Chapter 10

COORDINATE GEOMETRY

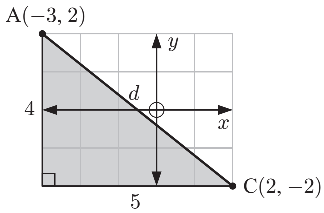
EXERCISE 10A.1

1 **a** The distance between A and B is 5 units.

b The distance between B and C is 4 units.

c The distance between C and D is 3 units.

d



Let the distance from A to C be d units.

$d^2 = 4^2 + 5^2$ {Pythagoras}

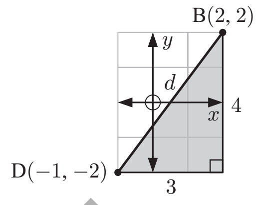
$\therefore\ d^2 = 16 + 25$

$\therefore\ d^2 = 41$

$\therefore\ d = \sqrt{41}$ {as $d > 0$}

$\therefore$ the distance from A to C is $\sqrt{41}$ units.

e



Let the distance from B to D be d units.

$d^2 = 3^2 + 4^2$ {Pythagoras}

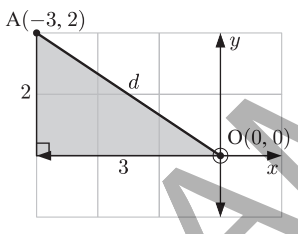
$\therefore\ d^2 = 9 + 16$

$\therefore\ d^2 = 25$

$\therefore\ d = \sqrt{25} = 5$ {as $d > 0$}

$\therefore$ the distance from B to D is 5 units.

f



Let the distance from O to A be d units.

$d^2 = 3^2 + 2^2$ {Pythagoras}

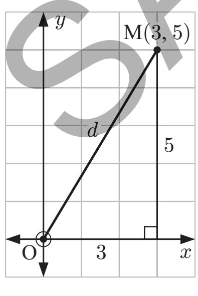
$\therefore\ d^2 = 9 + 4$

$\therefore\ d^2 = 13$

$\therefore\ d = \sqrt{13}$ {as $d > 0$}

$\therefore$ the distance from O to A is $\sqrt{13}$ units.

2 **a**



Let the distance from O to M be d units.

$d^2 = 3^2 + 5^2$ {Pythagoras}

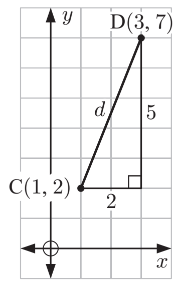
$\therefore\ d^2 = 9 + 25$

$\therefore\ d^2 = 34$

$\therefore\ d = \sqrt{34}$ {as $d > 0$}

$\therefore$ the distance from O to M is $\sqrt{34}$ units.

b



Let the distance from C to D be d units.

$d^2 = 2^2 + 5^2$ {Pythagoras}

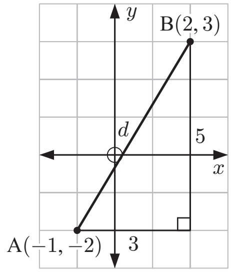
$\therefore\ d^2 = 4 + 25$

$\therefore\ d^2 = 29$

$\therefore\ d = \sqrt{29}$ {as $d > 0$}

$\therefore$ the distance from C to D is $\sqrt{29}$ units.

c



Let the distance from A to B be d units.

$d^2 = 3^2 + 5^2$ {Pythagoras}

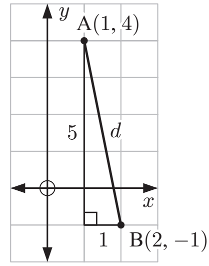
$\therefore\ d^2 = 9 + 25$

$\therefore\ d^2 = 34$

$\therefore\ d = \sqrt{34}$ {as $d > 0$}

$\therefore$ the distance from A to B is $\sqrt{34}$ units.

d



Let the distance from A to B be d units.

$d^2 = 1^2 + 5^2$ {Pythagoras}

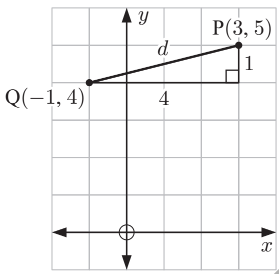
$\therefore\ d^2 = 1 + 25$

$\therefore\ d^2 = 26$

$\therefore\ d = \sqrt{26}$ {as $d > 0$}

$\therefore$ the distance from A to B is $\sqrt{26}$ units.

e



Let the distance from P to Q be d units.

$d^2 = 4^2 + 1^2$ {Pythagoras}

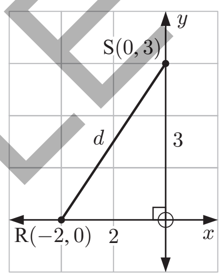
$\therefore\ d^2 = 16 + 1$

$\therefore\ d^2 = 17$

$\therefore\ d = \sqrt{17}$ {as $d > 0$}

$\therefore$ the distance from P to Q is $\sqrt{17}$ units.

f



Let the distance from R to S be d units.

$d^2 = 2^2 + 3^2$ {Pythagoras}

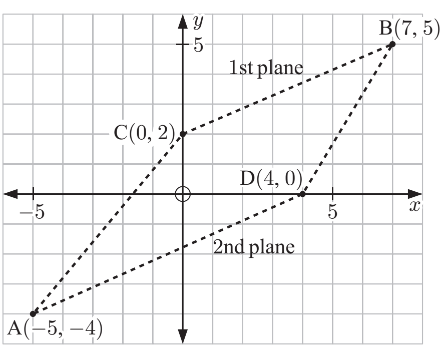
$\therefore\ d^2 = 4 + 9$

$\therefore\ d^2 = 13$

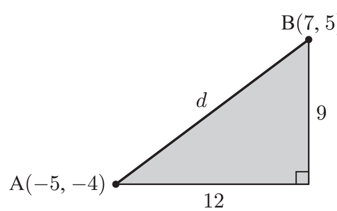
$\therefore\ d = \sqrt{13}$ {as $d > 0$}

$\therefore$ the distance from R to S is $\sqrt{13}$ units.

3 **a**



b



Let the distance from A to B be d units.

$d^2 = 12^2 + 9^2$ {Pythagoras}

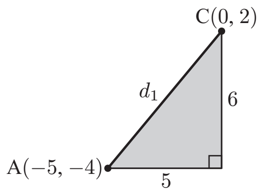
$\therefore\ d^2 = 144 + 81$

$\therefore\ d^2 = 225$

$\therefore\ d = \sqrt{225} = 15$ {as $d > 0$}

$\therefore$ the distance from A to B is 15 units.

c **i** Let the distance from A to C be d_1 units.



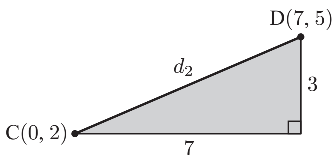
$d_1^2 = 5^2 + 6^2$ {Pythagoras}

$\therefore\ d_1^2 = 25 + 36$

$\therefore\ d_1^2 = 61$

$\therefore\ d_1 = \sqrt{61}$ {as $d_1 > 0$}

Let the distance from C to B be d_2 units.



$d_2^2 = 7^2 + 3^2$ {Pythagoras}

$\therefore\ d_2^2 = 49 + 9$

$\therefore\ d_2^2 = 58$

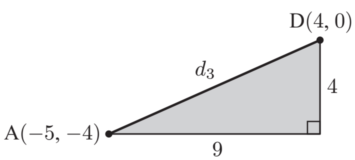
$\therefore\ d_2 = \sqrt{58}$ {as $d_2 > 0$}

$\therefore$ distance travelled by the first plane $= d_1$ units $+ d_2$ units

$= \sqrt{61}$ units $+ \sqrt{58}$ units

≈ 15.4 units

ii Let the distance from A to D be d_3 units.



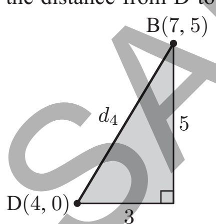
$d_3^2 = 9^2 + 4^2$ {Pythagoras}

$\therefore\ d_3^2 = 81 + 6$

$\therefore\ d_3^2 = 97$

$\therefore\ d_3 = \sqrt{97}$ {as $d_3 > 0$}

Let the distance from D to B be d_4 units.



$d_4^2 = 3^2 + 5^2$ {Pythagoras}

$\therefore\ d_4^2 = 9 + 25$

$\therefore\ d_4^2 = 34$

$\therefore\ d_4 = \sqrt{34}$ {as $d_4 > 0$}

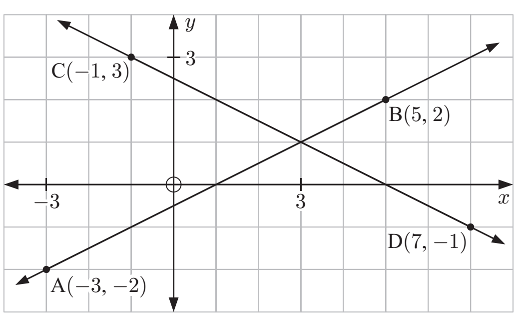
$\therefore$ distance travelled by the second plane $= d_3$ units $+ d_4$ units

$= \sqrt{97}$ units $+ \sqrt{34}$ units

≈ 15.7 units

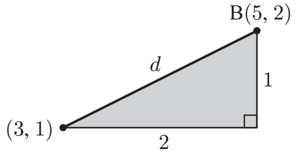
d The first plane has a shorter distance to travel.

4 **a, b**



b (AB) and (CD) intersect at (3, 1).

c From the graph in **a**, it is clear that B is the closest point to the intersection of (AB) and (CD). Let the distance between B and the point of intersection be d units.



$$d^2 = 2^2 + 1^2 \quad \{\text{Pythagoras}\}$$
$$\therefore\ d^2 = 4 + 1$$
$$\therefore\ d^2 = 5$$
$$\therefore\ d = \sqrt{5} \quad \{\text{as } d > 0\}$$
$\therefore$ the distance is $\sqrt{5}$ units.

EXERCISE 10A.2

1 **a** O(0, 0) P(3, −1)

$\uparrow x_1$ $\uparrow y_1$ $\uparrow x_2$ $\uparrow y_2$

$$\begin{aligned} OP &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(3-0)^2 + (-1-0)^2} \\ &= \sqrt{3^2 + (-1)^2} \\ &= \sqrt{9+1} \\ &= \sqrt{10} \text{ units} \end{aligned}$$

b A(2, 1) B(4, 4)

$\uparrow x_1$ $\uparrow y_1$ $\uparrow x_2$ $\uparrow y_2$

$$\begin{aligned} AB &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(4-2)^2 + (4-1)^2} \\ &= \sqrt{2^2 + 3^2} \\ &= \sqrt{4+9} \\ &= \sqrt{13} \text{ units} \end{aligned}$$

c C(−2, 1) D(2, 5)

$\uparrow x_1$ $\uparrow y_1$ $\uparrow x_2$ $\uparrow y_2$

$$\begin{aligned} CD &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(2 - -2)^2 + (5-1)^2} \\ &= \sqrt{4^2 + 4^2} \\ &= \sqrt{16+16} \\ &= \sqrt{32} \\ &= 4\sqrt{2} \text{ units} \end{aligned}$$

d E(4, 3) F(−1, −1)

$\uparrow x_1$ $\uparrow y_1$ $\uparrow x_2$ $\uparrow y_2$

$$\begin{aligned} EF &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(-1-4)^2 + (-1-3)^2} \\ &= \sqrt{(-5)^2 + (-4)^2} \\ &= \sqrt{25+16} \\ &= \sqrt{41} \text{ units} \end{aligned}$$

e G(0, −3) H(1, 4)

$\uparrow x_1$ $\uparrow y_1$ $\uparrow x_2$ $\uparrow y_2$

$$\begin{aligned} GH &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(1-0)^2 + (4 - -3)^2} \\ &= \sqrt{1^2 + 7^2} \\ &= \sqrt{1+49} \\ &= \sqrt{50} \\ &= 5\sqrt{2} \text{ units} \end{aligned}$$

f I(0, 2) J(−3, 0)

$\uparrow x_1$ $\uparrow y_1$ $\uparrow x_2$ $\uparrow y_2$

$$\begin{aligned} IJ &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(-3-0)^2 + (0-2)^2} \\ &= \sqrt{(-3)^2 + (-2)^2} \\ &= \sqrt{9+4} \\ &= \sqrt{13} \text{ units} \end{aligned}$$

g P(3, 9) Q(11, −1)

x_1 y_1 x_2 y_2

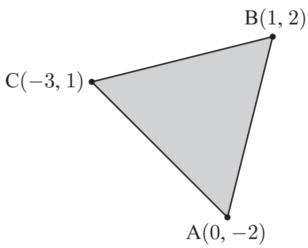
$$\begin{aligned}PQ &= \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}\\ &= \sqrt{(11-3)^2+(-1-9)^2}\\ &= \sqrt{8^2+(-10)^2}\\ &= \sqrt{64+100}\\ &= \sqrt{164}\\ &= 2\sqrt{41}\text{ units}\end{aligned}$$

h R($-\sqrt{2}$, 3) S($\sqrt{2}$, −1)

x_1 y_1 x_2 y_2

$$\begin{aligned}RS &= \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}\\ &= \sqrt{(\sqrt{2}--\sqrt{2})^2+(-1-3)^2}\\ &= \sqrt{(2\sqrt{2})^2+(-4)^2}\\ &= \sqrt{8+16}\\ &= \sqrt{24}\\ &= 2\sqrt{6}\text{ units}\end{aligned}$$

2 **a**



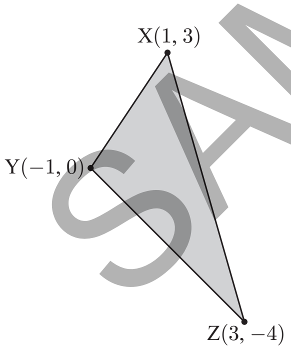
$$\begin{aligned}AB &= \sqrt{(1-0)^2+(2--2)^2}\\ &= \sqrt{1^2+4^2}\\ &= \sqrt{17}\text{ units}\end{aligned}$$

$$\begin{aligned}AC &= \sqrt{(-3-0)^2+(1--2)^2}\\ &= \sqrt{(-3)^2+3^2}\\ &= \sqrt{18}\text{ units}\end{aligned}$$

$$\begin{aligned}BC &= \sqrt{(-3-1)^2+(1-2)^2}\\ &= \sqrt{(-4)^2+(-1)^2}\\ &= \sqrt{17}\text{ units}\end{aligned}$$

Since two of the side lengths are equal, triangle ABC is isosceles.

b



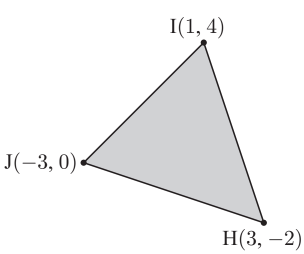
$$\begin{aligned}XY &= \sqrt{(-1-1)^2+(0-3)^2}\\ &= \sqrt{(-2)^2+(-3)^2}\\ &= \sqrt{13}\text{ units}\end{aligned}$$

$$\begin{aligned}XZ &= \sqrt{(3-1)^2+(-4-3)^2}\\ &= \sqrt{2^2+(-7)^2}\\ &= \sqrt{53}\text{ units}\end{aligned}$$

$$\begin{aligned}YZ &= \sqrt{(3--1)^2+(-4-0)^2}\\ &= \sqrt{4^2+(-4)^2}\\ &= \sqrt{32}\text{ units}\end{aligned}$$

Since none of the side lengths are equal, triangle XYZ is scalene.

c



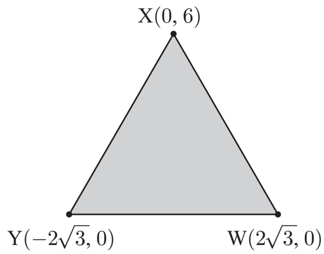
$$\begin{aligned}HI &= \sqrt{(1-3)^2+(4--2)^2}\\ &= \sqrt{(-2)^2+6^2}\\ &= \sqrt{40}\text{ units}\end{aligned}$$

$$\begin{aligned}HJ &= \sqrt{(-3-3)^2+(0--2)^2}\\ &= \sqrt{(-6)^2+2^2}\\ &= \sqrt{40}\text{ units}\end{aligned}$$

$$\begin{aligned}IJ &= \sqrt{(-3-1)^2+(0-4)^2}\\ &= \sqrt{(-4)^2+(-4)^2}\\ &= \sqrt{32}\text{ units}\end{aligned}$$

Since two of the side lengths are equal, triangle HIJ is isosceles.

d



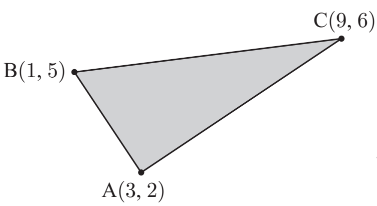
$$\begin{aligned} WX &= \sqrt{(0-2\sqrt{3})^2+(6-0)^2} \\ &= \sqrt{(-2\sqrt{3})^2+6^2} \\ &= \sqrt{48} \text{ units} \end{aligned}$$

$$\begin{aligned} WY &= \sqrt{(-2\sqrt{3}-2\sqrt{3})^2+(0-0)^2} \\ &= \sqrt{(-4\sqrt{3})^2+0^2} \\ &= \sqrt{48} \text{ units} \end{aligned}$$

$$\begin{aligned} XY &= \sqrt{(-2\sqrt{3}-0)^2+(0-6)^2} \\ &= \sqrt{(-2\sqrt{3})^2+(-6)^2} \\ &= \sqrt{48} \text{ units} \end{aligned}$$

Since all of the side lengths are equal, triangle WXY is equilateral.

3 **a**



$$\begin{aligned} AB &= \sqrt{(1-3)^2+(5-2)^2} \\ &= \sqrt{(-2)^2+3^2} \\ &= \sqrt{13} \text{ units} \end{aligned}$$

$$\begin{aligned} AC &= \sqrt{(9-3)^2+(6-2)^2} \\ &= \sqrt{6^2+4^2} \\ &= \sqrt{52} \text{ units} \end{aligned}$$

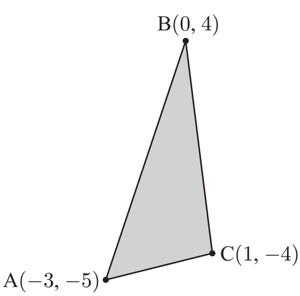
$$\begin{aligned} BC &= \sqrt{(9-1)^2+(6-5)^2} \\ &= \sqrt{8^2+1^2} \\ &= \sqrt{65} \text{ units} \end{aligned}$$

The shorter sides are [AB] and [AC].

Now, $$\begin{aligned} AB^2 + AC^2 &= 13 + 52 \\ &= 65 \\ &= BC^2 \end{aligned}$$

Using the converse of Pythagoras' theorem, triangle ABC is right angled. The right angle is opposite the longest side, so the right angle is at A.

b



$$\begin{aligned} AB &= \sqrt{(0--3)^2+(4--5)^2} \\ &= \sqrt{3^2+9^2} \\ &= \sqrt{90} \text{ units} \end{aligned}$$

$$\begin{aligned} AC &= \sqrt{(1--3)^2+(-4--5)^2} \\ &= \sqrt{4^2+1^2} \\ &= \sqrt{17} \text{ units} \end{aligned}$$

$$\begin{aligned} BC &= \sqrt{(1-0)^2+(-4-4)^2} \\ &= \sqrt{1^2+(-8)^2} \\ &= \sqrt{65} \text{ units} \end{aligned}$$

The shorter sides are [AC] and [BC].

Now, $$\begin{aligned} AC^2 + BC^2 &= 17 + 65 \\ &= 82 \\ &\neq AB^2 \end{aligned}$$

Using the converse of Pythagoras' theorem, triangle ABC is *not* right angled.