B(1, 1) \), \( C(4, -8) \)
   
   b. \( P(-4, 2) \), \( Q(-1, 3) \), \( R(5, 6) \)
   
   c. \( R(-2, 1) \), \( S(4, 11) \), \( T(-5, -4) \)
   
   d. \( X(7, -5) \), \( Y(2, -1) \), \( Z(-6, 5) \)

2. Find \( n \) given that:
   
   a. \( A(-7, -8) \), \( B(-1, 1) \), and \( C(3, n) \) are collinear.
   
   b. \( P(3, -11) \), \( Q(n, -2) \), and \( R(-5, 13) \) are collinear.

**RATES**

In the previous section we considered the gradients of straight lines and the gradient between points.

We see gradients every day in the real world when we consider the slope of a hill or ramp. The sign alongside indicates to drivers that the road ahead is steeply downhill.

Gradients are also important when we consider how quantities are related. If we draw the graph relating two quantities, the gradient of the line describes the rate at which one quantity changes relative to the other.

One of the most common examples of a rate is speed, which is the rate at which something is travelling.

For example, a cheetah sprinting after its prey can travel 20 m every second.

If we plot the distance the cheetah travels against the time taken, the gradient of the graph

\[
\frac{y\text{-step}}{x\text{-step}} = \frac{80}{4} = 20.
\]

In comparison, the speed of the cheetah

\[
= \frac{\text{distance travelled}}{\text{time taken}} = \frac{80 \text{ m}}{4 \text{ s}} = 20 \text{ m/s}^{-1}.
\]

So, the gradient of the graph gives the speed at which the cheetah is running.
EXERCISE 8D

1. The graph alongside displays the mass of various volumes of silver.
   a. Find the gradient of the line.
   b. Interpret the gradient found in a.
   c. i. What is the mass of 3 cm$^3$ of silver?
      ii. What is the volume of 100 g of silver?

2. A motorcyclist makes a day’s journey and plots her progress on the graph alongside. Find:
   a. the average speed for the whole trip
   b. the average speed from
      i. O to A
      ii. B to C
   c. the time interval over which her average speed was greatest.

3. Harriet buys a car. Every 500 km she records how much she has spent on petrol and upkeep costs. She then plots the results on the graph shown.
   a. How much did the car cost?
   b. What is the gradient of AB? What does this represent?
   c. Find the gradient of the straight line segment from A to D. What does this gradient mean?

4. The graphs alongside show the energy per litre of biodiesel and ethanol.
   a. Find the gradient of each line.
   b. Which type of fuel gives more energy per litre?

5. The graph alongside indicates the amount of tax paid for various incomes.
   a. What does the value at A mean?
   b. Find the gradients of the line segments AB and BC. What do these gradients indicate?
   c. What do you expect to happen for people who earn more than $42 000 p.a.?
VERTICAL LINES

The graph opposite shows a vertical line passing through (2, −1) and (2, 3).

The gradient of the line is \( \frac{3 - (-1)}{2 - 2} = \frac{4}{0} \) which is undefined.

All points on the line have \( x \)-coordinate 2, so the equation of the line is \( x = 2 \).

All vertical lines have equations of the form \( x = a \) where \( a \) is a constant.

The gradient of a vertical line is undefined.

HORIZONTAL LINES

This graph shows a horizontal line passing through (−3, 1) and (2, 1).

The gradient of the line is \( \frac{1 - 1}{2 - (-3)} = \frac{0}{5} = 0 \).

All points on the line have \( y \)-coordinate 1, so the equation of the line is \( y = 1 \).

All horizontal lines have equations of the form \( y = b \) where \( b \) is a constant.

The gradient of a horizontal line is zero.

EXERCISE 8E

1. Find the equations of the lines labelled A to D:

2. Identify as either a vertical or horizontal line and hence plot the graph of:
   a) \( x = 1 \)  
   b) \( y = 2 \)  
   c) \( x = -4 \)  
   d) \( y = -2 \)
3 Find the equation of the:
   a horizontal line through $(3, -4)$
   b vertical line which cuts the $x$-axis at 5
   c vertical line through $(-1, -3)$
   d horizontal line which cuts the $y$-axis at 2
   e $x$-axis
   f $y$-axis

4 Find the equation of the line passing through:
   a $(2, 2)$ and $(2, -2)$
   b $(2, -2)$ and $(-2, -2)$.

**EQUATIONS OF LINES**

The equation of a line is an equation which connects the $x$ and $y$ values for every point on the line.

**GRADIENT-INTERCEPT FORM**

Every straight line that is not vertical will cut the $y$-axis at a single point. The $y$-coordinate of this point is called the **$y$-intercept** of the line.

A line with gradient $m$ and $y$-intercept $c$ has equation $y = mx + c$.

We call this the **gradient-intercept form** of the equation of a line.

For example, the line alongside has

$$\text{gradient} = \frac{y\text{-step}}{x\text{-step}} = \frac{-2}{3}$$

and its $y$-intercept is 2.

So, its equation is $y = -\frac{2}{3}x + 2$.

**GENERAL FORM**

Another way to write the equation of a line is using the **general form** $ax + by + d = 0$.

We can rearrange equations from gradient-intercept form into general form by performing operations on both sides of the equation.

For example, if $y = -\frac{2}{3}x + 2$

then $3y = -2x + 6$ \{multiplying both sides by 3\}

$\therefore 2x + 3y = 6$ \{adding $2x$ to both sides\}

$\therefore 2x + 3y - 6 = 0$ \{subtracting 6 from both sides\}

So, the line with gradient-intercept form $y = -\frac{2}{3}x + 2$ has general form $2x + 3y - 6 = 0$. 
FINDING THE EQUATION OF A LINE

In order to find the equation, we need to know some information about the line.

Suppose we know the gradient of the line is 2 and that the line passes through (4, 1).

We suppose \((x, y)\) is any point on the line.

The gradient between \((4, 1)\) and \((x, y)\) is \(\frac{y - 1}{x - 4}\), and this gradient must equal 2.

So, \(\frac{y - 1}{x - 4} = 2\)

\[\therefore y - 1 = 2(x - 4)\] \{multiplying both sides by \((x - 4)\}\n
\[\therefore y - 1 = 2x - 8\] \{expanding the brackets\}

\[\therefore y = 2x - 7\] \{adding 1 to both sides\}

This is the equation of the line in gradient-intercept form.

We can find the equation of a line if we know:
- its gradient and the coordinates of any point on the line, or
- the coordinates of two distinct points on the line.

If a straight line has gradient \(m\) and passes through the point \((x_1, y_1)\) then its equation is \(\frac{y - y_1}{x - x_1} = m\).

We can rearrange this equation into either gradient-intercept or general form.

Example 15

Find, in gradient-intercept form, the equation of the line through \((-1, 3)\) with a gradient of 5.

The equation of the line is \(y = mx + c\) where \(m = 5\).

When \(x = -1, \ y = 3\)

\[3 = 5(-1) + c\]

\[\therefore 3 = c - 5\]

\[\therefore c = 8\]

Thus, \(y = 5x + 8\) is the equation.

The equation of the line is \(\frac{y - 3}{x - (-1)} = 5\)

\[\therefore y - 3 = 5(x + 1)\]

\[\therefore y - 3 = 5x + 5\]

\[\therefore y = 5x + 8\]
Example 16

Find, in general form, the equation of the line with gradient \(\frac{3}{4}\) which passes through \((5, -2)\).

The equation of the line is

\[
\frac{y - (-2)}{x - 5} = \frac{3}{4}
\]

\[
\Rightarrow y + 2 = \frac{3}{4}(x - 5)
\]

\[
4(y + 2) = 3(x - 5)
\]

\[
4y + 8 = 3x - 15
\]

\[
3x - 4y - 23 = 0
\]

Example 17

Find the equation of the line which passes through the points \(A(-1, 5)\) and \(B(2, 3)\).

The gradient of the line is

\[
\frac{3 - 5}{2 - (-1)} = \frac{-2}{3}
\]

Using \(A\), the equation is

\[
\frac{y - 5}{x - (-1)} = \frac{-2}{3}
\]

\[
\Rightarrow y - 5 = \frac{-2}{3}(x + 1)
\]

\[
\Rightarrow 3(y - 5) = -2(x + 1)
\]

\[
\Rightarrow 3y - 15 = -2x - 2
\]

\[
\Rightarrow 2x + 3y - 13 = 0
\]

EXERCISE 8F.1

1 Find the equation of the line with:

- \(a\) gradient 1 and \(y\)-intercept \(-2\)
- \(b\) gradient \(-1\) and \(y\)-intercept 4
- \(c\) gradient 2 and \(y\)-intercept 0
- \(d\) gradient \(-\frac{1}{2}\) and \(y\)-intercept 3.

2 Find, in gradient-intercept form, the equation of the line through:

- \(a\) \((-2, -5)\) with gradient 4
- \(b\) \((-1, -2)\) with gradient \(-3\)
- \(c\) \((-7, -3)\) with gradient \(-5\)
- \(d\) \((1, 4)\) with gradient \(\frac{1}{2}\)
- \(e\) \((-1, 3)\) with gradient \(-\frac{1}{3}\)
- \(f\) \((2, 6)\) with gradient 0.

We would get the same equations using point \(B\). Try it yourself.
3 Find, in general form, the equation of the line through:
   a (2, 5) having gradient $\frac{2}{3}$
   b (−1, 4) having gradient $\frac{4}{3}$
   c (5, 0) having gradient $−\frac{1}{3}$
   d (6, −2) having gradient $−\frac{2}{5}$
   e (−3, −1) having gradient 4
   f (5, −3) having gradient −2
   g (4, −5) having gradient $−3\frac{1}{2}$
   h (−7, −2) having gradient 6.

4 Find, in gradient-intercept form, the equation of the line which passes through the points:
   a A(2, 3) and B(4, 8)
   b A(0, 3) and B(−1, 5)
   c A(−1, −2) and B(4, −2)
   d C(−3, 1) and D(2, 0)
   e P(5, −1) and Q(−1, −2)
   f R(−1, −3) and S(−4, −1).

5 Find, in general form, the equation of the line which passes through:
   a (0, 1) and (3, 2)
   b (1, 4) and (0, −1)
   c (2, −1) and (−1, −4)
   d (0, −2) and (5, 2)
   e (3, 2) and (−1, 0)
   f (−1, −1) and (2, −3).

6 Find the equations of the illustrated lines:
   a Two points on the line are (0, 1) and (5, 4)
     \[ m = \frac{4 - 1}{5 - 0} = \frac{3}{5} \]
     and the y-intercept \[ c = 1 \]
     \[ \therefore \text{the equation is } y = \frac{3}{5}x + 1 \]  
     \{gradient-intercept form\}
   b Two points on the line are (2, 4) and (6, −1)
     \[ m = \frac{−1 - 4}{6 - 2} = \frac{−5}{4} \]
     As we do not know the y-intercept we use the general form.
     The equation is \[ \frac{y - 4}{x - 2} = \frac{5}{4} \]
     \[ \therefore 4(y - 4) = -5(x - 2) \]
     \[ \therefore 4y - 16 = -5x + 10 \]
     \[ \therefore 5x + 4y - 26 = 0 \]
Find the equation connecting the variables in:

$(0, 2)$ and $(4, 8)$ lie on the straight line

$\therefore$ the gradient $m = \frac{8 - 2}{4 - 0} = \frac{6}{4} = \frac{3}{2}$, and the $y$-intercept $c = 2$.

In this case $K$ is on the vertical axis and $t$ is on the horizontal axis.

$\therefore$ the equation is $K = \frac{3}{2}t + 2$.

Find the equation connecting the variables given:

**FINDING THE GRADIENT FROM AN EQUATION**

When the equation of a line is written in gradient-intercept form, we can find the gradient by looking at the coefficient of $x$.

For equations in general form, one method of finding the gradient is to rearrange the equation first.
Find the gradient of the line \(2x + 5y - 17 = 0\).

\[
2x + 5y - 17 = 0 \\
\therefore 5y = -2x + 17 \\
\therefore y = -\frac{2}{5}x + \frac{17}{5}
\]

So, the gradient is \(-\frac{2}{5}\).

You will learn in the exercise a faster way of finding the gradient of a line with equation written in general form.

**EXERCISE 8F.2**

1. Find the gradient of the line with equation:
   - a. \(y = 3x + 2\)
   - b. \(y = 3 - 2x\)
   - c. \(y = 0\)
   - d. \(x = 5\)
   - e. \(y = \frac{2x + 1}{3}\)
   - f. \(y = \frac{3 - 4x}{5}\)

2. Find the gradient of the line with equation:
   - a. \(3x + y - 7 = 0\)
   - b. \(2x - 7y = 8\)
   - c. \(2x + 7y - 8 = 0\)
   - d. \(3x - 4y = 11\)
   - e. \(4x + 11y - 9 = 0\)
   - f. \(7x - 9y = 63\)

3. **a** Find the gradient of the line with equation \(ax + by + d = 0\).
   **b** Hence find the gradient of the line with equation:
   - i. \(2x + 5y + 1 = 0\)
   - ii. \(3x - 2y = 0\)
   - iii. \(5x + 4y - 10 = 0\)
   - iv. \(-x + 3y - 2 = 0\)
   - v. \(-2x + y = -3\)
   - vi. \(x - 4y = 6\)

**DOES A POINT LIE ON A LINE?**

A point lies on a line if its coordinates satisfy the equation of the line.

**Example 21**

Does \((3, -2)\) lie on the line with equation \(5x - 2y = 20\)?

Substituting \((3, -2)\) into \(5x - 2y = 20\) gives \(5(3) - 2(-2) = 20\) or \(19 = 20\) which is false.

\(\therefore (3, -2)\) does not lie on the line.
EXERCISE 8F.3

1  a. Does $(3, 4)$ lie on the line with equation $3x - 2y - 1 = 0$?
   b. Does $(-2, 5)$ lie on the line with equation $5x + 3y = -5$?
   c. Does $(6, -\frac{1}{2})$ lie on the line with equation $3x - 8y - 22 = 0$?
   d. Does $(8, -\frac{2}{3})$ lie on the line with equation $x - 9y = 14$?

2  Find $k$ if:
   a. $(3, 4)$ lies on the line with equation $x - 2y - k = 0$
   b. $(1, 5)$ lies on the line with equation $4x - 2y = k$
   c. $(1, 5)$ lies on the line with equation $6x + 7y = k$
   d. $(-2, -3)$ lies on the line with equation $4x - 3y - k = 0$

3  Find $a$ given that:
   a. $(a, 3)$ lies on the line with equation $y = 2x - 1$
   b. $(-2, a)$ lies on the line with equation $y = 1 - 3x$
   c. $(a, 5)$ lies on the line with equation $y = 3x + 4$

4  A straight road is to pass through points $A(5, 3)$ and $B(1, 8)$.
   a. Find where this road meets the road given by:
      i. $x = 3$
      ii. $y = 4$
   b. If we wish to refer to the points on road $AB$ that are between $A$ and $B$, how can we indicate this?
   c. Does $C(23, -20)$ lie on the road?

G

GRAPHING LINES

DISCUSSION

Discuss the easiest way to graph a line when its equation is given in the form:

- $y = mx + c$ such as $y = 2x + 3$
- $ax + by + d = 0$ such as $2x + 3y - 12 = 0$.

GRAPHING FROM THE GRADIENT-INTERCEPT FORM

The easiest way to graph lines with equations given in gradient-intercept form is to use the $y$-intercept and one other point on the graph. The other point can be found by substitution or by using the gradient.
**Example 22**

Graph the line with equation \( y = \frac{5}{2}x - 2 \).

**Method 1:**
The \( y \)-intercept is \(-2\).
When \( x = 2 \), \( y = 5 - 2 = 3 \)
\( \therefore \) \((0, -2)\) and \((2, 3)\) lie on the line.

**Method 2:**
The \( y \)-intercept is \(-2\)
and the gradient \( = \frac{5}{2} \)
So, we start at \((0, -2)\) and move to another point by moving across 2, then up 5.

**Example 23**

Graph the line with equation \( 3x + 5y + 30 = 0 \).

When \( x = 0 \), \( 5y + 30 = 0 \)
\( \therefore \) \( y = -6 \)
So, the \( y \)-intercept is \(-6\).
When \( y = 0 \), \( 3x + 30 = 0 \)
\( \therefore \) \( x = -10 \)
So, the \( x \)-intercept is \(-10\).
EXERCISE 8G.1

1 Draw the graph of the line with equation:
   a \( y = \frac{1}{2}x + 2 \)
   b \( y = 2x + 1 \)
   c \( y = -x + 3 \)
   d \( y = -3x + 2 \)
   e \( y = -\frac{1}{2}x \)
   f \( y = -2x - 2 \)
   g \( y = \frac{3}{2}x \)
   h \( y = \frac{2}{3}x + 2 \)
   i \( y = -\frac{3}{4}x - 1 \)

2 Use axes intercepts to sketch the graphs of:
   a \( x + 2y = 8 \)
   b \( 4x + 3y - 12 = 0 \)
   c \( 2x - 3y = 6 \)
   d \( 3x - y - 6 = 0 \)
   e \( x + y = 5 \)
   f \( x - y = -5 \)
   g \( 2x - y + 4 = 0 \)
   h \( 9x - 2y = 9 \)
   i \( 3x + 4y = -15 \)

FINDING WHERE LINES MEET

When we graph two lines on the same set of axes, there are three possible situations which may occur:

Case 1: The lines meet in a single point of intersection. 
Case 2: The lines are parallel and never meet. There is no point of intersection. 
Case 3: The lines are coincident. There are infinitely many points of intersection.

We saw these situations in Chapter 5 when we solved simultaneous equations. In general there was a single solution, but in some special cases there was either no solution or infinitely many solutions.

Example 24

Use graphical methods to find where the lines \( x + y = 6 \) and \( 2x - y = 6 \) meet.

For \( x + y = 6 \)
   when \( x = 0 \), \( y = 6 \)
   when \( y = 0 \), \( x = 6 \)

For \( 2x - y = 6 \)
   when \( x = 0 \), \( -y = 6 \) \( \therefore y = -6 \)
   when \( y = 0 \), \( 2x = 6 \) \( \therefore x = 3 \)

The graphs meet at (4, 2).

Check: \( 4 + 2 = 6 \) ✓ and \( 2 \times 4 - 2 = 6 \) ✓
EXERCISE 8G.2

1 Use graphical methods to find the point of intersection of:

- **a** \( y = x - 3 \)
- **b** \( x - y - 1 = 0 \)
- **c** \( 4x + 3y + 12 = 0 \)
- **d** \( 3x + y + 3 = 0 \)
- **e** \( 3x + y = 9 \)
- **f** \( x - 3y = -9 \)
- **g** \( 2x - y = 6 \)
- **h** \( y = 2x - 4 \)
- **i** \( y = -x - 5 \)

2 How many points of intersection do the following pairs of lines have?

Explain, but do not graph them.

- **a** \( 3x + y - 5 = 0 \)
- **b** \( 3x + y + 5 = 0 \)
- **c** \( 3x - y + 5 = 0 \)
- **d** \( 3x + y - 8 = 0 \)
- **e** \( 6x + 2y + 10 = 0 \)
- **f** \( 3x - y + k = 0 \) where \( k \) is a constant.

ACTIVITY

Two candles are lit at the same time. The first candle is 20 cm long and burns at a rate of 2.5 mm per hour. The second candle is 24.5 cm long and burns at a rate of 3.5 mm per hour.

What to do:

1 Explain why the heights of the candles after \( t \) hours are given by \( h_1 = 200 - 2.5t \) mm for the first candle and \( h_2 = 245 - 3.5t \) mm for the second candle.

2 Use the equations in 1 to determine how long each candle will last.

3 Graph each equation on the same set of axes.

4 At what time will the candles have the same height?

5 If you want the candles to 'go out' together, which candle would you light first? How long after this would you light the other one?
USING TECHNOLOGY TO FIND WHERE LINES MEET

We can plot straight lines using a graphing package or a graphics calculator. We can also use technology to find the point of intersection of a pair of lines. This is often much easier than doing the algebra by hand, particularly when the answers are not integers.

However, most graphing packages and graphics calculators need to have the equations of the lines in the form \( y = mx + c \). This means that if the equation of a line is given in general form we will have to rearrange it into gradient-intercept form.

Suppose we wish to find the point of intersection of \( 2x - 3y - 5 = 0 \) and \( x - 4y + 1 = 0 \).

**Step 1:** Rearrange each equation into the form \( y = mx + c \).

\[
egin{align*}
2x - 3y - 5 &= 0 \\
\therefore 3y &= 2x - 5 \\
\therefore y &= \frac{2}{3}x - \frac{5}{3}
\end{align*}
\]

\[
\begin{align*}
x - 4y + 1 &= 0 \\
\therefore 4y &= x + 1 \\
\therefore y &= \frac{1}{4}x + \frac{1}{4}
\end{align*}
\]

**Step 2:** If you are using the graphing package, click on the icon and enter the two equations.
If you are using a graphics calculator, enter the two equations in the appropriate window. See the instructions at the start of the book for further help.

**Step 3:** Solve the equations using technology.

For this example, the point of intersection is \((4.6, 1.4)\).

**EXERCISE 8G.3**

1. Use technology to find the point of intersection of:
   
   a. \( y = x + 5 \)
   \( x + 2y - 1 = 0 \)
   
   b. \( 5x + 2y - 13 = 0 \)
   \( y = 3x + 1 \)
   
   c. \( 2x + y - 6 = 0 \)
   \( 4x - 3y - 5 = 0 \)
   
   d. \( 7x + 3y + 3 = 0 \)
   \( x - y - 4 = 0 \)

2. If you can, find the point(s) of intersection of the following using technology:
   
   a. \( y = 2x + 5 \)
   \( 2x - y - 2 = 0 \)
   
   b. \( 4x - 3y + 6 = 0 \)
   \( y = \frac{4}{5}x + 2 \)

   Explain your results.

3. A potter knows that if he makes \( x \) pots per day, his costs are \( y = 200 + 4x \) pounds. His income from selling the pots is \( y = 17x + 5 \) pounds. He always sells all the pots he makes.
   
   a. Graph these two equations using technology, and find their point of intersection.
   
   b. What does this point represent?
We have already seen that the midpoint \( M \) of the line segment \( AB \) is the point on the line segment that is halfway between \( A \) and \( B \).

The **perpendicular bisector** of \( AB \) is the set of **all** points which are the same distance from \( A \) and \( B \).

The perpendicular bisector is a line which passes through \( M \) and which is perpendicular to \( AB \). It divides the Cartesian plane into two regions: the set of points closer to \( A \) than to \( B \), and the set of points closer to \( B \) than to \( A \).

**Example 25**

Find the equation of the perpendicular bisector of \( AB \) given \( A(-1, 2) \) and \( B(3, 4) \).

The midpoint \( M \) of \( AB \) is \( \left( \frac{-1 + 3}{2}, \frac{2 + 4}{2} \right) \) or \( M(1, 3) \).

The gradient of \( AB \) is \( \frac{4 - 2}{3 - (-1)} = \frac{2}{4} = \frac{1}{2} \).

\[ \therefore \text{the gradient of the perpendicular bisector is } -\frac{2}{1} \]

The equation of the perpendicular bisector is

\[ \frac{y - 3}{x - 1} = -2 \]

\[ \therefore y - 3 = -2(x - 1) \]

\[ \therefore y - 3 = -2x + 2 \]

\[ \therefore y = -2x + 5 \]

**EXERCISE 8H**

1. Find the equation of the perpendicular bisector of \( AB \) given:
   a. \( A(3, -3) \) and \( B(1, -1) \)  
   b. \( A(1, 3) \) and \( B(-3, 5) \)  
   c. \( A(3, 1) \) and \( B(-3, 6) \)  
   d. \( A(4, -2) \) and \( B(4, 4) \).

2. Two post offices are located at \( P(3, 8) \) and \( Q(7, 2) \) on a Council map. Each post office services those houses which are closer to them than the other post office. Find the equation of the boundary between the regions.
3 Answer the **Opening Problem** on page 252.

4 The perpendicular bisector of a chord of a circle passes through the centre of the circle.
   A circle passes through points P(5, 7), Q(7, 1) and R(−1, 5).
   a Find the equations of the perpendicular bisectors of PQ and QR.
   b Solve the equations in a simultaneously to find the centre of the circle.

5 Triangle ABC has the vertices shown.
   a Find the coordinates of P, Q and R, the midpoints of AB, BC and AC respectively.
   b Find the equation of the perpendicular bisector of:
      i AB      ii BC      iii AC
   c Find the coordinates of X, the point of intersection of the perpendicular bisector of AB and the perpendicular bisector of BC.
   d Does the point X lie on the perpendicular bisector of AC?
   e What does your result from d suggest about the perpendicular bisectors of the sides of a triangle?
   f What is special about the point X in relation to the vertices of the triangle ABC?

---

**REVIEW SET 8A**

1 a Find the distance between the points A(−3, 2) and B(1, 5).
   b Find the gradient of the line perpendicular to a line with gradient \(\frac{3}{7}\).
   c Find the midpoint of the line segment joining C(−3, 1) to D(5, 7).

2 Find the axes intercepts and gradient of the line with equation \(5x - 2y + 10 = 0\).

3 Determine the equation of the illustrated line:

4 Find \(a\) given that P(−3, 4), Q(2, 6) and R(5, \(a\)) are collinear.

5 Find \(c\) if (−1, \(c\)) lies on the line with equation \(3x - 2y + 7 = 0\).

6 Determine the equation of the line:
   a with gradient −3 and \(y\)-intercept 4      b through the points (−3, 4) and (3, 1).

7 Use graphical methods to find the point of intersection of \(y = 2x - 9\) and \(x + 4y - 36 = 0\).
8 Find the distance between P(−4, 7) and Q(−1, 3).

9 a Find the gradient of the line \( y = -4x + 7 \).
   b Line L is perpendicular to \( y = -4x + 7 \). What is its gradient?
   c L passes through the point (−2, 1). Find the equation of L.

10 Use midpoints to find the fourth vertex of the given parallelogram:

11 Find \( k \) given that (−3, \( k \)) is 7 units away from (2, 4).

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**REVIEW SET 8B**

1 Determine the midpoint of the line segment joining K(3, 5) to L(7, −2).

2 Find the gradient of the following lines:

3 Find, in gradient-intercept form, the equation of the line through:
   a (2, −1) with gradient −3   b (3, −2) and (−1, 4).

4 Find where the following lines cut the axes:
   a \( y = -\frac{3}{2}x + 7 \)   b \( 5x - 3y - 12 = 0 \)

5 Does (2, −5) lie on the line with equation \( 3x + 4y + 14 = 0 \)?

6 If \( 3x + ky = 7 \) and \( y = 3 - 4x \) are the equations of two lines, find \( k \) if:
   a the lines are parallel   b the lines are perpendicular.

7 Use graphical methods to find where the line through A(−5, 0) and B(3, 10) meets the line with equation \( 3x + 2y - 7 = 0 \).

8 Find the equation of the:
   a horizontal line through (−4, 3)   b vertical line through (−6, 1).

9 Find the equation of the perpendicular bisector of the line segment joining P(7, −1) to Q(−3, 5).
10 The illustrated circle has centre $(3, 2)$ and radius 5. The points $A(8, 2)$, $B(6, -2)$ lie on the circle.
   a. Find the midpoint of chord $AB$.
   b. Hence, find the equation of the perpendicular bisector of the chord.
   c. Show that the point $(3, 2)$ lies on the perpendicular bisector found in b.
   d. What property of circles has been checked in c?

11 Farmer Huber has a triangular field with corners $A(-1, 1)$, $B(1, 5)$ and $C(5, 1)$. There are gates at M and N, the midpoints of AB and BC respectively. A straight path goes from M to N.
   a. Use gradients to show that the path is parallel to AC.
   b. Show that the path is half as long as the fenceline AC.

1 Find, in general form, the equation of the line through:
   a. $(1, -5)$ with gradient $\frac{2}{3}$
   b. $(2, -3)$ and $(-4, -5)$.

2 If $5x - 7y - 8 = 0$ and $3x + ky + 11 = 0$ are the equations of two lines, find the value of $k$ for which:
   a. the lines are parallel
   b. the lines are perpendicular.

3 A point $T$ on the $y$-axis is 3 units from the point $A(-1, 2)$. Find:
   a. the possible coordinates of $T$
   b. the equation of the line $AT$, given that $T$ has a positive $y$-coordinate.

4 A truck driver plots his day’s travel on the graph alongside.
   a. Find the gradient of $AB$.
   b. Find the gradient of $OC$.
   c. Interpret your answers to a and b.

5 Fully classify triangle $KLM$ for $K(-5, -2)$, $L(0, 1)$ and $M(3, -4)$.

6 Navigation signs are posted on the bank of a river at W, X and Y as shown alongside. The local council plans to place another sign at Z such that WXYZ is a parallelogram. Use midpoints to find the coordinates of Z.
7 Draw the graph of the line with equation:
   a  \( y = -\frac{1}{3}x + 4 \)  
   b  \( 5x - 2y + 1 = 0 \)

8 Two primary schools are located at P(5, 12) and Q(9, 4) on a council map. If the
   Local Education Authority wishes to zone the region so that children must attend the
   closest school to their place of residence, what is the equation of the line that forms
   this boundary?

9 Given A(6, 8), B(14, 6), C(–1, –3) and D(–9, –1):
   a Use the gradient to show that:
      i  AB is parallel to DC  
      ii  BC is parallel to AD.
   b  What kind of figure is ABCD?
   c  Check that AB = DC and BC = AD using the distance
      formula.
   d  Find the midpoints of diagonals: i AC  
      ii BD.
   e  What property of parallelograms has been checked in d?

10 Copy and complete:

<table>
<thead>
<tr>
<th>Equation of line</th>
<th>Gradient</th>
<th>x-intercept</th>
<th>y-intercept</th>
</tr>
</thead>
<tbody>
<tr>
<td>a  ( 5x - 2y - 10 = 0 )</td>
<td>( \frac{5}{2} )</td>
<td>( 2 )</td>
<td>( -5 )</td>
</tr>
<tr>
<td>b  ( 4x + 5y = 20 )</td>
<td>( \frac{4}{5} )</td>
<td>( -4 )</td>
<td>( 1 )</td>
</tr>
<tr>
<td>c  ( y = -2x + 5 )</td>
<td>( -2 )</td>
<td>( \frac{5}{2} )</td>
<td>( -\frac{5}{2} )</td>
</tr>
<tr>
<td>d  ( x = 8 )</td>
<td></td>
<td>( 8 )</td>
<td>( )</td>
</tr>
<tr>
<td>e  ( y = 5 )</td>
<td></td>
<td></td>
<td>( 5 )</td>
</tr>
<tr>
<td>f  ( x + y = 11 )</td>
<td>( 1 )</td>
<td>( 1 )</td>
<td>( -10 )</td>
</tr>
</tbody>
</table>

11 Consider the points A(1, 3), B(6, 3), C(3, –1) and D(–2, –1).
   a Use the distance formula to show that ABCD is a rhombus.
   b  Find the midpoints of AC and BD.
   c  Use gradients to show that AC and BD are perpendicular.

**REVIEWS SET 8D**

1 For the points P(1, –3) and Q(–4, 0), find:
   a  the distance between P and Q
   b  the gradient of PQ
   c  the equation of the line passing through P and Q.

2 Fully classify triangle ABC for A(5, –1), B(–2, 3) and C(0, 8).

3 Find \( k \) given that the line joining A(5, \( k \)) and B(2, 4) is perpendicular to the line
   with equation \( x - 3y - 7 = 0 \).

4 Use midpoints to find the fourth vertex K of parallelogram HIJK for H(3, 4),
   I(–3, –1), and J(4, 10).
5 Find the gradient of the line with equation:
   a $y = \frac{4 - 3x}{2}$
   b $5x + 3y + 6 = 0$

6 Find $c$ given that $P(5, 9)$, $Q(-2, c)$ and $R(-5, 4)$ are collinear.

7 Find the equation linking the variables in these graphs:
   a
   b

8 Find $t$ if:
   a $(-2, 4)$ lies on the line with equation $2x - 7y = t$
   b $(3, t)$ lies on the line with equation $4x + 5y = -1$.

9 Consider $P(1, 5)$, $Q(5, 7)$, and $R(3, 1)$.
   a Show that triangle PQR is isosceles.
   b Find the midpoint $M$ of QR.
   c Use the gradient to verify that PM is perpendicular to QR.
   d Draw a sketch to illustrate what you have found in a, b, and c.

10 The Circular Gardens are bounded by East Avenue and Diagonal Road. Diagonal Road intersects North Street at C and East Avenue at D. Diagonal Rd is tangential to the Circular Gardens at B.
   a Find the equation of:
      i East Avenue
      ii North Street
      iii Diagonal Road.
   b Where does Diagonal Road intersect:
      i East Avenue
      ii North Street?

Jalen monitors the amount of water in his rainwater tank during a storm.
   a How much water was in the tank before the storm?
   b When was it raining the hardest?
   c At what rate is the tank filling between C and D?
   d What is the average water collection rate during the whole storm?