# Chapter 8
## FORMULAE

### EXERCISE 8A

**1** **a** \$150 was deposited for 8 weeks

$\therefore$ amount deposited = \$150 × 8

$A = \text{initial amount} + \text{amount deposited}$

$\therefore A = 2000 + 150 \times 8$

**b** $A = 2000 + 150 \times w$

$\therefore A = 2000 + 150w$

**c** $A = 2000 + d \times w$

$\therefore A = 2000 + dw$

**d** $A = P + d \times w$

$\therefore A = P + dw$

**2** **a** Cost of \$60 per hour for 5 hours = \$60 × 5

$C = \text{call out fee} + \text{cost of hours of work}$

$\therefore C = 40 + 60 \times 5$

**b** $C = 40 + 60 \times t$

$\therefore C = 40 + 60t$

**c** $C = 40 + x \times t$

$\therefore C = 40 + xt$

**d** $C = F + x \times t$

$\therefore C = F + xt$

**3** **a** $P = 3 \times \text{number of correct answers} - 1 \times \text{number of incorrect answers}$

For 15 questions there are 10 correct answers and $(15 - 10)$ incorrect answers.

$\therefore P = 3 \times 10 - 1 \times (15 - 10)$

$\therefore P = 3 \times 10 - 5$

**b** For 20 questions there are $c$ correct answers and $(20 - c)$ incorrect answers.

$$\begin{aligned}\therefore P &= 3 \times c - 1 \times (20 - c)\\ &= 3c - (20 - c)\\ &= 4c - 20\end{aligned}$$

**c** For $a$ questions there are $c$ correct answers and $(a - c)$ incorrect answers.

$$\begin{aligned}\therefore P &= 3 \times c - 1 \times (a - c)\\ &= 3c - (a - c)\\ &= 4c - a\end{aligned}$$

**4** **a** If there are 4 performances, there are $(4 - 1)$ breaks.

$D = 4 \times 6 + (4 - 1) \times 2$

$\therefore D = 4 \times 6 + 2(4 - 1)$

**b** $D = 5 \times m + (5 - 1) \times 3$

$\therefore D = 5m + 3(5 - 1)$

**c** $D = 8 \times m + (8 - 1) \times b$

$\therefore D = 8m + b(8 - 1)$

**d** $D = p \times m + (p - 1) \times b$

$\therefore D = mp + b(p - 1)$

**5** **a** If a side of the paddock is 3 units, then it has $(3 - 1) = 2$ dividing fences each with 2 gates, and if another side is 2 units, then it has $(2 - 1) = 1$, dividing fences, each with 3 gates.

$\therefore G = 2 \times (3 - 1) + 3 \times (2 - 1)$

**b** $G = 3 \times (5 - 1) + 5 \times (3 - 1)$

**c** $G = 4 \times (4 - 1) + 4 \times (4 - 1)$

**d** $G = m \times (n - 1) + n \times (m - 1)$

$\therefore G = mn - m + nm - n$

$\therefore G = 2mn - m - n$

**6** **a** Radius of semi-circle $= \dfrac{2a}{2} = a$

$$\begin{aligned}\therefore \text{area of semi-circle} &= \tfrac{1}{2}(\text{area of circle})\\ &= \tfrac{1}{2} \times \pi r^2\\ &= \tfrac{1}{2} \times \pi a^2\\ &= \frac{\pi a^2}{2}\end{aligned}$$

$$\begin{aligned}\text{Area of rectangle} &= \text{length} \times \text{width}\\ &= 2a \times b\\ &= 2ab\end{aligned}$$

$\therefore$ total area is $A = 2ab + \dfrac{\pi a^2}{2}$

**b**

$$\begin{aligned}\text{Area of triangle} &= \tfrac{1}{2} \times \text{base} \times \text{height}\\ &= \tfrac{1}{2} \times a \times 2r\\ &= ar\end{aligned}$$

$$\begin{aligned}\text{Area of semi-circle} &= \tfrac{1}{2}(\text{area of circle})\\ &= \tfrac{1}{2} \times \pi r^2\\ &= \frac{\pi r^2}{2}\end{aligned}$$

$\therefore$ total area is $A = ar + \dfrac{\pi r^2}{2}$

**c**

$$\begin{aligned}\text{Area of rectangle} &= \text{length} \times \text{width}\\ &= aw\end{aligned}$$

Radius of semi-circles $= \dfrac{w}{2}$

$$\begin{aligned}\text{Area of 2 semi-circles} &= \text{area of circle}\\ &= \pi r^2\\ &= \pi\left(\frac{w}{2}\right)^2\\ &= \frac{\pi w^2}{4}\end{aligned}$$

$\therefore$ total area is $A = aw + \dfrac{\pi w^2}{4}$

**d**

$$\begin{aligned}\text{Area of rectangle} &= \text{length} \times \text{width}\\ &= ab\end{aligned}$$

Radius of semi-circle $= \dfrac{a}{2}$

$$\begin{aligned}\text{Area of semi-circle} &= \tfrac{1}{2} \times \pi r^2\\ &= \tfrac{1}{2} \times \pi \times \left(\frac{a}{2}\right)^2\\ &= \frac{\pi a^2}{8}\end{aligned}$$

$\therefore$ shaded area is $A = ab - \dfrac{\pi a^2}{8}$

**e**

$$\begin{aligned}\text{Area of rectangle} &= \text{length} \times \text{width}\\ &= a \times 2r\\ &= 2ar\end{aligned}$$

$$\begin{aligned}\text{Area of 2 semi-circles} &= \text{area of circle}\\ &= \pi r^2\end{aligned}$$

$\therefore$ shaded area is $A = 2ar - \pi r^2$

**f** Let the height of the triangle be $h$ units.

$$\begin{aligned}\therefore\ h^2 + a^2 &= b^2 \quad \{\text{Pythagoras}\}\\ \therefore\ h^2 &= b^2 - a^2\\ \therefore\ h &= \sqrt{b^2 - a^2} \quad \{\text{as } h > 0\}\end{aligned}$$

$$\begin{aligned}\text{Area of triangle} &= \tfrac{1}{2} \times \text{base} \times \text{height}\\ &= \tfrac{1}{2} \times a \times \sqrt{b^2 - a^2}\\ &= \frac{a\sqrt{b^2 - a^2}}{2}\end{aligned}$$

$\therefore$ radius of semi-circle $= \dfrac{\sqrt{b^2 - a^2}}{2}$

$$\begin{aligned}\therefore\ \text{area of semi-circle} &= \tfrac{1}{2} \times \text{area of circle}\\ &= \tfrac{1}{2} \times \pi r^2\\ &= \tfrac{1}{2} \times \pi \times \left(\frac{\sqrt{b^2 - a^2}}{2}\right)^2\\ &= \frac{\pi(b^2 - a^2)}{8}\end{aligned}$$

$\therefore$ shaded area is $A = \dfrac{\pi(b^2 - a^2)}{8} + \dfrac{a\sqrt{b^2 - a^2}}{2}$

**7** **a** Volume $=$ area of end $\times$ length

$\therefore\ V = Al$

**b**

$$\begin{aligned}\text{Volume} &= \text{area of end} \times \text{length}\\ &= \text{area of circle} \times \text{length}\\ &= \pi r^2 \times h\\ &= \pi\left(\frac{d}{2}\right)^2 \times h\end{aligned}$$

$\therefore\ V = \dfrac{\pi d^2 h}{4}$

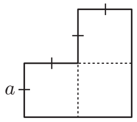
**c**

$$\begin{aligned}\text{Volume} &= \text{area of end} \times \text{length}\\ &= \text{area of triangle} \times \text{length}\\ &= \tfrac{1}{2} \times a \times b \times c\end{aligned}$$

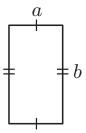
$\therefore\ V = \dfrac{abc}{2}$

**8** **a** The faces of this solid are:

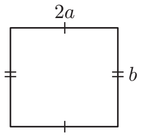
$2 \times$



$4 \times$

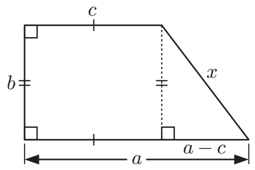


$2 \times$



$$\begin{aligned}\therefore\ \text{surface area} &= 2 \times 3a^2 + 4 \times ab + 2 \times 2ab\\ &= 6a^2 + 8ab\end{aligned}$$

**b**

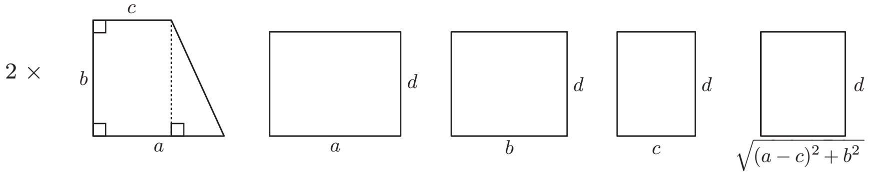


In the right angled triangle

$$x^2 = (a-c)^2 + b^2 \qquad \{\text{Pythagoras}\}$$

$$\therefore \quad x = \sqrt{(a-c)^2 + b^2} \qquad \{\text{as } x > 0\}$$

The faces of this solid are:



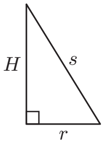
$$\therefore \quad \text{surface area} = 2 \times \left(\frac{a+c}{2}\right) b + ad + bd + cd + d\sqrt{(a-c)^2 + b^2}$$

$$= ab + bc + ad + bd + cd + d\sqrt{(a-c)^2 + b^2}$$

**c** Area of base = area of circle

$$= \pi r^2$$

Area of curved surface of a cylinder $= 2\pi rh$



Let the slant height of the cone be $s$.

$$s^2 = r^2 + H^2 \qquad \{\text{Pythagoras}\}$$

$$\therefore \quad s = \sqrt{r^2 + H^2} \qquad \{\text{as } s > 0\}$$

Area of curved surface of a cone $= \pi rs$

$$= \pi r\sqrt{r^2 + H^2}$$

$$\therefore \quad A = \pi r^2 + 2\pi rh + \pi r\sqrt{r^2 + H^2}$$

**9** Volume of the pipe = volume of large cylinder − volume of small cylinder

$$\therefore \quad V = \pi R^2 l - \pi r^2 l$$

$$\therefore \quad V = \pi l(R^2 - r^2)$$

$$\therefore \quad V = \pi l(R + r)(R - r) \qquad \{\text{difference of two squares}\}$$

## EXERCISE 8B

**1** **a** $C = 2\pi r$ where $r = 4.2$

$$\therefore \quad C = 2 \times \pi \times 4.2$$

$$\approx 26.4$$

$\therefore$ the circumference is approximately 26.4 cm.

**b** $C = 2\pi r$ where $C = 112$

$$\therefore \quad 112 = 2\pi r$$

$$\therefore \quad r = \frac{112}{2\pi}$$

$$\approx 17.8$$

$\therefore$ the circumference is approximately 17.8 cm.

**c** $C = 2\pi r$ where $C = 400$

$$\therefore \quad 400 = 2\pi r$$

$$\therefore \quad r = \frac{400}{2\pi}$$

$$\approx 63.662$$

Diameter $= 2 \times r \approx 127$

$\therefore$ the diameter is approximately 127 m.

**2** **a** $D = \frac{1}{2}gt^2$ where $g = 9.8$ and $t = 2$

$\therefore$ $D = \frac{1}{2} \times 9.8 \times 2^2$

$= 19.6$

$\therefore$ the stone has fallen 19.6 m.

**b** $D = \frac{1}{2}gt^2$ where $g = 9.8$ and $t = 4.8$

$\therefore$ $D = \frac{1}{2} \times 9.8 \times 4.8^2$

$= 112.896$

$\therefore$ the height of the cliff is approximately 113 m.

**3** **a** $A = \pi r^2$ where $r = 6.4$

$\therefore$ $A = \pi \times 6.4^2$

$\approx 129$

$\therefore$ area is approximately 129 cm$^2$.

**b** $A = \pi r^2$ where $A = 160$

$\therefore$ $\pi r^2 = 160$

$\therefore$ $r^2 = \frac{160}{\pi}$ $\{\div$ both sides by $\pi\}$

$\therefore$ $r = \sqrt{\frac{160}{\pi}}$ {as $r > 0$}

$\approx 7.14$

$\therefore$ radius is approximately 7.14 m.

**4** **a** $V = \pi r^2 h$ where $r = 8$ and $h = 21.2$

$\therefore$ $V = \pi \times 8^2 \times 21.2$

$\approx 4263$ cm$^3$

$\therefore$ volume is approximately 4263 cm$^3$.

**b** $V = \pi r^2 h$ where $r = 6$ and $V = 120$

$\therefore$ $\pi \times 6^2 \times h = 120$

$\therefore$ $h = \frac{120}{\pi \times 6^2}$ $\{\div$ both sides by $(\pi \times 6^2)\}$

$\approx 1.06$

$\therefore$ height is approximately 1.06 cm.

**c** $V = \pi r^2 h$ where $V = 470$ and $h = 6 \times 100 = 600$ cm

$\therefore$ $\pi \times r^2 \times 600 = 470$

$\therefore$ $r^2 = \frac{470}{\pi \times 600}$ $\{\div$ both sides by $(\pi \times 600)\}$

$\therefore$ $r = \sqrt{\frac{470}{\pi \times 600}}$ {as $r > 0$}

$\therefore$ $r \approx 0.4993$ cm

$\therefore$ $r \approx 4.99$ mm

$\therefore$ the radius is approximately 0.499 cm or 4.99 mm.

**5** **a** $A = 4\pi r^2$ where $r = 7.5$

$\therefore$ $A = 4 \times \pi \times 7.5^2$

$\approx 706.9$

$\therefore$ the area is approximately 707 cm$^2$.

**b** $A = 4\pi r^2$ where $A = 2$

$\therefore$ $4\pi r^2 = 2$

$\therefore$ $r^2 = \frac{2}{4\pi}$ $\{\div$ both sides by $4\pi\}$

$\therefore$ $r = \sqrt{\frac{1}{2\pi}}$ {as $r > 0$}

$\therefore$ $r \approx 0.3989$ m

$\therefore$ $r \approx 39.9$ cm

$\therefore$ the radius is approximately 39.9 cm.

**6** **a** $T = \frac{1}{5}\sqrt{l}$ where $l = 45$

$\therefore$ $T = \frac{1}{5} \times \sqrt{45}$

$\approx 1.34$

$\therefore$ the time is approximately 1.34 seconds.

**b** $T = \frac{1}{5}\sqrt{l}$ where $T = 1.8$

$\therefore$ $\frac{1}{5}\sqrt{l} = 1.8$

$\therefore$ $\sqrt{l} = 9$ $\{\times$ both sides by $5\}$

$\therefore$ $l = 81$ {square both sides}

$\therefore$ the length is 81 cm.

## EXERCISE 8C

**1** **a** $2x + 5y = 10$

$\therefore \quad 5y = 10 - 2x$

$\{-\ 2x$ from both sides$\}$

$\therefore \quad y = \dfrac{10 - 2x}{5}$

$\{\div$ both sides by $5\}$

$\therefore \quad y = \dfrac{{}^{2}\cancel{10}}{{}_{1}\cancel{5}} - \dfrac{2x}{5}$

$\therefore \quad y = 2 - \frac{2}{5}x$

**b** $3x + 4y = 20$

$\therefore \quad 4y = 20 - 3x$

$\{-\ 3x$ from both sides$\}$

$\therefore \quad y = \dfrac{20 - 3x}{4}$

$\{\div$ both sides by $4\}$

$\therefore \quad y = \dfrac{{}^{5}\cancel{20}}{{}_{1}\cancel{4}} - \dfrac{3x}{4}$

$\therefore \quad y = 5 - \frac{3}{4}x$

**c** $2x - y = 8$

$\therefore \quad -y = 8 - 2x$

$\{-\ 2x$ from both sides$\}$

$\therefore \quad y = 2x - 8$

$\{\times$ both sides by $-1\}$

**d** $2x + 7y = 14$

$\therefore \quad 7y = 14 - 2x$

$\{-\ 2x$ from both sides$\}$

$\therefore \quad y = \dfrac{14 - 2x}{7}$

$\{\div$ both sides by $7\}$

$\therefore \quad y = \dfrac{{}^{2}\cancel{14}}{{}_{1}\cancel{7}} - \dfrac{2x}{7}$

$\therefore \quad y = 2 - \frac{2}{7}x$

**e** $5x + 2y = 20$

$\therefore \quad 2y = 20 - 5x$

$\{-\ 5x$ from both sides$\}$

$\therefore \quad y = \dfrac{20 - 5x}{2}$

$\{\div$ both sides by $2\}$

$\therefore \quad y = \dfrac{{}^{10}\cancel{20}}{{}_{1}\cancel{2}} - \dfrac{5x}{2}$

$\therefore \quad y = 10 - \frac{5}{2}x$

**f** $2x - 3y = -12$

$\therefore \quad -3y = -12 - 2x$

$\{-\ 2x$ from both sides$\}$

$\therefore \quad y = \dfrac{2x + 12}{3}$

$\{\div$ both sides by $-3\}$

$\therefore \quad y = \dfrac{2x}{3} + \dfrac{\cancel{12}^{\,4}}{\cancel{3}_{\,1}}$

$\therefore \quad y = \frac{2}{3}x + 4$

**2** **a** $p + x = r$

$\therefore \quad x = r - p \quad \{-\ p$ from both sides$\}$

**b** $xy = z$

$\therefore \quad x = \dfrac{z}{y} \quad \{\div$ both sides by $y\}$

**c** $3x + a = d$

$\therefore \quad 3x = d - a \quad \{-\ a$ from both sides$\}$

$\therefore \quad x = \dfrac{d - a}{3} \quad \{\div$ both sides by $3\}$

**d** $5x + 2y = d$

$\therefore \quad 5x = d - 2y \quad \{-\ 2y$ from both sides$\}$

$\therefore \quad x = \dfrac{d - 2y}{5} \quad \{\div$ both sides by $5\}$

**e** $ax + by = p$

$\therefore \quad ax = p - by \quad \{-\ by$ from both sides$\}$

$\therefore \quad x = \dfrac{p - by}{a} \quad \{\div$ both sides by $a\}$

**f** $y = mx + c$

$\therefore \quad mx + c = y \quad \{$swap sides$\}$

$\therefore \quad mx = y - c \quad \{-\ c$ from both sides$\}$

$\therefore \quad x = \dfrac{y - c}{m} \quad \{\div$ both sides by $m\}$

**g** $2 + tx = s$

$\therefore \quad tx = s - 2 \quad \{-\ 2$ from both sides$\}$

$\therefore \quad x = \dfrac{s - 2}{t} \quad \{\div$ both sides by $t\}$

**h** $p + qx = m$

$\therefore \quad qx = m - p \quad \{-\ p$ from both sides$\}$

$\therefore \quad x = \dfrac{m - p}{q} \quad \{\div$ both sides by $q\}$

**i**
$$\begin{aligned} 6 &= a+bx \\ \therefore\ a+bx &= 6 && \{\text{swap sides}\} \\ \therefore\ bx &= 6-a && \{-\,a \text{ from both sides}\} \\ \therefore\ x &= \frac{6-a}{b} && \{\div \text{ both sides by } b\} \end{aligned}$$

**3** **a**
$$\begin{aligned} z &= t-5y \\ \therefore\ z+5y &= t && \{+\,5y \text{ to both sides}\} \\ \therefore\ 5y &= t-z && \{-\,z \text{ from both sides}\} \\ \therefore\ y &= \frac{t-z}{5} && \{\div \text{ both sides by } 5\} \end{aligned}$$

**b**
$$\begin{aligned} c-2y &= p \\ \therefore\ -2y &= p-c && \{-\,c \text{ from both sides}\} \\ \therefore\ y &= \frac{p-c}{-2} && \{\div \text{ both sides by } -2\} \\ \therefore\ y &= \frac{c-p}{2} && \left\{\times\ \frac{-1}{-1}\right\} \end{aligned}$$

**c**
$$\begin{aligned} a-3y &= t \\ \therefore\ -3y &= t-a && \{-\,a \text{ from both sides}\} \\ \therefore\ y &= \frac{t-a}{-3} && \{\div \text{ both sides by } -3\} \\ \therefore\ y &= \frac{a-t}{3} && \left\{\times\ \frac{-1}{-1}\right\} \end{aligned}$$

**d**
$$\begin{aligned} n-ky &= 5 \\ \therefore\ -ky &= 5-n && \{-\,n \text{ from both sides}\} \\ \therefore\ y &= \frac{5-n}{-k} && \{\div \text{ both sides by } -k\} \\ \therefore\ y &= \frac{n-5}{k} && \left\{\times\ \frac{-1}{-1}\right\} \end{aligned}$$

**e**
$$\begin{aligned} a-by &= n \\ \therefore\ -by &= n-a && \{-\,a \text{ from both sides}\} \\ \therefore\ y &= \frac{n-a}{-b} && \{\div \text{ both sides by } -b\} \\ \therefore\ y &= \frac{a-n}{b} && \left\{\times\ \frac{-1}{-1}\right\} \end{aligned}$$

**f**
$$\begin{aligned} p &= a-ny \\ \therefore\ p+ny &= a && \{+\,ny \text{ to both sides}\} \\ \therefore\ ny &= a-p && \{-\,p \text{ from both sides}\} \\ \therefore\ y &= \frac{a-p}{n} && \{\div \text{ both sides by } n\} \end{aligned}$$

**4** **a**
$$\begin{aligned} az &= \frac{b}{c} \\ \therefore\ z &= \frac{b}{ac} && \{\div \text{ both sides by } a\} \end{aligned}$$

**b**
$$\begin{aligned} \frac{a}{z} &= d \\ \therefore\ a &= dz && \{\times \text{ both sides by } z\} \\ \therefore\ \frac{a}{d} &= z && \{\div \text{ both sides by } d\} \\ \therefore\ z &= \frac{a}{d} && \{\text{swap sides}\} \end{aligned}$$

**c**
$$\begin{aligned} \frac{3}{d} &= \frac{2}{z} \\ \therefore\ \frac{3z}{d} &= 2 && \{\times \text{ both sides by } z\} \\ \therefore\ z &= \frac{2d}{3} && \{\times \text{ both sides by } \tfrac{d}{3}\} \end{aligned}$$

**d**
$$\begin{aligned} \frac{z}{2} &= \frac{a}{z} \\ \therefore\ \frac{z^2}{2} &= a && \{\times \text{ both sides by } z\} \\ \therefore\ z^2 &= 2a && \{\times \text{ both sides by } 2\} \\ \therefore\ z &= \pm\sqrt{2a} \end{aligned}$$

**e**
$$\begin{aligned} \frac{b}{z} &= \frac{z}{n} \\ \therefore\ bn &= z^2 && \{\times \text{ both sides by } nz\} \\ \therefore\ z &= \pm\sqrt{bn} \end{aligned}$$

**f**
$$\begin{aligned} \frac{m}{z} &= \frac{z}{a-b} \\ \therefore\ m &= \frac{z^2}{a-b} \\ & \{\times \text{ both sides by } z\} \\ \therefore\ m(a-b) &= z^2 \\ & \{\times \text{ both sides by } (a-b)\} \\ \therefore\ z &= \pm\sqrt{m(a-b)} \end{aligned}$$

**5** **a**
$$\begin{aligned} F &= ma \\ \therefore\ ma &= F && \{\text{swap sides}\} \\ \therefore\ a &= \frac{F}{m} && \{\div \text{ both sides by } m\} \end{aligned}$$

**b**
$$\begin{aligned} C &= 2\pi r \\ \therefore\ 2\pi r &= C && \{\text{swap sides}\} \\ \therefore\ r &= \frac{C}{2\pi} && \{\div \text{ both sides by } 2\pi\} \end{aligned}$$

**c**

$$\begin{aligned} V &= ldh \\ \therefore\ ldh &= V && \{\text{swap sides}\} \\ \therefore\ d &= \frac{V}{lh} && \{\div \text{ both sides by } lh\} \end{aligned}$$

**d**

$$\begin{aligned} A &= \frac{b}{K} \\ \therefore\ AK &= b && \{\times \text{ both sides by } K\} \\ \therefore\ K &= \frac{b}{A} && \{\div \text{ both sides by } A\} \end{aligned}$$

**e**

$$\begin{aligned} A &= \frac{bh}{2} \\ \therefore\ \frac{bh}{2} &= A && \{\text{swap sides}\} \\ \therefore\ bh &= 2A && \{\times \text{ both sides by } 2\} \\ \therefore\ h &= \frac{2A}{b} && \{\div \text{ both sides by } b\} \end{aligned}$$

**f**

$$\begin{aligned} I &= \frac{PRT}{100} \\ \therefore\ \frac{PRT}{100} &= I && \{\text{swap sides}\} \\ \therefore\ PRT &= 100I && \{\times \text{ both sides by } 100\} \\ \therefore\ T &= \frac{100I}{PR} && \{\div \text{ both sides by } PR\} \end{aligned}$$

**6**

$$\begin{aligned} A &= 2\pi r^2 + 2\pi rh \\ \therefore\ 2\pi r^2 + 2\pi rh &= A && \{\text{swap sides}\} \\ \therefore\ 2\pi rh &= A - 2\pi r^2 && \{- 2\pi r^2 \text{ from both sides}\} \\ \therefore\ h &= \frac{A - 2\pi r^2}{2\pi r} && \{\div \text{ both sides by } 2\pi r\} \\ \therefore\ h &= \frac{A}{2\pi r} - r \end{aligned}$$

**7** **a**

$$\begin{aligned} A &= \pi r^2, \quad r > 0 \\ \therefore\ \pi r^2 &= A && \{\text{swap sides}\} \\ \therefore\ r^2 &= \frac{A}{\pi} && \{\div \text{ both sides by } \pi\} \\ \therefore\ r &= \sqrt{\frac{A}{\pi}} && \{\text{as } \ r > 0\} \end{aligned}$$

**b**

$$\begin{aligned} N &= \frac{x^5}{a} \\ \therefore\ \frac{x^5}{a} &= N && \{\text{swap sides}\} \\ \therefore\ x^5 &= aN && \{\times \text{ both sides by } a\} \\ \therefore\ x &= \sqrt[5]{aN} && \{\text{fifth root both sides}\} \end{aligned}$$

**8** **a**

$$\begin{aligned} M &= 5k^3 \\ \therefore\ 5k^3 &= M && \{\text{swap sides}\} \\ \therefore\ k^3 &= \frac{M}{5} && \{\div \text{ both sides by } 5\} \\ \therefore\ k &= \sqrt[3]{\frac{M}{5}} && \{\text{cube root both sides}\} \end{aligned}$$

**b**

$$\begin{aligned} D &= \frac{n}{x^3} \\ \therefore\ Dx^3 &= n && \{\times \text{ both sides by } x^3\} \\ \therefore\ x^3 &= \frac{n}{D} && \{\div \text{ both sides by } D\} \\ \therefore\ x &= \sqrt[3]{\frac{n}{D}} && \{\text{cube root both sides}\} \end{aligned}$$

**c**

$$\begin{aligned} y &= 4x^2 - 7 \\ \therefore\ y + 7 &= 4x^2 && \{+ 7 \text{ to both sides}\} \\ \therefore\ \frac{y+7}{4} &= x^2 && \{\div \text{ both sides by } 4\} \\ \therefore\ x &= \pm\sqrt{\frac{y+7}{4}} \\ \therefore\ x &= \pm\frac{\sqrt{y+7}}{2} \end{aligned}$$

**d**

$$\begin{aligned} P^2 &= Q^2 + R^2 \\ \therefore\ P^2 - R^2 &= Q^2 && \{- R^2 \text{ from both sides}\} \\ \therefore\ Q &= \pm\sqrt{P^2 - R^2} \end{aligned}$$

**9** **a**

$$\begin{aligned} d &= \frac{\sqrt{a}}{n} \\ \therefore\ \frac{\sqrt{a}}{n} &= d && \{\text{swap sides}\} \\ \therefore\ \sqrt{a} &= dn && \{\times \text{ both sides by } n\} \\ \therefore\ a &= d^2n^2 && \{\text{square both sides}\} \end{aligned}$$

**b**

$$\begin{aligned} T &= \tfrac{1}{5}\sqrt{l} \\ \therefore\ \tfrac{1}{5}\sqrt{l} &= T && \{\text{swap sides}\} \\ \therefore\ \sqrt{l} &= 5T && \{\times \text{ both sides by } 5\} \\ \therefore\ l &= 25T^2 && \{\text{square both sides}\} \end{aligned}$$

**c**

$$c = \sqrt{a^2 - b^2}$$

$$\therefore\ c^2 = a^2 - b^2 \qquad \{\text{square both sides}\}$$

$$\therefore\ c^2 + b^2 = a^2 \qquad \{+\,b^2 \text{ to both sides}\}$$

$$\therefore\ a = \pm\sqrt{b^2 + c^2} \qquad \{\text{swap sides}\}$$

**d**

$$\frac{k}{a} = \frac{5}{\sqrt{d}}$$

$$\therefore\ k\sqrt{d} = 5a \qquad \{\times \text{ both sides by } a\sqrt{d}\}$$

$$\therefore\ \sqrt{d} = \frac{5a}{k} \qquad \{\div \text{ both sides by } k\}$$

$$\therefore\ d = \left(\frac{5a}{k}\right)^2 \qquad \{\text{square both sides}\}$$

$$\therefore\ d = \frac{25a^2}{k^2}$$

**e**

$$T = 2\pi\sqrt{\frac{l}{g}}$$

$$\therefore\ 2\pi\sqrt{\frac{l}{g}} = T \qquad \{\text{swap sides}\}$$

$$\therefore\ \sqrt{\frac{l}{g}} = \frac{T}{2\pi} \qquad \{\div \text{ both sides by } 2\pi\}$$

$$\therefore\ \frac{l}{g} = \left(\frac{T}{2\pi}\right)^2 \qquad \{\text{square both sides}\}$$

$$\therefore\ \frac{l}{g} = \frac{T^2}{4\pi^2}$$

$$\therefore\ l = \frac{gT^2}{4\pi^2} \qquad \{\times \text{ both sides by } g\}$$

**f**

$$A = 4\sqrt{\frac{a}{b}}$$

$$\therefore\ \frac{A}{4} = \sqrt{\frac{a}{b}} \qquad \{\div \text{ both sides by } 4\}$$

$$\therefore\ \left(\frac{A}{4}\right)^2 = \frac{a}{b} \qquad \{\text{square both sides}\}$$

$$\therefore\ \frac{A^2}{16} = \frac{a}{b}$$

$$\therefore\ bA^2 = 16a \qquad \{\times \text{ both sides by } 16b\}$$

$$\therefore\ b = \frac{16a}{A^2} \qquad \{\div \text{ both sides by } A^2\}$$

**10** **a**

$$3x + a = bx + c$$

$$\therefore\ 3x - bx = c - a \qquad \{-\,bx - a \text{ from both sides}\}$$

$$\therefore\ x(3 - b) = c - a \qquad \{x \text{ is a common factor on the LHS}\}$$

$$\therefore\ x = \frac{c - a}{3 - b} \qquad \{\div \text{ both sides by } (3 - b)\}$$

**b**

$$ax = c - bx$$

$$\therefore\ ax + bx = c \qquad \{+\,bx \text{ to both sides}\}$$

$$\therefore\ x(a + b) = c \qquad \{x \text{ is a common factor on the LHS}\}$$

$$\therefore\ x = \frac{c}{a + b} \qquad \{\div \text{ both sides by } (a + b)\}$$

**c**

$$mx + a = nx - 2$$

$$\therefore\ mx - nx = -a - 2 \qquad \{\text{writing the terms containing } x \text{ on the LHS}\}$$

$$\therefore\ x(m - n) = -a - 2 \qquad \{x \text{ is a common factor on the LHS}\}$$

$$\therefore\ x = \frac{-a - 2}{m - n} \qquad \{\div \text{ both sides by } (m - n)\}$$

$$\therefore\ x = \frac{a + 2}{n - m} \qquad \left\{\times\ \frac{-1}{-1}\right\}$$

**d**

$$8x + a = -bx$$

$$\therefore\ 8x + bx = -a \qquad \{\text{writing the terms containing } x \text{ on the LHS}\}$$

$$\therefore\ x(b + 8) = -a \qquad \{x \text{ is a common factor on the LHS}\}$$

$$\therefore\ x = \frac{-a}{b + 8} \qquad \{\div \text{ both sides by } (b + 8)\}$$

**e**
$$\begin{aligned} a - x &= b - cx \\ \therefore\ cx - x &= b - a && \{\text{writing the terms containing } x \text{ on the LHS}\} \\ \therefore\ x(c-1) &= b - a && \{x \text{ is a common factor on the LHS}\} \\ \therefore\ x &= \frac{b-a}{c-1} && \{\div \text{ both sides by } (c-1)\} \\ \text{or}\quad x &= \frac{a-b}{1-c} \end{aligned}$$

**f**
$$\begin{aligned} rx + d &= e - sx \\ \therefore\ rx + sx &= e - d && \{\text{writing the terms containing } x \text{ on the LHS}\} \\ \therefore\ x(r+s) &= e - d && \{x \text{ is a common factor on the LHS}\} \\ \therefore\ x &= \frac{e-d}{r+s} && \{\div \text{ both sides by } (r+s)\} \end{aligned}$$

**11** **a**
$$\begin{aligned} P &= \frac{2}{a+b} \\ \therefore\ P(a+b) &= 2 && \{\times \text{ both sides by } (a+b)\} \\ \therefore\ Pa + Pb &= 2 && \{\text{expanding}\} \\ \therefore\ Pa &= 2 - Pb && \{-\,Pb \text{ from both sides}\} \\ \therefore\ a &= \frac{2-Pb}{P} && \{\div \text{ both sides by } P\} \\ \text{or}\quad a &= \frac{2}{P} - b \end{aligned}$$

**b**
$$\begin{aligned} T &= \frac{8}{q+r} \\ \therefore\ T(q+r) &= 8 && \{\times \text{ both sides by } (q+r)\} \\ \therefore\ qT + rT &= 8 && \{\text{expanding}\} \\ \therefore\ rT &= 8 - qT && \{-\,qT \text{ from both sides}\} \\ \therefore\ r &= \frac{8-qT}{T} && \{\div \text{ both sides by } T\} \\ \text{or}\quad r &= \frac{8}{T} - q \end{aligned}$$

**c**
$$\begin{aligned} A &= \frac{B}{p-q} \\ \therefore\ A(p-q) &= B && \{\times \text{ both sides by } (p-q)\} \\ \therefore\ Ap - Aq &= B && \{\text{expanding}\} \\ \therefore\ -Aq &= -Ap + B && \{-\,Ap \text{ from both sides}\} \\ \therefore\ Aq &= Ap - B && \{\times \text{ both sides by } -1\} \\ \therefore\ q &= \frac{Ap-B}{A} && \{\div \text{ both sides by } A\} \\ \text{or}\quad q &= p - \frac{B}{A} \end{aligned}$$

**d**
$$\begin{aligned} A &= \frac{3}{2x+y} \\ \therefore\ A(2x+y) &= 3 && \{\times \text{ both sides by } (2x+y)\} \\ \therefore\ 2Ax + Ay &= 3 && \{\text{expanding}\} \\ \therefore\ 2Ax &= 3 - Ay && \{-\,Ay \text{ from both sides}\} \\ \therefore\ x &= \frac{3-Ay}{2A} && \{\div \text{ both sides by } 2A\} \end{aligned}$$

**e** $M = \dfrac{4}{x^2+y^2}, \quad y > 0$

$\therefore \; M(x^2+y^2) = 4$ {× both sides by $(x^2+y^2)$}

$\therefore \; Mx^2 + My^2 = 4$ {expanding}

$\therefore \; My^2 = 4 - Mx^2$ {$-\,Mx^2$ from both sides}

$\therefore \; y^2 = \dfrac{4 - Mx^2}{M}$ {÷ both sides by $M$}

$\therefore \; y = \sqrt{\dfrac{4 - Mx^2}{M}}$ {as $y > 0$}

or $y = \sqrt{\dfrac{4}{M} - x^2}$

**12** **a** $y = \dfrac{x}{x+1}$

$\therefore \; y(x+1) = x$ {× both sides by $(x+1)$}

$\therefore \; xy + y = x$ {expanding}

$\therefore \; xy - x = -y$ {writing the terms containing $x$ on the LHS}

$\therefore \; x - xy = y$ {× both sides by $-1$}

$\therefore \; x(1-y) = y$ {$x$ is a common factor on the LHS}

$\therefore \; x = \dfrac{y}{1-y}$ {÷ both sides by $(1-y)$}

**b** $y = \dfrac{x-3}{x+2}$

$\therefore \; y(x+2) = x - 3$ {× both sides by $(x+2)$}

$\therefore \; xy + 2y = x - 3$ {expanding}

$\therefore \; xy - x = -2y - 3$ {writing the terms containing $x$ on the LHS}

$\therefore \; x - xy = 2y + 3$ {× both sides by $-1$}

$\therefore \; x(1-y) = 2y + 3$ {$x$ is a common factor on the LHS}

$\therefore \; x = \dfrac{2y+3}{1-y}$ {÷ both sides by $(1-y)$}

**c** $y = \dfrac{3x-1}{x+3}$

$\therefore \; y(x+3) = 3x - 1$ {× both sides by $(x+3)$}

$\therefore \; xy + 3y = 3x - 1$ {expanding}

$\therefore \; xy - 3x = -3y - 1$ {writing the terms containing $x$ on the LHS}

$\therefore \; 3x - xy = 3y + 1$ {× both sides by $-1$}

$\therefore \; x(3-y) = 3y + 1$ {$x$ is a common factor on the LHS}

$\therefore \; x = \dfrac{3y+1}{3-y}$ {÷ both sides by $(3-y)$}

**d** $y = \dfrac{4x-1}{2-x}$

$\therefore \; y(2-x) = 4x - 1$ {× both sides by $(2-x)$}

$\therefore \; 2y - xy = 4x - 1$ {expanding}

$\therefore \; -xy - 4x = -2y - 1$ {writing the terms containing $x$ on the LHS}

$\therefore \; xy + 4x = 2y + 1$ {× both sides by $-1$}

$\therefore \; x(y+4) = 2y + 1$ {$x$ is a common factor on the LHS}

$\therefore \; x = \dfrac{2y+1}{y+4}$ {÷ both sides by $(y+4)$}

**e**

$$\begin{aligned}
y &= \frac{3x+7}{3-2x} \\
\therefore\quad y(3-2x) &= 3x+7 && \{\times \text{ both sides by } (3-2x)\} \\
\therefore\quad 3y-2xy &= 3x+7 && \{\text{expanding}\} \\
\therefore\quad -2xy-3x &= -3y+7 && \{\text{writing the terms containing } x \text{ on the LHS}\} \\
\therefore\quad 2xy+3x &= 3y-7 && \{\times \text{ both sides by } -1\} \\
\therefore\quad x(2y+3) &= 3y-7 && \{x \text{ is a common factor on the LHS}\} \\
\therefore\quad x &= \frac{3y-7}{2y+3} && \{\div \text{ both sides by } (2y+3)\}
\end{aligned}$$

**f**

$$\begin{aligned}
y &= 1+\frac{2}{x-3} \\
\therefore\quad y(x-3) &= (x-3)+2 && \{\times \text{ all terms by } (x-3)\} \\
\therefore\quad xy-3y &= x-1 && \{\text{expanding, collecting like terms}\} \\
\therefore\quad xy-x &= 3y-1 && \{\text{writing the terms containing } x \text{ on the LHS}\} \\
\therefore\quad x(y-1) &= 3y-1 && \{x \text{ is a common factor on the LHS}\} \\
\therefore\quad x &= \frac{3y-1}{y-1} && \{\div \text{ both sides by } (y-1)\}
\end{aligned}$$

**g**

$$\begin{aligned}
y &= -2+\frac{5}{x+4} \\
\therefore\quad y+2 &= \frac{5}{x+4} && \{+\,2 \text{ to both sides}\} \\
\therefore\quad (y+2)(x+4) &= 5 && \{\times \text{ both sides by } (x+4)\} \\
\therefore\quad x+4 &= \frac{5}{y+2} && \{\div \text{ both sides by } (y+2)\} \\
\therefore\quad x &= \frac{5}{y+2}-4 && \{-\,4 \text{ from both sides}\} \\
\therefore\quad x &= \frac{5}{y+2}-\frac{4(y+2)}{y+2} \\
\therefore\quad x &= \frac{5-4y-8}{y+2} && \{\text{expanding}\} \\
\therefore\quad x &= \frac{-4y-3}{y+2} && \{\text{collecting like terms}\}
\end{aligned}$$

**h**

$$\begin{aligned}
y &= -3-\frac{6}{x-2} \\
\therefore\quad y+3 &= \frac{-6}{x-2} && \{+\,3 \text{ to both sides}\} \\
\therefore\quad (y+3)(x-2) &= -6 && \{\times \text{ both sides by } (x-2)\} \\
\therefore\quad x-2 &= \frac{-6}{y+3} && \{\div \text{ both sides by } (y+3)\} \\
\therefore\quad x &= 2-\frac{6}{y+3} && \{+\,2 \text{ to both sides}\} \\
\therefore\quad x &= \frac{2(y+3)}{(y+3)}-\frac{6}{(y+3)} \\
\therefore\quad x &= \frac{2y+6-6}{y+3} && \{\text{expanding}\} \\
\therefore\quad x &= \frac{2y}{y+3} && \{\text{collecting like terms}\}
\end{aligned}$$

**13** $$N = \frac{1}{1-p^2}, \qquad N > 0,\ p > 0$$

$$\therefore\ N(1-p^2) = 1 \qquad \{\times \text{ both sides by } (1-p^2)\}$$

$$\therefore\ 1 - p^2 = \frac{1}{N} \qquad \{\div \text{ both sides by } N\}$$

$$\therefore\ -p^2 = \frac{1}{N} - 1 \qquad \{-\,1 \text{ from both sides}\}$$

$$\therefore\ p^2 = 1 - \frac{1}{N} \qquad \{\times \text{ both sides by } -1\}$$

$$\therefore\ p = \sqrt{1 - \frac{1}{N}} \qquad \{\text{as } N > 0,\ p > 0\}$$

## EXERCISE 8D

**1** **a** $$A = \frac{\theta}{360} \times \pi r^2$$

$$\therefore\ \frac{\theta}{360} \times \pi r^2 = A \qquad \{\text{swap sides}\}$$

$$\therefore\ \theta \times \pi r^2 = 360A \qquad \{\times \text{ both sides by } 360\}$$

$$\therefore\ \theta = \frac{360A}{\pi r^2} \qquad \{\div \text{ both sides by } \pi r^2, \text{ as } r > 0\}$$

**b** **i** If $r = 3$ and $A = 5$,

$$\theta = \frac{360 \times 5}{\pi \times 3^2}$$

$$\therefore\ \theta \approx 63.7^\circ$$

**ii** If $r = 7$ and $A = 45$,

$$\theta = \frac{360 \times 45}{\pi \times 7^2}$$

$$\therefore\ \theta \approx 105^\circ$$

**iii** If $r = 8.5$ and $A = 135$,

$$\theta = \frac{360 \times 135}{\pi \times 8.5^2}$$

$$\therefore\ \theta \approx 214^\circ$$

**2** **a** $$K = \frac{d^2}{2ab}$$

$$\therefore\ aK = \frac{d^2}{2b} \qquad \{\times \text{ both sides by } a\}$$

$$\therefore\ a = \frac{d^2}{2bK} \qquad \{\div \text{ both sides by } K\}$$

**b** **i** If $K = 112,\ d = 24,\ b = 2$,

$$a = \frac{24^2}{2 \times 2 \times 112}$$

$$\therefore\ a \approx 1.29$$

**ii** If $K = 400,\ d = 72,\ b = 0.4$,

$$a = \frac{72^2}{2 \times 0.4 \times 400}$$

$$\therefore\ a = 16.2$$

**3** **a** $$H = 1 + \sqrt{t}$$

$$\therefore\ \sqrt{t} + 1 = H \qquad \{\text{swap sides}\}$$

$$\therefore\ \sqrt{t} = H - 1 \qquad \{-\,1 \text{ from both sides}\}$$

$$\therefore\ t = (H-1)^2 \qquad \{\text{squaring both sides}\}$$

**b** **i** If $H = 2,\ t = (2-1)^2 = 1$.
So it will take 1 year for a bush to reach 2 m in height.

**ii** If $H = 3,\ t = (3-1)^2 = 4$.
So it will take 4 years for a bush to reach 3 m in height.

**iii** If $H = 3.5,\ t = (3.5-1)^2 = 6.25$.
So it will take $6\frac{1}{4}$ years for a bush to reach $3\frac{1}{2}$ m in height.

**4** **a**
$$
\begin{aligned}
V &= \tfrac{4}{3}\pi r^3 \\
\therefore\ \frac{V}{\frac{4}{3}\pi} &= r^3 && \{\div \text{ both sides by } \tfrac{4}{3}\pi\} \\
\therefore\ \frac{3V}{4\pi} &= r^3 \\
\therefore\ \sqrt[3]{\frac{3V}{4\pi}} &= r && \{\text{cube root both sides}\} \\
\therefore\ r &= \sqrt[3]{\frac{3V}{4\pi}} && \{\text{swap sides}\}
\end{aligned}
$$

**b** **i** If $V = 40$, $r = \sqrt[3]{\dfrac{3 \times 40}{4\pi}}$
$$= \sqrt[3]{\frac{120}{4\pi}}$$
$\therefore\ r \approx 2.12$

$\therefore$ radius is approximately 2.12 cm.

**ii** If $V = 800$, $r = \sqrt[3]{\dfrac{3 \times 800}{4\pi}}$
$$\approx \sqrt[3]{\frac{600}{\pi}}$$
$\therefore\ r \approx 5.76$

$\therefore$ radius is approximately 5.76 cm.

**iii** If $V = 1\,000\,000$, $r = \sqrt[3]{\dfrac{3 \times 1\,000\,000}{4\pi}}$
$$= \sqrt[3]{\frac{3\,000\,000}{4\pi}}$$
$\therefore\ r \approx 62.0$

$\therefore$ radius is approximately 62.0 cm.

**5** **a** $v^2 - u^2 = 2as,\ v \geqslant 0$
$$
\begin{aligned}
\therefore\ v^2 &= u^2 + 2as && \{+\,u^2 \text{ to both sides}\} \\
\therefore\ v &= \sqrt{u^2 + 2as} && \{\text{as } v \geqslant 0\}
\end{aligned}
$$

**b** **i** If $s = 100$, $u = 5$, and $a = 2$,
$$
\begin{aligned}
v &= \sqrt{5^2 + 2 \times 2 \times 100} \\
&= \sqrt{425} \\
&\approx 20.6
\end{aligned}
$$
$\therefore$ the final speed is approximately 20.6 m/s.

**ii** 1.5 km = 1500 m

If $s = 1500$, $u = 10$, and $a = 0.9$,
$$
\begin{aligned}
v &= \sqrt{10^2 + 2 \times 1500 \times 0.9} \\
&= \sqrt{2800} \\
&\approx 52.9
\end{aligned}
$$
$\therefore$ the final speed is approximately 52.9 m/s.

**6** **a** $P = \dfrac{w}{w+l} \times 100\%$

If $w = 10$ and $l = 7$, the winning percentage $P = \dfrac{10}{17} \times 100\%$
$$\approx 58.8\%$$

**b**
$$
\begin{aligned}
P &= \frac{w}{w+l} \times 100 \\
\therefore\ P(w+l) &= 100w && \{\times \text{ both sides by } (w+l)\} \\
\therefore\ Pw + Pl &= 100w && \{\text{expanding}\} \\
\therefore\ Pw - 100w &= -Pl && \{\text{writing the terms containing } w \text{ on the LHS}\} \\
\therefore\ w(P - 100) &= -Pl && \{w \text{ is a common factor on the LHS}\} \\
\therefore\ w &= \frac{-Pl}{P - 100} && \{\div \text{ both sides by } (P-100)\} \\
\therefore\ w &= \frac{Pl}{100 - P} && \left\{\times \frac{-1}{-1}\right\}
\end{aligned}
$$

**c** If $l = 15$ and $P = 37.5$,

$$\begin{aligned} w &= \frac{37.5 \times 15}{100 - 37.5} \\ &= 9 \end{aligned}$$

So, Martina has won 9 matches.

**d** Suppose Claude needs to win $x$ more consecutive matches.
Then $w = (84 + x)$, $l = 49$, and $P = 65$

$$\begin{aligned} \therefore\ 84 + x &= \frac{65 \times 49}{100 - 65} \\ \therefore\ 84 + x &= 91 \\ \therefore\ x &= 91 - 84 = 7 \end{aligned}$$

So, Claude must win 7 more consecutive matches.

**7** **a**

$$\begin{aligned} A &= 2(ab + ac + bc) \\ \therefore\ \frac{A}{2} &= ab + ac + bc && \{\div \text{ both sides by } 2\} \\ \therefore\ \frac{A}{2} - ab &= ac + bc && \{-\ ab \text{ from both sides}\} \\ \therefore\ ac + bc &= \frac{A}{2} - ab && \{\text{swap sides}\} \\ \therefore\ c(a + b) &= \frac{A}{2} - ab && \{c \text{ is a common factor on the LHS}\} \\ \therefore\ c &= \frac{\frac{A}{2} - ab}{a + b} && \{\div \text{ both sides by } (a + b)\} \\ \therefore\ c &= \frac{A - 2ab}{2(a + b)} && \left\{\times\ \frac{2}{2}\right\} \end{aligned}$$

**b** **i** $A = 180$, $a = 8$, $b = 6$

$$\begin{aligned} \therefore\ c &= \frac{180 - 2 \times 8 \times 6}{2(8 + 6)} \\ &= \frac{96}{2 \times 14} \\ &= 3 \end{aligned}$$

**ii** $A = 102$, $a = b = 3$

$$\begin{aligned} \therefore\ c &= \frac{102 - 2 \times 3 \times 3}{2(3 + 3)} \\ &= \frac{84}{12} \\ &= 7 \end{aligned}$$

**iii** $A = 531$, $a = 9$, $b = 12$

$$\begin{aligned} \therefore\ c &= \frac{531 - 2 \times 9 \times 12}{2(9 + 12)} \\ &= \frac{315}{42} \\ &= 7.5 \end{aligned}$$

**8** **a**

$$\begin{aligned} F &= G\,\frac{m_1 m_2}{d^2} \\ \therefore\ F &= \frac{6.67 \times 10^{-11} \times 5.97 \times 10^{24} \times 7.35 \times 10^{22}}{(3.82 \times 10^8)^2} \\ &\approx 20.1 \times 10^{19} \end{aligned}$$

$\therefore$ gravitational force $\approx 2.01 \times 10^{20}$ Newtons

**b**

$$\begin{aligned} F &= G\,\frac{m_1 m_2}{d^2}, && G,\ m_1,\ m_2,\ F > 0 \\ \therefore\ Fd^2 &= Gm_1 m_2 && \{\times \text{ both sides by } d^2\} \\ \therefore\ d^2 &= \frac{Gm_1 m_2}{F} && \{\div \text{ both sides by } F\} \\ \therefore\ d &= \sqrt{\frac{Gm_1 m_2}{F}} && \{\text{as } G,\ m_1,\ m_2,\ F > 0\} \end{aligned}$$

**c** **i**

$$\begin{aligned} d &= \sqrt{\frac{6.67 \times 10^{-11} \times 1.99 \times 10^{30} \times 5.97 \times 10^{24}}{3.54 \times 10^{22}}} \\ &\approx 1.50 \times 10^{11} \end{aligned}$$

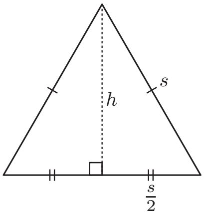
$\therefore$ distance $\approx 1.50 \times 10^{11}$ m.

**ii**

$$\begin{aligned} d &= \sqrt{\frac{6.67 \times 10^{-11} \times (2.32 \times 10^{26})^2}{1.76 \times 10^{14}}} \\ &\approx 1.43 \times 10^{14} \end{aligned}$$

$\therefore$ distance $\approx 1.43 \times 10^{14}$ m.

**9** **a** Surface area of a pyramid = area of base + area of 4 triangular sides



Using Pythagoras, $h^2 = s^2 - \left(\frac{s}{2}\right)^2$

$\therefore\ h^2 = s^2(1 - \frac{1}{4})$ {$s^2$ is a common factor on the RHS}

$\therefore\ h^2 = \frac{3}{4}s^2$

$\therefore\ h = \frac{\sqrt{3}s}{2}$ {as $s > 0$}

$\therefore\ A = s^2 + 4 \times \frac{\sqrt{3}s}{2} \times \frac{s}{2}$

$\therefore\ A = s^2 + \sqrt{3}s^2$

$\therefore\ A = s^2(1 + \sqrt{3})$ {$s^2$ is a common factor on the RHS}

**b** $A = s^2(1 + \sqrt{3}),\ s > 0$

$\therefore\ s^2(1 + \sqrt{3}) = A$ {swap sides}

$\therefore\ s^2 = \frac{A}{1 + \sqrt{3}}$ {÷ both sides by $(1 + \sqrt{3})$}

$\therefore\ s^2 = \frac{A}{1 + \sqrt{3}} \times \frac{1 - \sqrt{3}}{1 - \sqrt{3}}$

$\therefore\ s^2 = \frac{A(1 - \sqrt{3})}{-2}$

$\therefore\ s = \sqrt{\frac{A}{2}(\sqrt{3} - 1)}$ {as $s > 0$}

**c** **i** If $A = 50$ cm$^2$

$s = \sqrt{\frac{50}{2}(\sqrt{3} - 1)}$

$\approx 4.28$

$\therefore$ side length is approximately 4.28 cm.

**ii** If $A = 150$ cm$^2$

$s = \sqrt{\frac{150}{2}(\sqrt{3} - 1)}$

$\approx 7.41$

$\therefore$ side length is approximately 7.41 cm.

**iii** If $A = 600$ cm$^2$

$s = \sqrt{\frac{600}{2}(\sqrt{3} - 1)}$

$\approx 14.8$ $\therefore$ side length is approximately 14.8 cm.

**10** **a** $g$ goals and $b$ behinds

$= g \times 6 + b$

$= 6g + b$ behinds

For this property,

$6g + b = g \times b$

$\therefore\ 6g + b = gb$

**b** $6g + b = gb$

$\therefore\ b - gb = -6g$ {writing the terms containing $b$ on the LHS}

$\therefore\ b(1 - g) = -6g$ {$b$ is a common factor on the LHS}

$\therefore\ b = \frac{-6g}{1 - g}$ {÷ both sides by $(1 - g)$}

$\therefore\ b = \frac{6g}{g - 1}$ $\left\{\times\ \frac{-1}{-1}\right\}$

**c** When $g = 0,\ b = \frac{0}{-1} = 0$ ✓

When $g = 1,\ b = \frac{6}{0}$ which is undefined ✗

When $g = 2,\ b = \frac{12}{1} = 12$ ✓

When $g = 3,\ b = \frac{18}{2} = 9$ ✓

When $g = 4,\ b = \frac{24}{3} = 8$ ✓

When $g=5,\ b=\frac{30}{4}$ which is not a whole number ✗

When $g=6,\ b=\frac{36}{5}$ which is not a whole number ✗

When $g=7,\ b=\frac{42}{6}=7$ ✓

So other scores are: $0g\ 0b,\ 2g\ 12b,\ 3g\ 9b,\ 4g\ 8b,$ and $7g\ 7b$.

**11** **a** **i**
$$C=\sqrt{\frac{h}{H}}$$
$$\therefore\ C=\sqrt{\frac{1}{1.5}}\approx 0.816$$

**ii**
$$C=\sqrt{\frac{h}{H}}$$
$$\therefore\ C=\sqrt{\frac{1.1}{2}}\approx 0.742$$

**b**
$$C=\sqrt{\frac{h}{H}}$$
$$\therefore\ \sqrt{\frac{h}{H}}=C \qquad \{\text{swap sides}\}$$
$$\therefore\ \frac{h}{H}=C^2 \qquad \{\text{squaring both sides}\}$$
$$\therefore\ h=C^2H \qquad \{\times \text{ both sides by } H\}$$

**c** Tennis ball has $H=5$ m, $C=0.728$
$$\therefore\ h=5\times 0.728^2 \approx 2.65$$
$\therefore$ bounce height is approximately 2.65 m.

Golf ball has $H=5$ m, $C=0.858$
$$\therefore\ h=5\times 0.858^2 \approx 3.68$$
$\therefore$ bounce height is approximately 3.68 m.

Marble has $H=5$ m, $C=0.657$
$$\therefore\ h=5\times 0.657^2 \approx 2.16$$
$\therefore$ bounce height is approximately 2.16 m.

Superball has $H=5$ m, $C=0.893$
$$\therefore\ h=5\times 0.893^2 \approx 3.99$$
$\therefore$ bounce height is approximately 3.99 m.

**12** **a**
$$m=\frac{m_0}{\sqrt{1-\left(\frac{v}{c}\right)^2}}$$
$$\therefore\ m\sqrt{1-\left(\frac{v}{c}\right)^2}=m_0 \qquad \left\{\times \text{ both sides by } \sqrt{1-\left(\frac{v}{c}\right)^2}\right\}$$
$$\therefore\ \sqrt{1-\left(\frac{v}{c}\right)^2}=\frac{m_0}{m} \qquad \{\div \text{ both sides by } m\}$$
$$\therefore\ 1-\left(\frac{v}{c}\right)^2=\left(\frac{m_0}{m}\right)^2 \qquad \{\text{square both sides}\}$$
$$\therefore\ 1-\left(\frac{m_0}{m}\right)^2-\left(\frac{v}{c}\right)^2=0 \qquad \left\{-\left(\frac{m_0}{m}\right)^2 \text{ from both sides}\right\}$$
$$\therefore\ 1-\left(\frac{m_0}{m}\right)^2=\left(\frac{v}{c}\right)^2 \qquad \left\{+\left(\frac{v}{c}\right)^2 \text{ to both sides}\right\}$$
$$\therefore\ \frac{v^2}{c^2}=1-\frac{m_0^2}{m^2} \qquad \{\text{swap sides}\}$$
$$\therefore\ v^2=c^2\left(\frac{m^2-m_0^2}{m^2}\right) \qquad \{\times \text{ both sides by } c^2\}$$
$$\therefore\ v=\frac{c}{m}\sqrt{m^2-m_0^2} \qquad \{\sqrt{\ }\text{ both sides, speed } (v) \text{ is positive}\}$$

**b** If $m = 3m_0$, then

$$\begin{aligned} v &= \frac{c}{3m_0}\sqrt{(3m_0)^2 - {m_0}^2} \\ &= \frac{c}{3m_0}\sqrt{8{m_0}^2} \\ &= \frac{c\cancel{m_0}\sqrt{8}}{3\cancel{m_0}} \end{aligned}$$

$$\therefore\ v = \frac{\sqrt{8}}{3}c$$

or $v = \dfrac{2\sqrt{2}}{3}c$

**c** If $m = 30m_0$ and $c = 3 \times 10^8$, then

$$\begin{aligned} v &= \frac{3 \times 10^8}{30m_0}\sqrt{(30m_0)^2 - {m_0}^2} \\ &= \frac{3 \times 10^8}{30m_0}\sqrt{899{m_0}^2} \\ &= \frac{{}^{1}\cancel{3} \times 10^{\cancel{8}^{7}} \times \sqrt{899} \times \cancel{m_0}}{{}_{1}\cancel{10}\,\cancel{30}\cancel{m_0}} \\ &\approx 29.8 \times 10^7 \\ &\approx 2.98 \times 10^8 \end{aligned}$$

$\therefore$ the speed was approximately $3 \times 10^8$ m/s.

## EXERCISE 8E

**1** The sequence increases by 2 each time, which suggests that the formula contains $2n$.
$2n$ generates the set $\{2, 4, 6, 8, 10, ....\}$ but our sequence is 1 less than these values.
$\therefore$ the $n$th term is $2n - 1$.

**2** **a** The sequence increases by 2 each time.
$2n$ generates the set $\{2, 4, 6, 8, 10, ....\}$
Our sequence is 5 more than these values.
$\therefore$ the $n$th term is $2n + 5$.

**b** The sequence increases by 3 each time.
$3n$ generates the set $\{3, 6, 9, 12, 15, ....\}$
Our sequence is 1 more than these values.
$\therefore$ the $n$th term is $3n + 1$.

**c** The sequence increases by 4 each time.
$4n$ generates the set $\{4, 8, 12, 16, 20, ....\}$
Our sequence is 1 more than these values.
$\therefore$ the $n$th term is $4n + 1$.

**d** The sequence increases by 4 each time.
$4n$ generates the set $\{4, 8, 12, 16, 20, ....\}$
Our sequence is 1 less than these values.
$\therefore$ the $n$th term is $4n - 1$.

**e** The sequence increases by 7 each time.
$7n$ generates the set $\{7, 14, 21, 28, 35, ....\}$
Our sequence is 5 less than these values.
$\therefore$ the $n$th term is $7n - 5$.

**f** The sequence is $2^1, 2^2, 2^3, 2^4, 2^5, ....$
$\therefore$ the $n$th term is $2^n$.

**g** The sequence is $2^0, 2^1, 2^2, 2^3, 2^4, 2^5, ....$
The 1st term is $2^0$.
$\therefore$ the $n$th term is $2^{n-1}$.

**h** The sequence is $\dfrac{1}{3^1}, \dfrac{1}{3^2}, \dfrac{1}{3^3}, \dfrac{1}{3^4}, ....$
$\therefore$ the $n$th term is $\dfrac{1}{3^n}$.

**i** The sequence is $\dfrac{1}{5^0}, \dfrac{1}{5^1}, \dfrac{1}{5^2}, \dfrac{1}{5^3}, ....$
The 1st term is $\dfrac{1}{5^0}$.
$\therefore$ the $n$th term is $\dfrac{1}{5^{n-1}}$.

**3** **a** 1st diagram has 6 matchsticks.
2nd diagram has $6 + 5 = 11$ matchsticks.
3rd diagram has $11 + 5 = 16$ matchsticks.
4th diagram has $16 + 5 = 21$ matchsticks.
5th diagram has $21 + 5 = 26$ matchsticks.

**b** The sequence is $\{6, 11, 16, 21, 26, 31, 36, 41, 46, 51, ....\}$
$\therefore$ there are 51 matchsticks in the 10th diagram.

**c** The sequence is $\{6, 11, 16, 21, 26, ....\}$ which increases by 5 each time.
$5n$ generates $\{5, 10, 15, 20, 25, ....\}$
Our sequence is 1 more than these values.
So, the number of matchsticks in the $n$th diagram is $5n + 1$.

**4** **a** 1st diagram has 7 matchsticks.
2nd diagram has $7+3=10$ matchsticks.
3rd diagram has $10+3=13$ matchsticks.
4th diagram has $13+3=16$ matchsticks.
5th diagram has $16+3=19$ matchsticks.

**b** The sequence is $\{7, 10, 13, 16, 19, 22, 25, 28, 31, 34, ....\}$
$\therefore$ there are 34 matchsticks in the 10th diagram.

**c** The sequence increases by 3 each time.
$3n$ generates $\{3, 6, 9, 12, 15, 18, ....\}$
Our sequence is 4 more than these values.
So, the number of matchsticks in the $n$th diagram is $3n+4$.

**5** **a** **i** $1+3=4 = 2^2$
**ii** $1+3+5=9 = 3^2$
**iii** $1+3+5+7=16=4^2$
**iv** $1+3+5+7+9=25=5^2$

**b** $S_1=1=1^2,\ S_2=2^2,\ S_3=3^2,\ S_4=4^2,\ S_5=5^2$
$\therefore\ S_n=n^2$

**6** **a** **i** $1+2=3 = 4-1=2^2-1$
**ii** $1+2+4=7 = 8-1=2^3-1$
**iii** $1+2+4+8=15=16-1=2^4-1$
**iv** $1+2+4+8+16=31=32-1=2^5-1$

**b** $S_1=1=2^1-1,\ S_2=2^2-1,\ S_3=2^3-1,\ S_4=2^4-1,\ S_5=2^5-1$
$\therefore\ S_n=2^n-1$

**7** **a** $S_1=\dfrac{1}{1\times 2}=\tfrac{1}{2}$

$$\begin{aligned}S_2&=\frac{1}{1\times 2}+\frac{1}{2\times 3}\\&=\tfrac{1}{2}+\tfrac{1}{6}\\&=\tfrac{2}{3}\end{aligned}$$

$$\begin{aligned}S_3&=\frac{1}{1\times 2}+\frac{1}{2\times 3}+\frac{1}{3\times 4}\\&=\tfrac{1}{2}+\tfrac{1}{6}+\tfrac{1}{12}\\&=\tfrac{3}{4}\end{aligned}$$

$$\begin{aligned}S_4&=\frac{1}{1\times 2}+\frac{1}{2\times 3}+\frac{1}{3\times 4}+\frac{1}{4\times 5}\\&=\tfrac{1}{2}+\tfrac{1}{6}+\tfrac{1}{12}+\tfrac{1}{20}\\&=\tfrac{4}{5}\end{aligned}$$

**b** **i** $$\begin{aligned}S_{10}&=\frac{10}{10+1}\\&=\tfrac{10}{11}\end{aligned}$$

**ii** $S_n=\dfrac{n}{n+1}$

**8** **a** $$\begin{aligned}S_1&=\frac{1(1+1)(2(1)+1)}{6}\\&=\frac{1\times 2\times 3}{6}\\&=1\\&=1^2\quad ✓\end{aligned}$$

$$\begin{aligned}S_2&=\frac{2(2+1)(2(2)+1)}{6}\\&=\frac{2\times 3\times 5}{6}\\&=5\\&=1^2+2^2\quad ✓\end{aligned}$$

$$\begin{aligned}S_3&=\frac{3(3+1)(2(3)+1)}{6}\\&=\frac{3\times 4\times 7}{6}\\&=14\\&=1^2+2^2+3^2\quad ✓\end{aligned}$$

$$\begin{aligned}S_4&=\frac{4(4+1)(2(4)+1)}{6}\\&=\frac{4\times 5\times 9}{6}\\&=30\\&=1^2+2^2+3^2+4^2\quad ✓\end{aligned}$$

**b** For $1^2+2^2+3^2+....+100^2$, $n=100$

$$\begin{aligned}\therefore\ S_{100} &= \frac{100(100+1)(2(100)+1)}{6}\\ &= \frac{100\times 101\times 201}{6}\\ &= \frac{2\,030\,100}{6}\\ &= 338\,350\end{aligned}$$

$\therefore$ the sum of the squares of the first one hundred integers is 338 350.

## REVIEW SET 8

**1** **a** **i** $V = 6\times 8$ litres

**ii** $V = n\times 8$
$= 8n$ litres

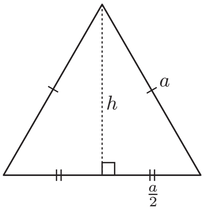
**iii** $V = n\times l$
$= ln$ litres

**b** $V = 25 + ln$ litres

**2** **a**
$$\begin{aligned} s &= \frac{d}{t}\\ \therefore\ s &= \frac{540\text{ km}}{6\text{ h}}\\ &= 90\text{ km/h}\end{aligned}$$

**b**
$$\begin{aligned} s &= \frac{d}{t}\\ \therefore\ 600 &= \frac{d\text{ km}}{6.5\text{ h}}\\ \therefore\ 600\times 6.5 &= d && \{\times \text{ both sides by } 6.5\}\\ \therefore\ d &= 600\times 6.5 && \{\text{swap sides}\}\\ &= 3900\text{ km}\end{aligned}$$

**3** Let the height of the triangle be $h$ units.



$$\begin{aligned} h^2+\left(\frac{a}{2}\right)^2 &= a^2 && \{\text{Pythagoras}\}\\ \therefore\ h^2 &= a^2-\frac{a^2}{4} && \{-\left(\frac{a}{2}\right)^2 \text{ from both sides}\}\\ \therefore\ h^2 &= \frac{3a^2}{4}\\ \therefore\ h &= \frac{a\sqrt{3}}{2}\end{aligned}$$

Total surface area = area of 2 triangular ends + area of 3 rectangular faces

$$\begin{aligned} &= 2\times\tfrac{1}{2}\times a\times\frac{a\sqrt{3}}{2}+3\times a\times b\\ \therefore\ A &= \frac{\sqrt{3}}{2}a^2+3ab\end{aligned}$$

**4** **a**
$$\begin{aligned} mx+n &= 3p\\ \therefore\ mx &= 3p-n && \{-\,n \text{ from both sides}\}\\ \therefore\ x &= \frac{3p-n}{m} && \{\div \text{ both sides by } m\}\end{aligned}$$

**b**
$$\begin{aligned} \frac{7}{y} &= \frac{5}{x}\\ \therefore\ 7x &= 5y && \{\times \text{ both sides by } xy\}\\ \therefore\ x &= \frac{5y}{7} && \{\div \text{ both sides by } 7\}\end{aligned}$$

**5** **a**
$$\begin{aligned} T &= \sqrt{k-l^2}\\ \therefore\ \sqrt{k-l^2} &= T && \{\text{swap sides}\}\\ \therefore\ k-l^2 &= T^2 && \{\text{square both sides}\}\\ \therefore\ k &= T^2+l^2 && \{+\,l^2 \text{ to both sides}\}\end{aligned}$$

**b**
$$\begin{aligned} P &= 2k^2-r, && k<0\\ \therefore\ 2k^2-r &= P && \{\text{swap sides}\}\\ \therefore\ 2k^2 &= P+r && \{+\,r \text{ to both sides}\}\\ \therefore\ k^2 &= \frac{P+r}{2} && \{\div \text{ both sides by } 2\}\\ \therefore\ k &= -\sqrt{\frac{P+r}{2}} && \{\text{as } k<0\}\end{aligned}$$

**6** **a**

$$\begin{aligned} L &= \sqrt{a^2+b^2+c^2} \\ \therefore\ L &= \sqrt{2^2+6^2+9^2} \\ &= 11 \end{aligned}$$

$\therefore$ the length of diagonal is 11 cm.

**b**

$$\begin{aligned} \sqrt{a^2+b^2+c^2} &= L \\ \therefore\ a^2+b^2+c^2 &= L^2 && \{\text{square both sides}\} \\ \therefore\ c^2 &= L^2-a^2-b^2 && \{-\,a^2-b^2 \text{ from both sides}\} \\ \therefore\ c &= \sqrt{L^2-a^2-b^2} && \{\text{as } c>0\} \end{aligned}$$

**c** $a=8,\ b=12,\ L=17$

$$\begin{aligned} \therefore\ c &= \sqrt{17^2-8^2-12^2} \\ &= 9 \end{aligned}$$

So, the height of the prism is 9 cm.

**7**

$$\begin{aligned} y &= \frac{2x-3}{x-2} \\ \therefore\ y(x-2) &= 2x-3 && \{\times \text{ both sides by } (x-2)\} \\ \therefore\ xy-2y &= 2x-3 && \{\text{expanding}\} \\ \therefore\ xy-2x &= 2y-3 && \{\text{writing the terms containing } x \text{ on the LHS}\} \\ \therefore\ x(y-2) &= 2y-3 && \{x \text{ is a common factor on the LHS}\} \\ \therefore\ x &= \frac{2y-3}{y-2} && \{\div \text{ both sides by } (y-2)\} \end{aligned}$$

**8** **a**

$$I = \frac{E}{r+R} \text{ amperes}$$

$E=24,\ r=0.5,\ R=2.5$

$$\begin{aligned} \therefore\ I &= \frac{24}{0.5+2.5} \\ &= \frac{24}{3} \\ &= 8 \end{aligned}$$

$\therefore$ the current is 8 amperes.

**b**

$$\begin{aligned} I &= \frac{E}{R+r} \\ \therefore\ I(R+r) &= E && \{\times \text{ both sides by } (R+r)\} \\ \therefore\ IR+Ir &= E && \{\text{expanding}\} \\ \therefore\ Ir &= E-IR && \{-\,IR \text{ from both sides}\} \\ \therefore\ r &= \frac{E-IR}{I} && \{\div \text{ both sides by } I\} \\ \text{or}\quad r &= \frac{E}{I}-R \end{aligned}$$

**c** $I=1.5,\ E=7.725,\ R=5$

$$\begin{aligned} \therefore\ r &= \frac{7.725}{1.5}-5 \\ &= 0.15 \end{aligned}$$

$\therefore$ the resistance is 0.15 ohms.

**9** **a** 1st diagram has 4 matchsticks.
2nd diagram has $4+3=7$ matchsticks.
3rd diagram has $7+3=10$ matchsticks.

**b** The sequence increases by 3 each time.
$3n$ generates $\{3, 6, 9, 12, 15, ....\}$
Our sequence is 1 more than these values.
$\therefore$ the number of matchsticks in the $n$th diagram is $3n+1$.

**10** **a** The next 3 terms are 9, 11, 13.

**b** The sequence increases by 2 each time.
$2n$ generates $\{2, 4, 6, 8, 10, 12, ....\}$
Our sequence is 1 less than these values.
$\therefore$ the $n$th term is $2n-1$.

**11** **a**

$$T_4 = \frac{1}{\underset{\substack{\nearrow \\ 2\times4-1}}{7} \times \underset{\substack{\nwarrow \\ 2\times4+1}}{9}}, \qquad T_5 = \frac{1}{\underset{\substack{\nearrow \\ 2\times5-1}}{9} \times \underset{\substack{\nwarrow \\ 2\times5+1}}{11}}, \qquad T_6 = \frac{1}{\underset{\substack{\nearrow \\ 2\times6-1}}{11} \times \underset{\substack{\nwarrow \\ 2\times6+1}}{13}}$$

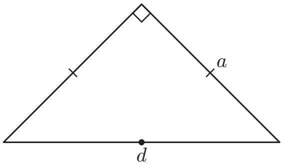
**b** $$T_{20} = \frac{1}{\underset{\substack{\uparrow \\ 2\times 20-1}}{39}\times\underset{\substack{\uparrow \\ 2\times 20+1}}{41}}$$

**c** $$T_n = \frac{1}{(2n-1)(2n+1)}$$

## PRACTICE TEST 8A

**1** Area = area of triangle + area of semi-circle

$$= \tfrac{1}{2} \times \text{base} \times \text{height} + \tfrac{1}{2} \times \pi \times r^2$$



$$\begin{aligned}
d^2 &= a^2 + a^2 && \{\text{Pythagoras}\}\\
\therefore\ d^2 &= 2a^2\\
\therefore\ d &= \sqrt{2}a && \{\text{as } d > 0\}\\
\therefore\ r &= \frac{\sqrt{2}a}{2} && \{r = \tfrac{1}{2}d\}\\
\therefore\ A &= \tfrac{1}{2}a \times a + \tfrac{1}{2} \times \pi \times \left(\frac{\sqrt{2}a}{2}\right)^2\\
&= \tfrac{1}{2}a^2 + \tfrac{\pi}{4}a^2
\end{aligned}$$

$\therefore$ the answer is **E**.

**2** When $a = 5$ and $b = -2$,

$$\begin{aligned}
T &= 3a - 2b^2\\
&= 3 \times 5 - 2 \times (-2) \times (-2)\\
&= 15 - 8\\
&= 7
\end{aligned}$$

$\therefore$ the answer is **A**.

**3** When $a = 8$, $b = 15$, and $A = 69$,

$$\begin{aligned}
\left(\frac{8+15}{2}\right) \times h &= 69\\
\therefore\ \tfrac{23}{2} \times h &= 69\\
\therefore\ h &= \frac{2 \times 69}{23} && \{\times \text{ both sides by } \tfrac{2}{23}\}\\
\therefore\ h &= 6 && \therefore \text{ the answer is } \textbf{B}.
\end{aligned}$$

**4** $$\begin{aligned}
3x + 5y &= 2\\
\therefore\ 5y &= 2 - 3x && \{-\,3x \text{ from both sides}\}\\
\therefore\ y &= \frac{2-3x}{5} && \{\div \text{ both sides by } 5\}\\
\therefore\ y &= \tfrac{2}{5} - \tfrac{3}{5}x && \therefore \text{ the answer is } \textbf{C}.
\end{aligned}$$

**5** **A** $$\begin{aligned}
\frac{k}{5} &= x^2\\
\therefore\ k &= 5x^2 && \{\times \text{ both sides by } 5\}\\
&\neq \frac{x^2}{5} && ✗
\end{aligned}$$

**B** $$\begin{aligned}
kx^2 &= 5\\
\therefore\ k &= \frac{5}{x^2} && \{\div \text{ both sides by } x^2\}\\
&\neq \frac{x^2}{5} && ✗
\end{aligned}$$

**C** $$\begin{aligned}
\frac{k}{x} &= \frac{x}{25}\\
\therefore\ k &= \frac{x^2}{25} && \{\times \text{ both sides by } x\}\\
&\neq \frac{x^2}{5} && ✗
\end{aligned}$$

**D** $$\begin{aligned}
\frac{x}{k} &= \frac{x}{5}\\
\therefore\ 5x &= kx && \{\times \text{ both sides by } 5k\}\\
\therefore\ 5 &= k && \{\div \text{ both sides by } x\}\\
\therefore\ k &= 5 && \{\text{swap sides}\}\\
&\neq \frac{x^2}{5} && ✗
\end{aligned}$$

**E** $$\begin{aligned}
\frac{x}{5} &= \frac{k}{x}\\
\frac{x^2}{5} &= k && \{\times \text{ both sides by } x\}\\
\therefore\ k &= \frac{x^2}{5} \quad ✓ && \{\text{swap sides}\}
\end{aligned}$$

$\therefore$ the answer is **E**.

**6**
$$P = \frac{y}{m^3}$$
$\therefore\ Pm^3 = y$ {× both sides by $m^3$}

$\therefore\ m^3 = \frac{y}{P}$ {÷ both sides by $P$}

$\therefore\ m = \sqrt[3]{\frac{y}{P}}$ {cube root both sides}

$\therefore$ the answer is **D**.

**7**
$$y = \frac{x+4}{2x-3}$$
$\therefore\ y(2x-3) = x+4$ {× both sides by $(2x-3)$}

$\therefore\ 2xy - 3y = x+4$ {expanding}

$\therefore\ 2xy - x = 3y+4$ {writing the terms containing $x$ on the LHS}

$\therefore\ x(2y-1) = 3y+4$ {$x$ is a common factor on the LHS}

$\therefore\ x = \frac{3y+4}{2y-1}$ {÷ both sides by $(2y-1)$}

$\therefore$ the answer is **A**.

**8**
$$D = 50 \times \frac{a}{a+12}$$
$\therefore\ (a+12)D = 50a$ {× both sides by $(a+12)$}

$\therefore\ aD + 12D = 50a$ {expanding}

$\therefore\ aD - 50a = -12D$ {writing the terms containing $a$ on the LHS}

$\therefore\ a(D-50) = -12D$ {$a$ is a common factor on the LHS}

$\therefore\ a = \frac{-12D}{D-50}$ {÷ both sides by $(D-50)$}

$\therefore\ a = \frac{12D}{50-D}$ $\left\{\times\ \frac{-1}{-1}\right\}$

$\therefore$ the answer is **D**.

**9** If $D = 20$, $a = \frac{12 \times 20}{50 - 20}$

$= \frac{240}{30}$

$= 8$

$\therefore$ Charlie is 8 years old.

$\therefore$ the answer is **C**.

**10** Sequence is $\{5, 8, 11, ....\}$

The sequence increases by 3 each time.

$3n$ generates $\{3, 6, 9, ....\}$

Our sequence is 2 more than these values.

$\therefore$ the number of matchsticks in the $n$th diagram is $3n + 2$.

$\therefore$ the answer is **E**.

## PRACTICE TEST 8B

**1** **a** $B = 15 + 25 \times 5$

**b** $B = c + 25 \times p$

$\therefore\ B = c + 25p$

**c** $B = c + m \times p$

$\therefore\ B = c + mp$

**2** **a** A side with 3 pieces has 2 corner pieces and $(3-2)$ edge pieces.

A side with 5 pieces has 2 corner pieces and $(5-2)$ edge pieces.

$\therefore$ for a $3 \times 5$ puzzle, $E = 2(3-2) + 2(5-2)$

$= 2 \times 1 + 2 \times 3$

$= 8$

**b** $E = 2(4-2) + 2(8-2)$

$= 2 \times 2 + 2 \times 6$

$= 16$

**c** $E = 2(m-2) + 2(n-2)$

**3** **a** If $p = 19,\ q = -3,\ r = 6,$

$$\begin{aligned} M &= 19 - (-3) \times 6 \\ &= 19 + 18 \\ &= 37 \end{aligned}$$

**b** If $M = -2,\ p = 14,\ q = 2,$

$$\begin{aligned} -2 &= 14 - 2r \\ \therefore\ 2r - 2 &= 14 && \{+\,2r \text{ to both sides}\} \\ \therefore\ 2r &= 16 && \{+\,2 \text{ to both sides}\} \\ \therefore\ r &= 8 && \{\div \text{ both sides by } 2\} \end{aligned}$$

**4** Volume = volume of base + volume of upper layer

$$\begin{aligned} V &= 3x \times x \times y + x \times x \times y \\ &= 3x^2y + x^2y \\ \therefore\ V &= 4x^2y \end{aligned}$$

**5** **a**

$$\begin{aligned} B &= ad - f \\ \therefore\ ad - f &= B && \{\text{swap sides}\} \\ \therefore\ ad &= B + f && \{+\,f \text{ to both sides}\} \\ \therefore\ a &= \frac{B+f}{d} && \{\div \text{ both sides by } d\} \end{aligned}$$

**b**

$$\begin{aligned} \frac{Q}{\sqrt{a}} &= \frac{t}{3} \\ \therefore\ 3Q &= \sqrt{a}\,t && \{\times \text{ both sides by } 3\sqrt{a}\,\} \\ \therefore\ \sqrt{a}\,t &= 3Q && \{\text{swap sides}\} \\ \therefore\ \sqrt{a} &= \frac{3Q}{t} && \{\div \text{ both sides by } t\} \\ \therefore\ a &= \frac{9Q^2}{t^2} && \{\text{square both sides}\} \end{aligned}$$

**6**

$$\begin{aligned} G &= \sqrt{\frac{5}{h+1}} \\ \therefore\ G^2 &= \frac{5}{h+1} && \{\text{square both sides}\} \\ \therefore\ G^2(h+1) &= 5 && \{\times \text{ both sides by } (h+1)\} \\ \therefore\ h+1 &= \frac{5}{G^2} && \{\div \text{ both sides by } G^2\} \\ \therefore\ h &= \frac{5}{G^2} - 1 && \{-\,1 \text{ from both sides}\} \\ \text{or}\quad h &= \frac{5-G^2}{G^2} \end{aligned}$$

**7** **a**

$$\begin{aligned} ab &= a + b \\ \therefore\ ab - b &= a \\ & \{-\,b \text{ from both sides}\} \\ \therefore\ b(a-1) &= a \\ & \{b \text{ is a common factor on the LHS}\} \\ \therefore\ b &= \frac{a}{a-1} \\ & \{\div \text{ both sides by } (a-1)\} \end{aligned}$$

**b** If $a = 3,\ b = \dfrac{3}{3-1}$

$$= \frac{3}{2}$$

*Check*: $ab = 3 \times \dfrac{3}{2} = \dfrac{9}{2}$

$a + b = 3 + \dfrac{3}{2} = \dfrac{9}{2}$ ✓

**8**

$$\begin{aligned} y &= \frac{4x-3}{3x+2} \\ \therefore\ y(3x+2) &= 4x - 3 && \{\times \text{ both sides by } (3x+2)\} \\ \therefore\ 3xy + 2y &= 4x - 3 && \{\text{expanding}\} \\ \therefore\ 3xy - 4x &= -2y - 3 && \{\text{writing the terms containing } x \text{ on the LHS}\} \\ \therefore\ x(3y-4) &= -2y - 3 && \{x \text{ is a common factor on the LHS}\} \\ \therefore\ x &= \frac{-2y-3}{3y-4} && \{\div \text{ both sides by } (3y-4)\} \\ \therefore\ x &= \frac{2y+3}{4-3y} && \left\{\times\ \frac{-1}{-1}\right\} \end{aligned}$$

**9** There are 8 matchsticks in the first diagram, $8+5=13$ matchsticks in the second diagram, and $13+5=18$ matchsticks in the third diagram.

So the sequence is $\{8, 13, 18, ....\}$ which increases by 5 each time.

$5n$ generates $\{5, 10, 15, ....\}$

Our sequence is 3 more than these values.

$\therefore$ the number of matchsticks in the $n$th diagram is $5n+3$.

**10** **a** **i** $2+4=6 \;=2\times 3$

**ii** $2+4+6=12=3\times 4$

**iii** $2+4+6+8=20=4\times 5$

**iv** $2+4+6+8+10=30=\underset{\substack{\nearrow\\ \text{5 terms}}}{5}\times\underset{\substack{\nwarrow\\ (5+1)}}{6}$

**b** $S_1=2=1\times 2,\quad S_2=2\times 3,\quad S_3=3\times 4,\quad S_4=4\times 5,\quad S_5=5\times 6$

$\therefore\quad S_n=n(n+1)$

## PRACTICE TEST 8C

**1** **a** $r=5,\ \ s=7$

$$\begin{aligned} A&=\pi r^2+\pi rs\\ &=\pi 5^2+\pi\times 5\times 7\\ &=60\pi\\ &\approx 188\end{aligned}$$

$\therefore$ the surface area is approximately 188 cm$^2$.

**b**

$$\begin{aligned} A&=\pi r^2+\pi rs\\ \pi rs+\pi r^2&=A && \{\text{swap sides}\}\\ \therefore\ \ \pi rs&=A-\pi r^2 && \{-\ \pi r^2 \text{ from both sides}\}\\ \therefore\ \ s&=\frac{A-\pi r^2}{\pi r} && \{\div \text{ both sides by } \pi r\}\\ \text{or}\ \ s&=\frac{A}{\pi r}-r\end{aligned}$$

**c** **i** $r=4,\ \ A=150$

$$\begin{aligned}\therefore\ \ s&=\frac{150}{\pi\times 4}-4\\ &\approx 7.94\end{aligned}$$

$\therefore$ the slant height is approximately 7.94 cm.

**ii** $r=7.5,\ \ A=400$

$$\begin{aligned}\therefore\ \ s&=\frac{400}{\pi\times 7.5}-7.5\\ &\approx 9.48\end{aligned}$$

$\therefore$ the slant height is approximately 9.48 cm.

**2** $y=\dfrac{6x-2}{x-1}$

**a** **i** When $x=2$, $y=\dfrac{6\times 2-2}{2-1}$

$\therefore\ \ y=10$

**ii** When $x=-3$, $y=\dfrac{6\times(-3)-2}{-3-1}$

$=\dfrac{-18-2}{-4}$

$\therefore\ \ y=5$

**b**

$$\begin{aligned} y&=\frac{6x-2}{x-1}\\ \therefore\ \ y(x-1)&=6x-2 && \{\times \text{ both sides by } (x-1)\}\\ \therefore\ \ xy-y&=6x-2 && \{\text{expanding}\}\\ \therefore\ \ xy-6x&=y-2 && \{\text{writing the terms containing } x \text{ on the LHS}\}\\ \therefore\ \ x(y-6)&=y-2 && \{x \text{ is a common factor on the LHS}\}\\ \therefore\ \ x&=\frac{y-2}{y-6} && \{\div \text{ both sides by } (y-6)\}\end{aligned}$$

**c** **i** When $y=10$, $x=\dfrac{10-2}{10-6}$

$\therefore\ \ x=2$

**ii** When $y=5$, $x=\dfrac{5-2}{5-6}$

$\therefore\ \ x=-3$

**d** **a i** and **c i** show $x=2,\ y=10$. **a ii** and **c ii** show $x=-3,\ y=5$.

**3** **a** $E = \frac{1}{2}mv^2$ where $m = 80$, $v = 5$

$\therefore\ E = \frac{1}{2} \times 80 \times 5^2$

$= 1000$

$\therefore$ the kinetic energy is 1000 joules.

**b** $E = \frac{1}{2}mv^2$, $v \geqslant 0$

$\therefore\ \dfrac{2E}{m} = v^2$ {$\times$ both sides by $\dfrac{2}{m}$}

$\therefore\ v^2 = \dfrac{2E}{m}$ {swap sides}

$\therefore\ v = \sqrt{\dfrac{2E}{m}}$ {as $v \geqslant 0$}

**c** When $m = 25$ and $E = 800$,

$v = \sqrt{\dfrac{2 \times 800}{25}}$

$= \dfrac{40}{5}$

$= 8$

$\therefore$ the speed of the wombat is 8 m/s.

**4** **a** **i** $1 + 2 = 3$

**ii** $1 + 2 + 3 = 6$

**iii** $1 + 2 + 3 + 4 = 10$

**b** **i** $1^3 + 2^3 = 1 + 8 = 9 = 3^2$

**ii** $1^3 + 2^3 + 3^3 = 9 + 27 = 36 = 6^2$

**iii** $1^3 + 2^3 + 3^3 + 4^3 = 36 + 64 = 100 = 10^2$

**c** $C_n = {S_n}^2$

**5** **a** **i** $K = \frac{5}{9}(F - 32) + 273.15$

When $F = 50$,

$K = \frac{5}{9}(50 - 32) + 273.15$

$= 283.15$

So, 50°F = 283.15 K.

**ii** When $F = -130$,

$K = \frac{5}{9}(-130 - 32) + 273.15$

$= -90 + 273.15$

$= 183.15$

So, −130°F = 183.15 K.

**iii** When $F = 150$,

$K = \frac{5}{9}(150 - 32) + 273.15$

$\approx 65.56 + 273.15$

$\approx 338.71$

So, 150°F ≈ 338.71 K.

**b** $K = \frac{5}{9}(F - 32) + 273.15$

$\frac{5}{9}(F - 32) + 273.15 = K$ {swap sides}

$\therefore\ \frac{5}{9}(F - 32) = K - 273.15$ {$-$ 273.15 from both sides}

$\therefore\ F - 32 = \frac{9}{5}(K - 273.15)$ {$\times$ both sides by $\frac{9}{5}$}

$\therefore\ F = \frac{9}{5}(K - 273.15) + 32$ {+ 32 to both sides}

$\therefore\ F = \frac{9}{5}K - 459.67$ {expanding, collecting like terms}

**c** **i** When $K = 313.15$,

$F = \frac{9}{5} \times 313.15 - 459.67$

$= 104$

So, 313.15 K = 104°F.

**ii** When $K = 0$, $F = \frac{9}{5} \times 0 - 459.67$

So, 0 K = −459.67°F.

**iii** When $K = 200$, $F = \frac{9}{5} \times 200 - 459.67$

$= -99.67$

So, 200 K = −99.67°F.