Chapter 16

PROPERTIES OF CURVES

EXERCISE 16A

1 **a** We seek the tangent to $y = x - 2x^2 + 3$ at $x = 2$.

When $x = 2$, $y = 2 - 2(2)^2 + 3 = -3$

$\therefore$ the point of contact is $(2, -3)$.

Now $\dfrac{dy}{dx} = 1 - 4x$, so at $x = 2$,

$\dfrac{dy}{dx} = 1 - 8 = -7$

$\therefore$ the tangent has equation

$$\frac{y-(-3)}{x-2} = -7$$

$\therefore$ $y + 3 = -7(x-2)$

$\therefore$ $y = -7x + 14 - 3$

$\therefore$ $y = -7x + 11$

b We seek the tangent to

$y = \sqrt{x} + 1 = x^{\frac{1}{2}} + 1$ at $x = 4$.

When $x = 4$, $y = \sqrt{4} + 1 = 3$

$\therefore$ the point of contact is $(4, 3)$.

Now $\dfrac{dy}{dx} = \dfrac{1}{2\sqrt{x}}$, so at $x = 4$,

$\dfrac{dy}{dx} = \dfrac{1}{2\sqrt{4}} = \frac{1}{4}$

$\therefore$ the tangent has equation

$$\frac{y-3}{x-4} = \frac{1}{4}$$

$\therefore$ $4y - 12 = x - 4$

$\therefore$ $4y = x + 8$

c We seek the tangent to $y = x^3 - 5x$ at $x = 1$.

When $x = 1$, $y = 1^3 - 5(1) = -4$

$\therefore$ the point of contact is $(1, -4)$.

Now $\dfrac{dy}{dx} = 3x^2 - 5$, so at $x = 1$,

$\dfrac{dy}{dx} = 3 - 5 = -2$

$\therefore$ the tangent has equation

$$\frac{y-(-4)}{x-1} = -2$$

$\therefore$ $y + 4 = -2x + 2$

$\therefore$ $y = -2x - 2$

d We seek the tangent to $y = \dfrac{4}{\sqrt{x}}$ at $(1, 4)$.

Now $y = \dfrac{4}{\sqrt{x}} = 4x^{-\frac{1}{2}}$

$\therefore$ $\dfrac{dy}{dx} = -2x^{-\frac{3}{2}}$ so at $x = 1$,

$\dfrac{dy}{dx} = -2\left(1^{-\frac{3}{2}}\right) = -2$

$\therefore$ the tangent has equation

$$\frac{y-4}{x-1} = -2$$

$\therefore$ $y - 4 = -2x + 2$

$\therefore$ $y = -2x + 6$

e We seek the tangent to

$y = \dfrac{3}{x} - \dfrac{1}{x^2} = 3x^{-1} - x^{-2}$ at $(-1, -4)$.

Now $\dfrac{dy}{dx} = -3x^{-2} + 2x^{-3}$

$= -\dfrac{3}{x^2} + \dfrac{2}{x^3}$ so at $(1, -4)$,

$\dfrac{dy}{dx} = -\dfrac{3}{(-1)^2} + \dfrac{2}{(-1)^3}$

$= -3 - 2$

$= -5$

$\therefore$ the tangent has equation

$$\frac{y-(-4)}{x-(-1)} = -5$$

$\therefore$ $y + 4 = -5x - 5$

$\therefore$ $y = -5x - 9$

f We seek the tangent to

$y = 3x^2 - \dfrac{1}{x} = 3x^2 - x^{-1}$ at $x = -1$.

When $x = -1$, $y = 3(-1)^2 - \frac{1}{(-1)} = 4$

$\therefore$ the point of contact is $(-1, 4)$.

Now $\dfrac{dy}{dx} = 6x + x^{-2}$

$= 6x + \dfrac{1}{x^2}$ so at $x = -1$,

$\dfrac{dy}{dx} = 6(-1) + \dfrac{1}{(-1)^2} = -5$

$\therefore$ the tangent has equation

$$\frac{y-4}{x-(-1)} = -5$$

$\therefore$ $y - 4 = -5x - 5$

$\therefore$ $y = -5x - 1$

2 **a** We seek the normal to $y = x^2$ at $(3, 9)$.

Now $\dfrac{dy}{dx} = 2x$ so at $x = 3$,

$\dfrac{dy}{dx} = 2(3) = 6 = \frac{6}{1}$

$\therefore$ the normal at (3, 9) has gradient $-\frac{1}{6}$, so the equation of the normal is

$$\frac{y-9}{x-3} = -\tfrac{1}{6}$$

$\therefore \quad 6y - 54 = -x + 3$

$\therefore \quad 6y = -x + 57$

b We seek the normal to $y = x^3 - 5x + 2$ at $x = -2$.

When $x = -2$, $y = (-2)^3 - 5(-2) + 2 = 4$

and so the point of contact is $(-2, 4)$.

Now $\dfrac{dy}{dx} = 3x^2 - 5$ so at $x = -2$,

$\dfrac{dy}{dx} = 3(-2)^2 - 5 = 7$

$\therefore$ the normal at $(-2, 4)$ has gradient $-\frac{1}{7}$, so the equation of the normal is

$$\frac{y-4}{x-(-2)} = -\tfrac{1}{7}$$

$\therefore \quad 7y - 28 = -(x+2)$

$\therefore \quad 7y = -x + 26$

c We seek the normal to $y = \dfrac{5}{\sqrt{x}} - \sqrt{x}$ at $(1, 4)$.

Now $y = 5x^{-\frac{1}{2}} - x^{\frac{1}{2}}$

$\therefore \dfrac{dy}{dx} = -\frac{5}{2}x^{-\frac{3}{2}} - \frac{1}{2}x^{-\frac{1}{2}}$ so at $x = 1$,

$\dfrac{dy}{dx} = -\frac{5}{2}\left(1^{-\frac{3}{2}}\right) - \frac{1}{2}\left(1^{-\frac{1}{2}}\right)$

$= -\frac{5}{2} - \frac{1}{2} = -3$

$\therefore$ the normal at $(1, 4)$ has gradient $\frac{1}{3}$, so the equation of the normal is

$$\frac{y-4}{x-1} = \tfrac{1}{3}$$

$\therefore \quad 3y - 12 = x - 1$

$\therefore \quad 3y = x + 11$

d We seek the normal to $y = 8\sqrt{x} - \dfrac{1}{x^2}$ at $x = 1$.

When $x = 1$, $y = 8\sqrt{1} - \dfrac{1}{1^2} = 7$

$\therefore$ the point of contact is $(1, 7)$.

Now $y = 8\sqrt{x} - \dfrac{1}{x^2} = 8x^{\frac{1}{2}} - x^{-2}$

$\therefore \dfrac{dy}{dx} = 4x^{-\frac{1}{2}} + 2x^{-3}$ so at $x = 1$,

$\dfrac{dy}{dx} = 4 + 2 = 6$

$\therefore$ the normal at $(1, 7)$ has gradient $-\frac{1}{6}$, so the equation of the normal is

$$\frac{y-7}{x-1} = -\tfrac{1}{6}$$

$\therefore \quad 6y - 42 = -x + 1$

$\therefore \quad 6y = -x + 43$

3 **a** $y = 2x^3 + 3x^2 - 12x + 1$

$\therefore \dfrac{dy}{dx} = 6x^2 + 6x - 12$

Horizontal tangents have gradient $= 0$

so $6x^2 + 6x - 12 = 0$

$\therefore \quad x^2 + x - 2 = 0$

$\therefore \quad (x+2)(x-1) = 0$

$\therefore \quad x = -2$ or $x = 1$

Now at $x = -2$,

$y = 2(-2)^3 + 3(-2)^2 - 12(-2) + 1$
$= 21$

and at $x = 1$,

$y = 2(1)^3 + 3(1)^2 - 12(1) + 1$
$= -6$

$\therefore$ the points of contact are $(-2, 21)$ and $(1, -6)$

$\therefore$ the tangents are $y = 21$ and $y = -6$.

b Now $y = 2\sqrt{x} + \dfrac{1}{\sqrt{x}} = 2x^{\frac{1}{2}} + x^{-\frac{1}{2}}$

$\therefore \dfrac{dy}{dx} = x^{-\frac{1}{2}} - \frac{1}{2}x^{-\frac{3}{2}} = \dfrac{1}{\sqrt{x}} - \dfrac{1}{2x\sqrt{x}}$

Horizontal tangents have gradient $= 0$

$\therefore \quad \dfrac{1}{\sqrt{x}} - \dfrac{1}{2x\sqrt{x}} = 0$

$\therefore \quad \dfrac{2x-1}{2x\sqrt{x}} = 0$

$\therefore \quad 2x - 1 = 0$

$\therefore \quad x = \frac{1}{2}$

Now at $x = \frac{1}{2}$,

$$y = 2\sqrt{\tfrac{1}{2}} + \frac{1}{\sqrt{\frac{1}{2}}} = \frac{2\left(\frac{1}{2}\right) + 1}{\sqrt{\frac{1}{2}}} = \frac{2}{\sqrt{\frac{1}{2}}} = 2\sqrt{2}$$

$\therefore$ the only horizontal tangent touches at the curve at $(\frac{1}{2}, 2\sqrt{2})$.

c Now $y = 2x^3 + kx^2 - 3$

$\therefore \dfrac{dy}{dx} = 6x^2 + 2kx$

When $x = 2$, $\dfrac{dy}{dx} = 4$

$\therefore 6(2)^2 + 2k(2) = 4$

$\therefore 24 + 4k = 4$

$\therefore 4k = -20$

$\therefore k = -5$

d Now $y = 1 - 3x + 12x^2 - 8x^3$

$\therefore \dfrac{dy}{dx} = -3 + 24x - 24x^2$

When $x = 1$, $\dfrac{dy}{dx} = -3 + 24 - 24 = -3$

$\therefore$ the tangent at (1, 2) has gradient -3

The tangents to the curve have gradient -3 when $-3 + 24x - 24x^2 = -3$

$\therefore 24x^2 - 24x = 0$

$\therefore 24x(x - 1) = 0$

$\therefore$ when $x = 0$ or $x = 1$

So the other x-value for which the tangent to the curve has gradient -3 is $x = 0$,

and when $x = 0$, $y = 1 - 0 + 0 - 0 = 1$

$\therefore$ the tangent to the curve at (0, 1) is parallel to the tangent at (1, 2).

This tangent has equation $\dfrac{y - 1}{x - 0} = -3$

or $y = -3x + 1$.

4 **a** Now $y = x^2 + ax + b$

$\therefore \dfrac{dy}{dx} = 2x + a$

At $x = 1$, $\dfrac{dy}{dx} = 2 + a$

$\therefore$ the gradient of the tangent to the curve at $x = 1$ will be $2 + a$.

However the equation of the tangent is $2x + y = 6$ or $y = -2x + 6$

and so the gradient of the tangent is -2.

$\therefore 2 + a = -2$

$\therefore a = -4$

So, the curve is $y = x^2 - 4x + b$.

We also know that the tangent contacts the curve when $x = 1$.

$\therefore 1^2 - 4(1) + b = -2(1) + 6$

$\therefore 1 - 4 + b = 4$

$\therefore b = 7$

$\therefore a = -4, \quad b = 7$

b Now $y = a\sqrt{x} + \dfrac{b}{\sqrt{x}} = ax^{\frac{1}{2}} + bx^{-\frac{1}{2}}$

$\therefore \dfrac{dy}{dx} = \dfrac{a}{2}x^{-\frac{1}{2}} - \dfrac{b}{2}x^{-\frac{3}{2}}$

$\therefore$ at $x = 4$, $\dfrac{dy}{dx} = \dfrac{a}{2}\left(4^{-\frac{1}{2}}\right) - \dfrac{b}{2}\left(4^{-\frac{3}{2}}\right)$

$= \dfrac{a}{2}\left(\dfrac{1}{2}\right) - \dfrac{b}{2}\left(\dfrac{1}{8}\right)$

$= \dfrac{a}{4} - \dfrac{b}{16}$

$\therefore$ the gradient of the tangent to the curve at $x = 4$ will be $\dfrac{a}{4} - \dfrac{b}{16} = \dfrac{4a - b}{16}$

However the equation of the *normal* is $4x + y = 22$ or $y = -4x + 22$.

$\therefore$ the normal has gradient -4.

$\therefore$ the tangent has gradient $\frac{1}{4}$, and so

$\dfrac{4a - b}{16} = \frac{1}{4}$

$\therefore 4a - b = 4$

$\therefore b = 4a - 4$ .... (1)

Also, at $x = 4$ the normal line intersects the curve.

$\therefore a\sqrt{4} + \dfrac{b}{\sqrt{4}} = -4(4) + 22$

$\therefore 2a + \dfrac{b}{2} = 6$

Consequently, $2a + \dfrac{4a - 4}{2} = 6$ {using (1)}

$\therefore 2a + 2a - 2 = 6$

$\therefore 4a = 8$

$\therefore a = 2$

and so $b = 4(2) - 4 = 4$ {from (1)}

c $y = 2x^2 - 1$

$\therefore \dfrac{dy}{dx} = 4x$

$\therefore$ at the point where $x = a$, $\dfrac{dy}{dx} = 4a$

$\therefore$ the gradient of the tangent at the point where $x = a$ is $4a$.

Also, at $x = a$, $y = 2a^2 - 1$.

$\therefore$ the tangent has equation

$$\frac{y - (2a^2 - 1)}{x - a} = 4a$$

$\therefore y - 2a^2 + 1 = 4a(x - a)$

$\therefore y - 2a^2 + 1 = 4ax - 4a^2$

$\therefore 4ax - y = 2a^2 + 1$

5 **a** $y = \sqrt{2x+1}$

When $x = 4$, $y = \sqrt{2(4)+1} = 3$,

so the point of contact is $(4, 3)$.

Now $\dfrac{dy}{dx} = \frac{1}{2}(2x+1)^{-\frac{1}{2}}(2) = \dfrac{1}{\sqrt{2x+1}}$

$\therefore$ at $x = 4$, $\dfrac{dy}{dx} = \dfrac{1}{\sqrt{2(4)+1}} = \frac{1}{3}$

$\therefore$ the tangent has equation $\dfrac{y-3}{x-4} = \frac{1}{3}$

or $3y = x + 5$.

b $y = \dfrac{1}{2-x} = (2-x)^{-1}$

$\therefore$ at $x = -1$, $y = \dfrac{1}{2-(-1)} = \frac{1}{3}$

So the point of contact is $(-1, \frac{1}{3})$.

Now $\dfrac{dy}{dx} = -1(2-x)^{-2}(-1) = \dfrac{1}{(2-x)^2}$

$\therefore$ at $x = -1$, $\dfrac{dy}{dx} = \dfrac{1}{(2-(-1))^2} = \frac{1}{9}$

$\therefore$ the tangent has equation

$$\frac{y-\frac{1}{3}}{x-(-1)} = \frac{1}{9}$$

$\therefore$ $9y - 3 = x + 1$

$\therefore$ $9y = x + 4$

c We seek the tangent to $f(x) = \dfrac{x}{1-3x}$ at $(-1, -\frac{1}{4})$.

$f(x)$ is a quotient where

$u = x$ and $v = 1 - 3x$

$\therefore$ $u' = 1$ and $v' = -3$

Now $f'(x) = \dfrac{u'v - uv'}{v^2}$ {quotient rule}

$\therefore$ $f'(x) = \dfrac{1(1-3x) - x(-3)}{(1-3x)^2}$

$= \dfrac{1}{(1-3x)^2}$

$\therefore$ $f'(-1) = \dfrac{1}{(1-3(-1))^2} = \frac{1}{16}$

$\therefore$ the tangent has equation

$$\frac{y-\left(-\frac{1}{4}\right)}{x-(-1)} = \frac{1}{16}$$

$\therefore$ $16y + 4 = x + 1$

$\therefore$ $16y = x - 3$

d We seek the tangent to $f(x) = \dfrac{x^2}{1-x}$ at $(2, -4)$.

$f(x)$ is a quotient where

$u = x^2$ and $v = 1 - x$

$\therefore$ $u' = 2x$ and $v' = -1$

Now $f'(x) = \dfrac{u'v - uv'}{v^2}$ {quotient rule}

$\therefore$ $f'(x) = \dfrac{2x(1-x) - x^2(-1)}{(1-x)^2}$

$= \dfrac{2x - 2x^2 + x^2}{(1-x)^2} = \dfrac{2x - x^2}{(1-x)^2}$

$\therefore$ $f'(2) = \dfrac{2(2) - 2^2}{(1-2)^2} = \dfrac{4-4}{1} = 0$

As the tangent has gradient 0, it is horizontal.

$\therefore$ its equation is $y = c$

Since the contact point is $(2, -4)$, the tangent has equation $y = -4$.

6 **a** We seek the normal to $y = \dfrac{1}{(x^2+1)^2}$ at $(1, \frac{1}{4})$.

As $y = (x^2+1)^{-2}$,

$\dfrac{dy}{dx} = -2(x^2+1)^{-3}(2x) = \dfrac{-4x}{(x^2+1)^3}$

$\therefore$ at $x = 1$, $\dfrac{dy}{dx} = \dfrac{-4}{(1+1)^3} = \frac{-4}{8} = -\frac{1}{2}$

$\therefore$ the normal at $(1, \frac{1}{4})$ has gradient 2.

So the equation of the normal is

$$\frac{y-\frac{1}{4}}{x-1} = 2$$

$\therefore$ $y - \frac{1}{4} = 2x - 2$

$\therefore$ $y = 2x - \frac{7}{4}$

b $y = \dfrac{1}{\sqrt{3-2x}}$

$\therefore$ at $x = -3$, $y = \dfrac{1}{\sqrt{3-2(-3)}} = \frac{1}{3}$

$\therefore$ the point of contact is $(-3, \frac{1}{3})$

Now $y = (3-2x)^{-\frac{1}{2}}$

$\therefore$ $\dfrac{dy}{dx} = -\frac{1}{2}(3-2x)^{-\frac{3}{2}}(-2) = (3-2x)^{-\frac{3}{2}}$

$\therefore$ at $x = -3$, $\dfrac{dy}{dx} = (3-2(-3))^{-\frac{3}{2}}$

$= 9^{-\frac{3}{2}} = 3^{-3} = \frac{1}{27}$

$\therefore$ the normal at $(-3, \frac{1}{3})$ has gradient -27.

So the equation of the normal is

$$\frac{y-\frac{1}{3}}{x-(-3)} = -27$$

$\therefore$ $y - \frac{1}{3} = -27(x+3)$

$\therefore$ $y = -27x - \frac{242}{3}$

c $f(x) = \sqrt{x}(1-x)^2$

Since $f(4) = \sqrt{4}(1-4)^2 = 18$,
the point of contact is $(4, 18)$.
Now $f(x)$ is a product where

$u = x^{\frac{1}{2}}$ and $v = (1-x)^2$

$\therefore\ u' = \frac{1}{2}x^{-\frac{1}{2}}$ and $v' = 2(1-x)(-1) = -2(1-x)$

Now $f'(x) = u'v + uv'$ {product rule}

$\therefore\ f'(x) = \frac{1}{2}x^{-\frac{1}{2}}(1-x)^2 - x^{\frac{1}{2}}2(1-x)$

$\therefore\ f'(4) = \frac{1}{2\sqrt{4}}(1-4)^2 - \sqrt{4}(2)(1-4)$
$= \frac{1}{4}(9) - 2(2)(-3) = \frac{57}{4}$

$\therefore$ the normal at $(4, 18)$ has gradient $-\frac{4}{57}$.
So, the equation of the normal is

$$\frac{y-18}{x-4} = -\frac{4}{57}$$

$\therefore\ 57(y-18) = -4(x-4)$
$\therefore\ 57y = -4x + 1042$

d $f(x) = \dfrac{x^2-1}{2x+3}$

Since $f(-1) = \dfrac{(-1)^2-1}{2(-1)+3} = \dfrac{0}{1} = 0$,
the point of contact is $(-1, 0)$.
Now $f(x)$ is a quotient where

$u = x^2 - 1$ and $v = 2x+3$

$\therefore\ u' = 2x$ and $v' = 2$

Now $f'(x) = \dfrac{u'v - uv'}{v^2}$
$= \dfrac{2x(2x+3) - (x^2-1)(2)}{(2x+3)^2}$

$\therefore\ f'(-1)$
$= \dfrac{2(-1)(-2+3) - ((-1)^2-1)(2)}{(2(-1)+3)^2}$
$= \dfrac{-2(1) - (0)(2)}{(1)^2} = -2$

$\therefore$ the normal at $(-1, 0)$ has gradient $\frac{1}{2}$.
So, the equation of the normal is

$$\frac{y-0}{x-(-1)} = \frac{1}{2}$$

or $2y = x + 1$

7 The tangent has equation $3x + y = 5$ or $y = -3x + 5$
$\therefore$ the tangent has gradient -3 .... (1)
Also, at $x = -1$, $y = -3(-1) + 5 = 8$
$\therefore$ the tangent contacts the curve at $(-1, 8)$ .... (2)

Now $y = a(1-bx)^{\frac{1}{2}}$, so $\dfrac{dy}{dx} = \frac{1}{2}a(1-bx)^{-\frac{1}{2}}(-b)$

$\therefore\ -3 = \frac{1}{2}a(1+b)^{-\frac{1}{2}}(-b)$ {using (1)}

$\therefore\ 6 = \dfrac{ab}{\sqrt{1+b}}$ .... (3)

Using (2), $(-1, 8)$ must lie on the curve $y = a\sqrt{1-bx}$

$\therefore\ 8 = a\sqrt{1+b}$ .... (4)

$\therefore\ \dfrac{6\sqrt{1+b}}{b} = \dfrac{8}{\sqrt{1+b}}$ {equating a s in (3) and (4)}

$\therefore\ 6(1+b) = 8b$
$\therefore\ 6 + 6b = 8b$
$\therefore\ 6 = 2b$
$\therefore\ b = 3$ and $a = \frac{8}{\sqrt{4}} = 4$

8 **a** $f(x) = e^{-x}$
$\therefore\ f(1) = e^{-1}$
$\therefore$ the point of contact is $\left(1, \dfrac{1}{e}\right)$.

Now $f'(x) = -e^{-x}$
$\therefore\ f'(1) = -e^{-1} = -\dfrac{1}{e}$

So, the gradient of the tangent is $-\dfrac{1}{e}$

$\therefore$ the tangent has equation $\dfrac{y - \frac{1}{e}}{x-1} = -\dfrac{1}{e}$

$\therefore\ e\left(y - \dfrac{1}{e}\right) = -(x-1)$
$\therefore\ ey - 1 = -x + 1$
$\therefore\ x + ey = 2$
or $y = -\dfrac{1}{e}x + \dfrac{2}{e}$

b $y = \ln(2-x)$

so when $x=-1$, $y=\ln 3$

$\therefore$ the point of contact is $(-1, \ln 3)$.

Now $\dfrac{dy}{dx} = \dfrac{-1}{2-x}$

$\therefore$ when $x=-1$, $\dfrac{dy}{dx} = -\dfrac{1}{2+1} = -\frac{1}{3}$

So, the gradient of the tangent is $-\frac{1}{3}$.

$\therefore$ the tangent has equation $\dfrac{y-\ln 3}{x+1} = -\frac{1}{3}$

$\therefore$ $3(y-\ln 3) = -(x+1)$

$\therefore$ $3y - 3\ln 3 = -x-1$

$\therefore$ $x+3y = 3\ln 3 - 1$

c $y = \ln\sqrt{x}$

$= \ln x^{\frac{1}{2}}$

$= \frac{1}{2}\ln x$

$\therefore$ when $y=-1$, $-1 = \frac{1}{2}\ln x$

$\therefore$ $\ln x = -2$

$\therefore$ $x = e^{-2}$

$\therefore$ $x = \dfrac{1}{e^2}$ $\therefore$ the point of contact is $\left(\dfrac{1}{e^2}, -1\right)$

Now $\dfrac{dy}{dx} = \frac{1}{2}\,\dfrac{1}{x} = \dfrac{1}{2x}$, so at the point of contact, $\dfrac{dy}{dx} = \dfrac{1}{2e^{-2}} = \dfrac{e^2}{2}$

$\therefore$ the tangent has gradient $\dfrac{e^2}{2}$ and the normal has gradient $-\dfrac{2}{e^2}$

$\therefore$ the normal has equation $\dfrac{y+1}{x-\dfrac{1}{e^2}} = -\dfrac{2}{e^2}$

$\therefore$ $e^2(y+1) = -2\left(x - \dfrac{1}{e^2}\right)$

$\therefore$ $e^2 y + e^2 = -2x + \dfrac{2}{e^2}$

$\therefore$ $2x + e^2 y = -e^2 + \dfrac{2}{e^2}$ or $y = -\dfrac{2}{e^2}x + \dfrac{2}{e^4} - 1$

9 $y = \dfrac{\cos x}{1+\sin x}$ $\therefore$

$$\frac{dy}{dx} = \frac{(-\sin x)(1+\sin x) - \cos x(\cos x)}{(1+\sin x)^2}$$
$$= \frac{-\sin x - \sin^2 x - \cos^2 x}{(1+\sin x)^2}$$
$$= \frac{-1-\sin x}{(1+\sin x)^2} \qquad \{\sin^2 x + \cos^2 x = 1\}$$
$$= -\frac{(1+\sin x)}{(1+\sin x)^2}$$
$$= \frac{-1}{1+\sin x}$$

Since $\dfrac{-1}{1+\sin x}$ never equals 0, there are no horizontal tangents.

10 **a** $y = \sin x$ $\therefore$ $\dfrac{dy}{dx} = \cos x$

When $x=0$, $\dfrac{dy}{dx} = \cos 0 = 1$

$\therefore$ the tangent has equation $\dfrac{y-0}{x-0} = 1$

or $y = x$

b $y = \tan x$ $\therefore$ $\dfrac{dy}{dx} = \dfrac{1}{\cos^2 x}$

When $x=0$, $\dfrac{dy}{dx} = \dfrac{1}{\cos^2 0} = 1$

$\therefore$ the tangent has equation $\dfrac{y-0}{x-0} = 1$

or $y = x$