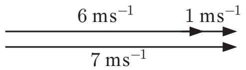
Chapter 15
VECTOR APPLICATIONS

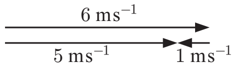
EXERCISE 15A

1 **a**



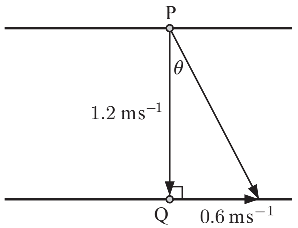
If the athlete is assisted by a wind of 1 m s^{-1} his speed will be 7 m s^{-1}.

b



If the athlete runs into a head wind of 1 m s^{-1} his speed will be 5 m s^{-1}.

2 **a**



$$\begin{aligned}(\text{actual speed})^2 &= (\text{swimming speed})^2 + (\text{current})^2 \\ &= 1.2^2 + 0.6^2 \\ &= 1.8\end{aligned}$$

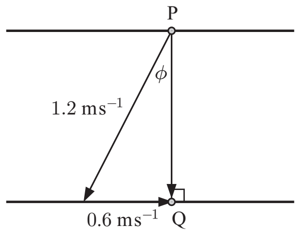
$\therefore$ actual speed $= \sqrt{1.8} \approx 1.34 \text{ m s}^{-1}$

$\tan\theta = \dfrac{0.6}{1.2}$

$\therefore \theta \approx 26.6^\circ$

$\therefore$ Mary's actual velocity is approximately 1.34 m s^{-1} in the direction 26.6° to the left of her intended line.

b **i**



Mary needs to aim to the right of Q so the current will correct her direction.

$\sin\phi = \dfrac{0.6}{1.2}$

$\therefore \phi = 30^\circ$

$\therefore$ Mary should aim to swim 30° to the right of Q.

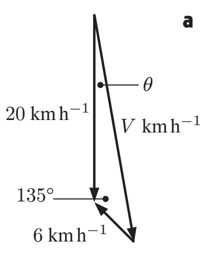
ii

$$(\text{swimming speed})^2 = (\text{actual speed})^2 + (\text{current})^2$$

$$\therefore (\text{actual speed})^2 = 1.2^2 - 0.6^2 = 1.08$$

$\therefore$ Mary's actual speed $= \sqrt{1.08} \approx 1.04 \text{ m s}^{-1}$

3



a Using the cosine rule,

$V^2 = 20^2 + 6^2 - 2 \times 20 \times 6 \times \cos 135^\circ$

$\therefore V \approx 24.6$

$\therefore$ the equivalent speed in still water is 24.6 km h^{-1}.

b Using the sine rule,

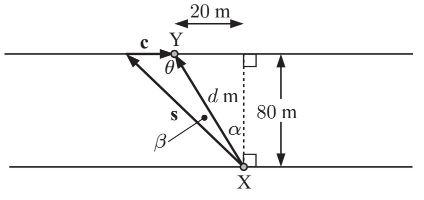
$\dfrac{\sin\theta}{6} \approx \dfrac{\sin 135^\circ}{24.61}$

$\therefore \theta \approx \sin^{-1}\left(\dfrac{6 \times \sin 135^\circ}{24.61}\right)$

$\therefore \theta \approx 9.93^\circ$

$\therefore$ the boat should head 9.93° east of south.

4



a $d^2 = 80^2 + 20^2$ {Pythagoras}

$\therefore d = \sqrt{80^2 + 20^2}$ $\{d > 0\}$

$\therefore d \approx 82.5$

$\therefore$ the distance from X to Y is about 82.5 m.

b $\alpha = \tan^{-1}(\frac{20}{80}) \approx 14.04°$

$\therefore \quad \theta \approx 90° + 14.04° \quad$ {exterior angle of $\triangle$}

$\therefore \quad \theta \approx 104.04°$

In t seconds, Stephanie can swim $1.8t$ metres, and the current will move $0.3t$ metres.

$\therefore \quad |\mathbf{s}| = 1.8t \quad$ and $\quad |\mathbf{c}| = 0.3t$

Using the sine rule,

$$\frac{\sin\beta}{0.3t} = \frac{\sin\theta}{1.8t}$$

$$\beta \approx \sin^{-1}\left(\frac{0.3 \times \sin 104.04°}{1.8}\right)$$

$\therefore \quad \beta \approx 9.31°$

$\therefore \quad \alpha + \beta \approx 23.3°$

$\therefore$ Stephanie should head 23.3° to the left of the perpendicular across the river.

c $\tan(\alpha+\beta) = \dfrac{20 + 0.3t}{80}$

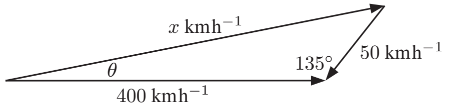
$\therefore \quad 20 + 0.3t \approx 80\tan(23.34°)$

$\therefore \quad t \approx \dfrac{80\tan(23.34°) - 20}{0.3}$

$\therefore \quad t \approx 48.4$

$\therefore$ Stephanie will take 48.4 seconds to cross the river.

5



a Using the cosine rule,

$x^2 = 50^2 + 400^2 - 2 \times 50 \times 400\cos 135°$

$\therefore \quad x \approx 436.79$

The aeroplane should fly so that its speed in still air would be 437 km h^{-1}.

The wind slows the aeroplane down to 400 km h^{-1}.

b Using the sine rule,

$\dfrac{\sin\theta}{50} \approx \dfrac{\sin 135°}{436.79}$

$\therefore \quad \theta \approx 4.64°$

The aeroplane should head 4.64° north of east.

EXERCISE 15B

1 **a** Given A(2, 1, 1), B(4, 3, 0), and C(1, 3, −2), $\overrightarrow{AB} = \begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix}$ and $\overrightarrow{AC} = \begin{pmatrix} -1 \\ 2 \\ -3 \end{pmatrix}$.

$$\therefore \quad \overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 2 & -1 \\ -1 & 2 & -3 \end{vmatrix} = \begin{vmatrix} 2 & -1 \\ 2 & -3 \end{vmatrix}\mathbf{i} - \begin{vmatrix} 2 & -1 \\ -1 & -3 \end{vmatrix}\mathbf{j} + \begin{vmatrix} 2 & 2 \\ -1 & 2 \end{vmatrix}\mathbf{k}$$

$$= -4\mathbf{i} + 7\mathbf{j} + 6\mathbf{k}$$

$\therefore \quad \text{area} = \frac{1}{2}|-4\mathbf{i} + 7\mathbf{j} + 6\mathbf{k}| \qquad \{\text{area} = \frac{1}{2}|\overrightarrow{AB} \times \overrightarrow{AC}|\}$

$= \frac{1}{2}\sqrt{(-4)^2 + 7^2 + 6^2}$

$= \frac{1}{2}\sqrt{101}$ units^2

b Given A(0, 0, 0), B(−1, 2, 3), and C(1, 2, 6), $\overrightarrow{AB} = \begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix}$ and $\overrightarrow{AC} = \begin{pmatrix} 1 \\ 2 \\ 6 \end{pmatrix}$.

$$\therefore \quad \overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1 & 2 & 3 \\ 1 & 2 & 6 \end{vmatrix} = \begin{vmatrix} 2 & 3 \\ 2 & 6 \end{vmatrix}\mathbf{i} - \begin{vmatrix} -1 & 3 \\ 1 & 6 \end{vmatrix}\mathbf{j} + \begin{vmatrix} -1 & 2 \\ 1 & 2 \end{vmatrix}\mathbf{k}$$

$$= 6\mathbf{i} + 9\mathbf{j} - 4\mathbf{k}$$

$\therefore \quad \text{area} = \frac{1}{2}|6\mathbf{i} + 9\mathbf{j} - 4\mathbf{k}| \qquad \{\text{area} = \frac{1}{2}|\overrightarrow{AB} \times \overrightarrow{AC}|\}$

$= \frac{1}{2}\sqrt{6^2 + 9^2 + (-4)^2}$

$= \frac{1}{2}\sqrt{133}$ units^2

c Given A(1, 3, 2), B(2, −1, 0), and C(1, 10, 6), $\overrightarrow{AB} = \begin{pmatrix} 1 \\ -4 \\ -2 \end{pmatrix}$ and $\overrightarrow{AC} = \begin{pmatrix} 0 \\ 7 \\ 4 \end{pmatrix}$.

$$\therefore \quad \overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -4 & -2 \\ 0 & 7 & 4 \end{vmatrix} = \begin{vmatrix} -4 & -2 \\ 7 & 4 \end{vmatrix} \mathbf{i} - \begin{vmatrix} 1 & -2 \\ 0 & 4 \end{vmatrix} \mathbf{j} + \begin{vmatrix} 1 & -4 \\ 0 & 7 \end{vmatrix} \mathbf{k}$$

$$= -2\mathbf{i} - 4\mathbf{j} + 7\mathbf{k}$$

$$\therefore \quad \text{area} = \tfrac{1}{2}\,|-2\mathbf{i} - 4\mathbf{j} + 7\mathbf{k}| = \tfrac{1}{2}\sqrt{(-2)^2 + (-4)^2 + 7^2} = \tfrac{1}{2}\sqrt{69} \text{ units}^2$$

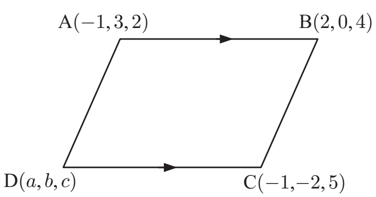
2 Given A(−1, 2, 2), B(2, −1, 4), and C(0, 1, 0), $\overrightarrow{AB} = \begin{pmatrix} 3 \\ -3 \\ 2 \end{pmatrix}$ and $\overrightarrow{AC} = \begin{pmatrix} 1 \\ -1 \\ -2 \end{pmatrix}$.

$$\therefore \quad \overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & -3 & 2 \\ 1 & -1 & -2 \end{vmatrix} = \begin{vmatrix} -3 & 2 \\ -1 & -2 \end{vmatrix} \mathbf{i} - \begin{vmatrix} 3 & 2 \\ 1 & -2 \end{vmatrix} \mathbf{j} + \begin{vmatrix} 3 & -3 \\ 1 & -1 \end{vmatrix} \mathbf{k}$$

$$= 8\mathbf{i} + 8\mathbf{j}$$

$$\therefore \quad \text{area of parallelogram} = |\,8\mathbf{i} + 8\mathbf{j}\,| = \sqrt{8^2 + 8^2} = 8\sqrt{2} \text{ units}^2$$

3 **a**



Suppose D is at (a, b, c).

Since $\overrightarrow{AB} = \overrightarrow{DC}$,

$$\begin{pmatrix} 3 \\ -3 \\ 2 \end{pmatrix} = \begin{pmatrix} -1-a \\ -2-b \\ 5-c \end{pmatrix}$$

$\therefore \quad -1 - a = 3, \quad -2 - b = -3, \quad$ and $\quad 5 - c = 2$

$\therefore \quad a = -4, \quad b = 1, \quad$ and $\quad c = 3$

$\therefore$ D is at (−4, 1, 3).

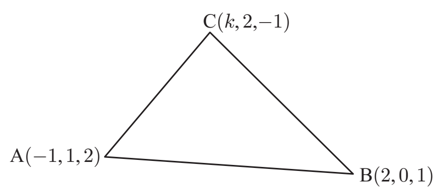
b $\overrightarrow{BC} = \begin{pmatrix} -3 \\ -2 \\ 1 \end{pmatrix}$ and $\overrightarrow{BA} = \begin{pmatrix} -3 \\ 3 \\ -2 \end{pmatrix}$

$$\therefore \quad \overrightarrow{BC} \times \overrightarrow{BA} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -3 & -2 & 1 \\ -3 & 3 & -2 \end{vmatrix} = \begin{vmatrix} -2 & 1 \\ 3 & -2 \end{vmatrix} \mathbf{i} - \begin{vmatrix} -3 & 1 \\ -3 & -2 \end{vmatrix} \mathbf{j} + \begin{vmatrix} -3 & -2 \\ -3 & 3 \end{vmatrix} \mathbf{k}$$

$$= \mathbf{i} - 9\mathbf{j} - 15\mathbf{k}$$

$$\therefore \quad \text{area} = |\mathbf{i} - 9\mathbf{j} - 15\mathbf{k}| = \sqrt{1^2 + (-9)^2 + (-15)^2} = \sqrt{307} \text{ units}^2$$

4 Now $\overrightarrow{AB} = \begin{pmatrix} 3 \\ -1 \\ -1 \end{pmatrix}$ and $\overrightarrow{AC} = \begin{pmatrix} k+1 \\ 1 \\ -3 \end{pmatrix}$



Area of $\triangle$ABC $= \frac{1}{2}\,|\,\overrightarrow{AC} \times \overrightarrow{AB}\,|$

$$\therefore \quad \sqrt{88} = \tfrac{1}{2}\left|\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ k+1 & 1 & -3 \\ 3 & -1 & -1 \end{vmatrix}\right| = \tfrac{1}{2}\left|\begin{vmatrix} 1 & -3 \\ -1 & -1 \end{vmatrix} \mathbf{i} - \begin{vmatrix} k+1 & -3 \\ 3 & -1 \end{vmatrix} \mathbf{j} + \begin{vmatrix} k+1 & 1 \\ 3 & -1 \end{vmatrix} \mathbf{k}\right|$$

$$\therefore \quad \sqrt{352} = |(-1-3)\mathbf{i} - (-(k+1) - -9)\mathbf{j} + (-(k+1) - 3)\mathbf{k}|$$

$$\therefore \quad \sqrt{352} = |-4\mathbf{i} + (k-8)\mathbf{j} + (-k-4)\mathbf{k}|$$

$$\therefore \quad \sqrt{352} = \sqrt{16 + (k-8)^2 + (-k-4)^2}$$

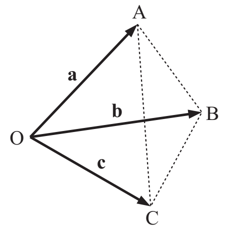
$$\therefore \quad 352 = 16 + k^2 - 16k + 64 + k^2 + 8k + 16$$

$$\therefore \quad 2k^2 - 8k - 256 = 0$$

$$\therefore \quad k^2 - 4k - 128 = 0$$

$$\therefore \quad k = \frac{4 \pm \sqrt{16 + 4(1)(128)}}{2} = 2 \pm \sqrt{132} = 2 \pm 2\sqrt{33}$$

5



Total surface area S of the tetrahedron is the sum of the areas of the 4 triangular faces.

Now $\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB} = -\mathbf{a} + \mathbf{b} = \mathbf{b} - \mathbf{a}$

and $\overrightarrow{AC} = \overrightarrow{AO} + \overrightarrow{OC} = -\mathbf{a} + \mathbf{c} = \mathbf{c} - \mathbf{a}$

$$\therefore \quad S = \tfrac{1}{2}|\mathbf{a}\times\mathbf{b}| + \tfrac{1}{2}|\mathbf{a}\times\mathbf{c}| + \tfrac{1}{2}|\mathbf{b}\times\mathbf{c}| + \tfrac{1}{2}|(\mathbf{b}-\mathbf{a})\times(\mathbf{c}-\mathbf{a})|$$
$$= \tfrac{1}{2}\{|\mathbf{a}\times\mathbf{b}| + |\mathbf{a}\times\mathbf{c}| + |\mathbf{b}\times\mathbf{c}| + |(\mathbf{b}-\mathbf{a})\times(\mathbf{c}-\mathbf{a})|\}$$

EXERCISE 15C

1 **a** **i** $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3 \\ -4 \end{pmatrix} + \lambda\begin{pmatrix} 1 \\ 4 \end{pmatrix}, \quad \lambda \in \mathbb{R}$

ii $x = 3 + \lambda, \quad y = -4 + 4\lambda, \quad \lambda \in \mathbb{R}$

iii $\lambda = x - 3 = \dfrac{y+4}{4}$

$\therefore \ 4x - 12 = y + 4$

$\therefore \ 4x - y = 16$

b **i** If the line has direction vector $\mathbf{b}$ perpendicular to $\begin{pmatrix} 5 \\ 2 \end{pmatrix}$, then

$\mathbf{b} \bullet \begin{pmatrix} 5 \\ 2 \end{pmatrix} = 0$

$\therefore \ \mathbf{b} = \begin{pmatrix} -2 \\ 5 \end{pmatrix}$ is a reasonable choice

$\therefore \ \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ 2 \end{pmatrix} + \lambda\begin{pmatrix} -2 \\ 5 \end{pmatrix}, \quad \lambda \in \mathbb{R}$

ii $x = 5 - 2\lambda, \quad y = 2 + 5\lambda, \quad \lambda \in \mathbb{R}$

iii $\lambda = \dfrac{x-5}{-2} = \dfrac{y-2}{5}$

$\therefore \ 5x - 25 = -2y + 4$

$\therefore \ 5x + 2y = 29$

c **i** $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -6 \\ 0 \end{pmatrix} + \lambda\begin{pmatrix} 3 \\ 7 \end{pmatrix}, \quad \lambda \in \mathbb{R}$

ii $x = -6 + 3\lambda, \quad y = 7\lambda, \quad \lambda \in \mathbb{R}$

iii $\lambda = \dfrac{x+6}{3} = \dfrac{y}{7}$

$\therefore \ 7x + 42 = 3y$

$\therefore \ 7x - 3y = -42$

d **i** Take $(-1, 11)$ as our fixed point,

so $\mathbf{a} = \begin{pmatrix} -1 \\ 11 \end{pmatrix}$.

The direction vector $\mathbf{b} = \begin{pmatrix} -3-(-1) \\ 12-11 \end{pmatrix}$

$= \begin{pmatrix} -2 \\ 1 \end{pmatrix}$

$\therefore \ \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -1 \\ 11 \end{pmatrix} + \lambda\begin{pmatrix} -2 \\ 1 \end{pmatrix}, \quad \lambda \in \mathbb{R}$

ii $x = -1 - 2\lambda, \quad y = 11 + \lambda, \quad \lambda \in \mathbb{R}$

iii $\lambda = \dfrac{x+1}{-2} = y - 11$

$\therefore \ x + 1 = -2y + 22$

$\therefore \ x + 2y = 21$

2 **a** $x = -1 + 2\lambda, \quad y = 4 - \lambda, \quad \lambda \in \mathbb{R}$

b When $\lambda = 0$, $x = -1 + 2(0) = -1$ and $y = 4 - 0 = 4$ $\quad \therefore$ the point is $(-1, 4)$.

When $\lambda = 1$, $x = -1 + 2(1) = 1$ and $y = 4 - 1 = 3$ $\quad \therefore$ the point is $(1, 3)$.

When $\lambda = 3$, $x = -1 + 2(3) = 5$ and $y = 4 - 3 = 1$ $\quad \therefore$ the point is $(5, 1)$.

When $\lambda = -1$, $x = -1 + 2(-1) = -3$ and $y = 4 - (-1) = 5$ $\quad \therefore$ the point is $(-3, 5)$.

When $\lambda = -4$, $x = -1 + 2(-4) = -9$ and $y = 4 - (-4) = 8$ $\quad \therefore$ the point is $(-9, 8)$.

3 **a** If $\lambda + 2 = 3$ and $1 - 3\lambda = -2$,

then $\lambda = 1$ and $-3\lambda = -3$

$\therefore \ \lambda = 1$

Since $\lambda = 1$ in each case, $(3, -2)$ lies on the line.

b If $(k, 4)$ lies on $x = 1 - 2\lambda, \quad y = 1 + \lambda$, then

$k = 1 - 2\lambda$ and $4 = 1 + \lambda$

$\therefore \ \lambda = 3$

and $k = 1 - 6$

$\therefore \ k = -5$

4 **a** When $\lambda = 1$, $\mathbf{r} = \begin{pmatrix} 1 \\ 5 \end{pmatrix} + 1 \times \begin{pmatrix} -1 \\ 3 \end{pmatrix} = \begin{pmatrix} 1-1 \\ 5+3 \end{pmatrix} = \begin{pmatrix} 0 \\ 8 \end{pmatrix}$

$\therefore$ the point is $(0, 8)$.

b $\begin{pmatrix} 1 \\ -3 \end{pmatrix}$ is a non-zero scalar multiple of $\begin{pmatrix} -1 \\ 3 \end{pmatrix}$. It is parallel and in the opposite direction, so it could also be used to describe the direction of the line.

c The line passes through point $(0, 8)$ and has direction vector $\begin{pmatrix} 1 \\ -3 \end{pmatrix}$.

$\therefore$ $\mathbf{r} = \begin{pmatrix} 0 \\ 8 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ -3 \end{pmatrix}$, $\mu \in \mathbb{R}$ is an alternative vector equation for the line.

5 **a** **i** $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \\ -7 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$, $\lambda \in \mathbb{R}$

ii $x = 1 + 2\lambda$, $y = 3 + \lambda$, $z = -7 + 3\lambda$, $\lambda \in \mathbb{R}$ **iii** $\lambda = \dfrac{x-1}{2} = y - 3 = \dfrac{z+7}{3}$

b **i** $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}$, $\lambda \in \mathbb{R}$

ii $x = \lambda$, $y = 1 + \lambda$, $z = 2 - 2\lambda$, $\lambda \in \mathbb{R}$ **iii** $\lambda = x = y - 1 = \dfrac{-z+2}{2}$

c **i** Since the line is parallel to the X-axis, it has direction vector $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

$\therefore$ $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -2 \\ 2 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$, $\lambda \in \mathbb{R}$

ii $x = -2 + \lambda$, $y = 2$, $z = 1$, $\lambda \in \mathbb{R}$ **iii** $y = 2$, $z = 1$

d **i** $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$, $\lambda \in \mathbb{R}$

ii $x = 2\lambda$, $y = 2 - \lambda$, $z = -1 + 3\lambda$, $\lambda \in \mathbb{R}$ **iii** $\lambda = \dfrac{x}{2} = -y + 2 = \dfrac{z+1}{3}$

e **i** Since the line is perpendicular to the XOY plane, it has direction vector $\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

$\therefore$ $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$, $\lambda \in \mathbb{R}$

ii $x = 3$, $y = 2$, $z = -1 + \lambda$, $\lambda \in \mathbb{R}$ **iii** $x = 3$, $y = 2$

6 **a** $\overrightarrow{AB} = \begin{pmatrix} -1-1 \\ 3-2 \\ 2-1 \end{pmatrix} = \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix}$ $\therefore$ $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix}$, $\lambda \in \mathbb{R}$

b $\overrightarrow{CD} = \begin{pmatrix} 3-0 \\ 1-1 \\ -1-3 \end{pmatrix} = \begin{pmatrix} 3 \\ 0 \\ -4 \end{pmatrix}$ $\therefore$ $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 0 \\ -4 \end{pmatrix}$, $\lambda \in \mathbb{R}$

c $\overrightarrow{EF} = \begin{pmatrix} 1-1 \\ -1-2 \\ 5-5 \end{pmatrix} = \begin{pmatrix} 0 \\ -3 \\ 0 \end{pmatrix}$ $\therefore$ $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ -3 \\ 0 \end{pmatrix}$, $\lambda \in \mathbb{R}$

d $\overrightarrow{GH} = \begin{pmatrix} 5-0 \\ -1-1 \\ 3--1 \end{pmatrix} = \begin{pmatrix} 5 \\ -2 \\ 4 \end{pmatrix}$ $\therefore$ $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 5 \\ -2 \\ 4 \end{pmatrix}$, $\lambda \in \mathbb{R}$

7 **a** $\begin{pmatrix} -2 \\ 0 \\ 3 \end{pmatrix}$ **b** $\begin{pmatrix} -1 \\ 1 \\ -3 \end{pmatrix}$

c $\lambda = \dfrac{x-2}{3} = \dfrac{y+1}{2} = z-1$

$\therefore\ x-2=3\lambda$ and $y+1=2\lambda$ and $z-1=\lambda$

$\therefore\ x=2+3\lambda \quad y=-1+2\lambda \quad z=1+\lambda$

$\therefore$ direction vector is $\begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$

d $\mu = \dfrac{1-x}{2} = \dfrac{y}{4} = \dfrac{z-3}{3}$

$\therefore\ 2\mu = 1-x$ and $y=4\mu$ and $z-3=3\mu$

$\therefore\ x=1-2\mu \quad y=4\mu \quad z=3+3\mu$

$\therefore$ direction vector is $\begin{pmatrix} -2 \\ 4 \\ 3 \end{pmatrix}$

8 Given $x=1-\lambda$, $y=3+\lambda$, $z=3-2\lambda$:

a The line meets the XOY plane when $z=0$ $\therefore\ 3-2\lambda=0$

$\therefore\ \lambda=\frac{3}{2}$

Then $x=1-\frac{3}{2}=-\frac{1}{2}$ and $y=3+\frac{3}{2}=\frac{9}{2}$, so the point is $\left(-\frac{1}{2}, \frac{9}{2}, 0\right)$.

b The line meets the YOZ plane when $x=0$ $\therefore\ 1-\lambda=0$

$\therefore\ \lambda=1$

Then $y=3+1=4$ and $z=3-2=1$, so the point is $(0, 4, 1)$.

c The line meets the XOZ plane when $y=0$ $\therefore\ 3+\lambda=0$

$\therefore\ \lambda=-3$

Then $x=1-(-3)=4$ and $z=3-2(-3)=9$, so the point is $(4, 0, 9)$.

9 **a** When $\lambda=0$, $x=x_0$, $y=y_0$, and $z=z_0$

$\therefore\ (x_0, y_0, z_0)$

b $\begin{pmatrix} l \\ m \\ n \end{pmatrix}$

c $\lambda = \dfrac{x-x_0}{l} = \dfrac{y-y_0}{m} = \dfrac{z-z_0}{n}$, $l, m, n \neq 0$

10 Given a line with equations $x=2-\lambda$, $y=3+2\lambda$, and $z=1+\lambda$,
the distance to the point $(1, 0, -2)$ is $\sqrt{(2-\lambda-1)^2+(3+2\lambda-0)^2+(1+\lambda+2)^2}$.

But this distance $=5\sqrt{3}$ units

$$\therefore\ \sqrt{(1-\lambda)^2+(3+2\lambda)^2+(\lambda+3)^2}=5\sqrt{3}$$

$$\therefore\ (1-\lambda)^2+(3+2\lambda)^2+(\lambda+3)^2=75$$

$$\therefore\ 1-2\lambda+\lambda^2+9+12\lambda+4\lambda^2+\lambda^2+6\lambda+9=75$$

$$\therefore\ 6\lambda^2+16\lambda-56=0$$

$$\therefore\ 3\lambda^2+8\lambda-28=0$$

$$\therefore\ (3\lambda+14)(\lambda-2)=0$$

$$\therefore\ \lambda=-\tfrac{14}{3} \text{ or } \lambda=2$$

When $\lambda=2$, the point is $(0, 7, 3)$, and when $\lambda=-\frac{14}{3}$, the point is $\left(\frac{20}{3}, -\frac{19}{3}, -\frac{11}{3}\right)$.