29 Consider $f(x) = x + \sin x$, $x \geqslant 0$.

a Sketch $y = f(x)$ for $0 \leqslant x \leqslant 2\pi$.

b Find the area enclosed by the curve, the x-axis, and $x = \frac{\pi}{2}$.

c Suppose $f(x) = x + \sin x$ and $g(x) = x^2$ meet at $x = a$, $a > 0$. Find a correct to 3 decimal places.

d Find the area enclosed by $y = x + \sin x$ and $y = x^2$.

30 Consider $f(t) = \sin^2 t - \sin t$ where $0 \leqslant t \leqslant 2$.

a Sketch the graph of $y = f(t)$ over the given domain.

b Find the area enclosed by the curve, the t-axis, and $t = 2$.

31 The shaded area is 24 units^2. Find k.
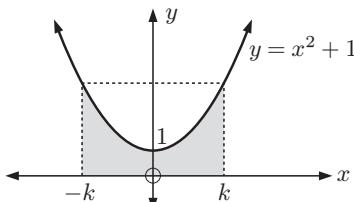
32 At time t seconds a particle is moving in a straight line with acceleration $4\cos t$ m s^{-2}. At $t = 0$ its velocity is 2 m s^{-1}.

a Find the velocity at $t = 4$ seconds.

b Find the total distance travelled in the interval $t = 0$ to $t = 5$ seconds. Give your answer to two decimal places.

33 Let $f(x) = 2\cos x - \sin x + 2x \sin x$, $0 \leqslant x \leqslant 2$.

a Show that $f'(x) = (2x - 1)\cos x$.

b Find the minimum value of $f(x)$ for $0 \leqslant x \leqslant 2$.

34 Find a given that $0 < a < 2\pi$ and $\int_a^{a+2} \sin x \, dx = 0.3$.

TRIAL EXAMINATION 1

Paper 1 - No calculators   (1 hour 30 minutes)

Section A

1 The first three terms of an infinite geometric sequence are 24, x, and 6, where $x > 0$.

a Write down x.   **b** Find the common ratio r.

c Find u_5.   **d** Find the sum to infinity of the sequence.

(7 marks)

2 Let $h(x) = e^{-x} \cos x$.

a Find $h'(x)$.   **b** Find $h'(\frac{\pi}{2})$.

c Find the equation of the tangent to the curve at the point where $x = \frac{\pi}{2}$.   (8 marks)

3 Adrian and Bevan are competing in an archery competition. Let A be the event of Adrian hitting the target, and B be the event of Bevan hitting the target. The tree diagram below shows the probabilities for the events A and B.

a Write down the values of x, y, and z.

b Find the probability that Bevan hits the target.

c Find $P(A' \mid B)$.
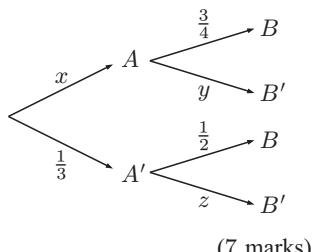
(7 marks)

4 a If $\sin 2\theta = \tan \theta$, show that either $\sin \theta = 0$ or $\cos \theta = \pm\frac{1}{\sqrt{2}}$.

b Hence, find θ such that $\sin 2\theta = \tan \theta$ for $-\pi \leqslant \theta \leqslant \pi$.   (6 marks)

5
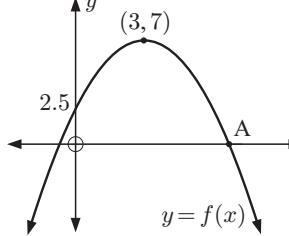
$f(x)$ is a quadratic function with vertex $(3, 7)$ and y-intercept 2.5. $f(x)$ can be written in the form $a(x - h)^2 + k$.

a Find h and k.   **b** Find a.

c Find the coordinates of A.   (7 marks)

6 Let $f'(x) = 2x - \dfrac{3}{\sqrt{x}}$ and $f(4) = 3$. Find $f(x)$.   (4 marks)

7 Random variable X has the following probability distribution:

x	1	2	3	4
$P(X = x)$	m	0.15	$2m$	n

a Find the value of $3m + n$.

b Write $E(X)$ in terms of m only.

**c** Find m if $E(X) = 2.7$.   (6 marks)

Section B

8 Points A$(2, -1, 3)$ and B$(1, 2, -4)$ lie on line L_1.

A second line L_2 is parallel to $\begin{pmatrix} 4 \\ 2m \\ m \end{pmatrix}$ and is perpendicular to L_1.

a Find $\overrightarrow{BA}$.

b Find a vector equation for L_1 in the form $\mathbf{r} = \mathbf{p} + t\mathbf{q}$.

c Find m.

d If L_2 passes through the point C$(2, 3, k)$, find a vector equation for L_2.

e If L_1 and L_2 intersect, find the value of k.   (16 marks)

9
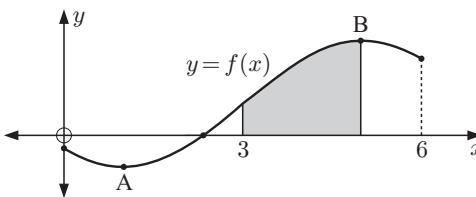
The graph shows the function $f(x) = -4\cos\left(\frac{\pi}{4}(x - 1)\right) + 2$, $0 \leqslant x \leqslant 6$. A and B are turning points of the function.

a Find the y-intercept.   **b** Find the x-intercept.

c Show that $f'(x) = \pi \sin\left(\frac{\pi}{4}(x - 1)\right)$.

d By considering $f'(x) = 0$, find the x-coordinates of A and B.

e Find the area of the shaded region in terms of π.   (15 marks)

10 $f(x) = 2x - 1$ and $g(x) = 3x^2 - 1$.

a Find $(g \circ f)(x)$.

b $g \circ f$ is translated through $\begin{pmatrix} -1 \\ 4 \end{pmatrix}$ to produce the function $h(x)$. Find $h(x)$.

c Write $h(x)$ in the form $a(x - h)^2 + k$.

d State the coordinates of the vertex of:

i $g(x)$ **ii** $h(x)$.

e Find c given that $y = 2x + c$ is a tangent to $h(x)$.   (14 marks)

Paper 2 - Calculators   (1 hour 30 minutes)

Section A

1 In the diagram, $\stackrel{\frown}{AOB}$ measures θ, and arc length AXB is 14.3 m.

  a Find θ, in radians.

  b Find the shaded area.
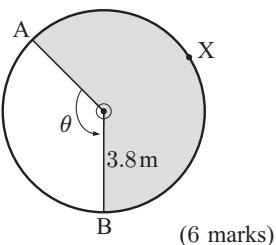
                            (6 marks)

2 Consider the binomial expansion of $(2x^2 - 1)^{12}$.

  a Find an expression for the $(r+1)$th term.

  b Find the coefficient of the x^{10} term.   (6 marks)

3 The number of children in the families of students in a class is shown in the following table:

Number of children	1	2	3	4	5	8
Frequency	4	11	7	8	2	1

  a Find the mean and median of the data.

  b What is the standard deviation?

  c What is the interquartile range?   (6 marks)

**4** $u_n = 19 - 2n$ is the nth term of a sequence.

  a Explain why the sequence is arithmetic.

  b Write down the common difference.

  c Is -55 a term of the sequence?

  d Find the sum of the first n terms of the sequence.

                            (8 marks)

5 The angle between $\mathbf{a} = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 3 \\ 1 \\ t \end{pmatrix}$ is $60°$.

  Find t, correct to 3 decimal places, given $t > 0$.   (5 marks)

6 The graph of $f(x) = 2x^3 + bx^2 + cx$ is shown alongside, but is not drawn to scale.

  a Use points A and C to find the values of b and c.

  b Find the x-coordinate of B, that is, k, given that $k > 1$.

                            (6 marks)
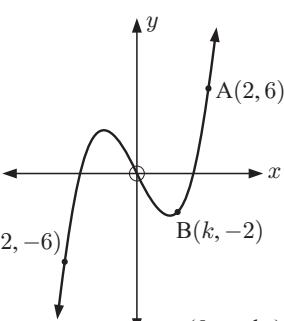
**7** Two variables x and y have the experimental values shown:

x	2	3	4	5	6	8
y	4.5	6.7	8.9	10.9	13.2	17.2

  a Explain why x and y have a positive correlation.

  b Find the equation of the linear regression line.

  c Describe the correlation between x and y.

  d Use the equation of the linear regression line to predict the value of y when $x = 7$.

  e Is it reliable to use the regression line to find y when $x = 12$?   (8 marks)

Section B

8 The graph of $f(x) = x^2 \ln(9 - x^2)$, $-3 < x < 3$ is shown.
It cuts the x-axis at a and b and has a maximum turning point at $x = c$.
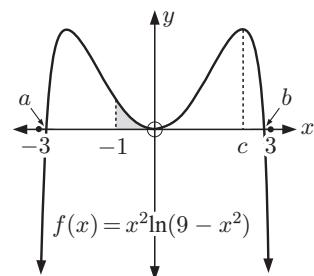
  a Find the values of a and b correct to 3 decimal places.

  b Find the value of c.

  c Find the equations of the asymptotes of the graph. Explain your answers.

  d Find the area of the shaded region.

  e If the region shaded is rotated $360°$ about the x-axis, what is the volume of the solid generated?   (13 marks)

9 The height of a Ferris wheel seat above ground level is given by
$H(t) = a \sin(b(t - c)) + d$,
where a, b, c, and d are constants.

The Ferris wheel starts moving clockwise when the seat is at A, 3 m above ground level.
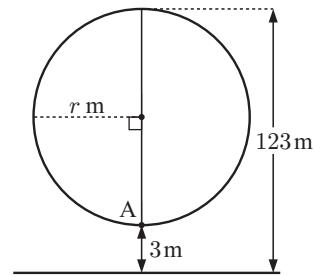
In 20 minutes the seat is returned to A.

  a Find the radius of the wheel.

  b Find the values of a and d.

  c Explain why $b = \frac{\pi}{10}$.

  d Determine the value of c.

  e How high is the seat above the ground after 8 minutes of motion?

  f After how long is the seat 100 m above ground level?

                            (18 marks)

10
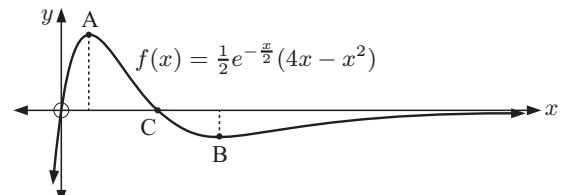
Consider the graph of the function $f(x) = \frac{1}{2}e^{-\frac{x}{2}}(4x - x^2)$.

  a From the graph, state the equation of any asymptote.

  b State the x-intercept(s) of the graph of f.

  c Without finding $f'(x)$, find the coordinates of the turning points at A and B, correct to 3 significant figures.

  d If $y = x^2 e^{-\frac{x}{2}}$, show that $\dfrac{dy}{dx} = \frac{1}{2}e^{-\frac{x}{2}}(4x - x^2)$.

  e Hence, find $\int e^{-\frac{x}{2}}(4x - x^2)\,dx$.   (14 marks)