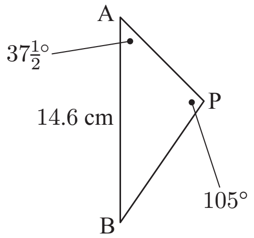


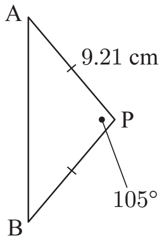
Using the sine rule,

$$\frac{BP}{\sin 37\frac{1}{2}°} \approx \frac{14.6}{\sin 105°}$$

$$\therefore \quad BP \approx \frac{14.6 \sin 37\frac{1}{2}°}{\sin 105°}$$

$$\therefore \quad BP \approx 9.21 \text{ cm}$$

d By the same reasoning as in **c**, $AP \approx 9.21$ cm



Area of $\triangle ABP$

$\approx \frac{1}{2} \times 9.21 \times 9.21 \times \sin 105°$

$\approx 40.9 \text{ cm}^2$

14 **a** There are 6 equal angles in the centre of the hexagon.

$$\therefore \quad A\widehat{O}B = \frac{360°}{6} = 60°$$

b $OA = OB$, so

$$O\widehat{A}B = O\widehat{B}A = \left(\frac{180 - 60}{2}\right)^{\circ} = 60°$$

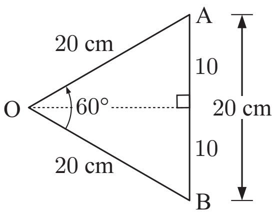
All the angles are $60°$, so $\triangle OAB$ is equilateral.

c The sign is made up of 6 equilateral triangles.

The area of each triangle

$= \frac{1}{2} \times 20 \times 20 \times \sin 60°$

$\approx 173.2 \text{ cm}^2$



$\therefore$ total area of figure $= 6 \times 173.2$

$= 1039.2$

$\approx 1040 \text{ cm}^2$

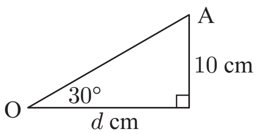
d **i** The height of the sign $y =$ length $OA \times 2$

$= 40$ cm

ii Now $\tan 30° = \dfrac{10}{d}$

$\therefore \quad d = \dfrac{10}{\tan 30°}$

≈ 17.32



The width of the sign $x \approx 2 \times 17.32 \approx 34.6$ cm

iii Area $=$ height $\times$ width

$\approx 40 \times 34.6$

$\approx 1385.6 \text{ cm}^2$

$\approx 1390 \text{ cm}^2$

iv Wasted area $=$ area of rectangle $-$ area of hexagon

$\approx 1385.6 - 1039.2$

$\approx 346 \text{ cm}^2$

$\therefore$ proportion wasted $\approx \dfrac{346}{1386} \times 100\%$

$= 25\%$

v The wasted area $\approx 346 \text{ cm}^2$

$\approx 0.0346 \text{ m}^2$

$\therefore$ the cost of the wasted material

$\approx €350 \times 0.0346$

$\approx €12.11$

SOLUTIONS TO TOPIC 6 (MATHEMATICAL MODELS)

SHORT QUESTIONS

1 **a** gradient $m = \dfrac{y\text{-step}}{x\text{-step}} = \dfrac{2-1}{2-0} = \dfrac{1}{2}$

y-intercept $= 1$

$\therefore \quad C(t) = \frac{1}{2}t + 1$

b $C(23) = \frac{1}{2}(23) + 1 = \$12.50$

$\therefore$ the call costs \$12.50.

c If $C(t) = \$18.31$

then $\frac{1}{2}t + 1 = 18.31$

$\therefore \quad \frac{1}{2}t = 17.31$

$\therefore \quad t = 34.62$

$\therefore$ the call lasts about 35 minutes.

2 **a** $h(t) = 1 - 2t^2$

$\therefore \quad h(0) = 1 - 2(0)^2$

$= 1$

b If $h(t) = 0$

then $1 - 2t^2 = 0$

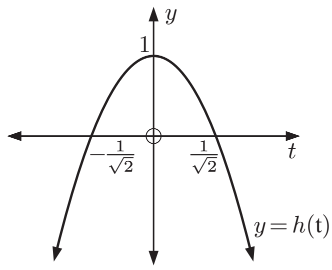
$\therefore \quad 2t^2 = 1$

$\therefore \quad t^2 = \frac{1}{2}$

$\therefore \quad t = \pm\frac{1}{\sqrt{2}}$

c Since $h(t)$ is a quadratic, its domain $= \{t \mid t \in \mathbb{R}\}$.

d



From the graph of $h(t)$, we observe the maximum is at $(0, 1)$.

$\therefore$ the range of $h(t)$ is $\{y \mid y \leqslant 1, \ y \in \mathbb{R}\}$.

3 **a** **i** The percentage of Carbon-14 remaining after 4 thousand years $\approx 61\%$.

ii It will take about 5600 years for the percentage of Carbon-14 to fall to 50%.

b When $t = 19$, $P = 100 \times (1.1318)^{-19}$

≈ 9.51

After 19 thousand years there will be $\approx 9.51\%$ remaining.

c The asymptote has equation $P = 0$.

4 **a** $f(2) = 15 - 2(2) = 11$ **b** $g(-2) = 2^{-2} + 1 = 1\frac{1}{4}$

c $g(x) = f(x)$ when $2^x + 1 = 15 - 2x$

Using technology, $x = 3$

5 **a** We see that for every increase of 5 bags of rice, the price decreases by 2000 rupiah.

$\therefore \quad P(b)$ is a linear function with gradient $\frac{-2000}{5} = -400$

$\therefore \quad P(b) = -400b + c$ for some constant c.

Now $P(50) = 30\,000$

$\therefore \quad -400 \times 50 + c = 30\,000$

$\therefore \quad c = 50\,000$

$\therefore \quad P(b) = -400b + 50\,000$

b $P(60) = -400 \times 60 + 50\,000 = 26\,000$

$\therefore$ the total cost $= 60 \times 26\,000 = 1\,560\,000$ rupiah

6 **a** The x-intercepts are at 2 and -1, so $y = a(x-2)(x+1)$

The graph passes through $(3, 12)$

$$\therefore\ a(1)(4) = 12$$
$$\therefore\ a = 3$$
$$\text{So,}\quad y = 3(x-2)(x+1)$$
$$= 3(x^2 - x - 2)$$
$$= 3x^2 - 3x - 6$$

b The axis of symmetry is $x = -\dfrac{b}{2a} = \frac{3}{6} = \frac{1}{2}$

Now when $x = \frac{1}{2}$, $y = 3(\frac{1}{2})^2 - 3(\frac{1}{2}) - 6$

$= \frac{3}{4} - \frac{3}{2} - 6 = -\frac{27}{4}$

$\therefore$ the vertex is at $(\frac{1}{2}, -\frac{27}{4})$.

7 **a** The graph passes through $(0, 20)$ and $(2, 35)$.

$$\therefore\ a + b = 20 \quad .... (1)$$
$$\text{and}\quad 4a + b = 35 \quad .... (2)$$

b Subtracting (1) from (2), $3a = 15$

$$\therefore\ a = 5$$
$$\text{Using (1),}\quad b = 15$$

c $y = 5 \times 2^x + 15$

When $x = 1$, $y = 5 \times 2 + 15 = 25$

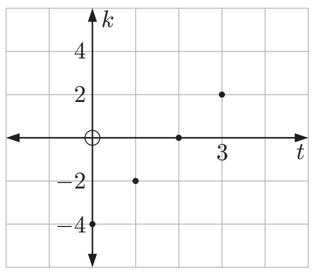
$$\therefore\ p = 25$$

When $x = 3$, $y = 5 \times 2^3 + 15 = 55$

$$\therefore\ q = 55$$

8 **a** Domain $= \{0, 1, 2, 3\}$ **b** Range $= \{-4, -2, 0, 2\}$

c



9 **a** $S_n = \dfrac{n}{2}(2a + (n-1)d)$

$$\therefore\ \frac{n}{2}(2 \times 7 + (n-1)(-5)) = -1001$$
$$\therefore\ n(14 - 5n + 5) = -2002$$
$$\therefore\ 5n^2 - 19n - 2002 = 0$$

b Using technology, $n = -\frac{91}{5}$ or 22

But $n \in \mathbb{Z}^+$, so $n = 22$

10 **a** $N(0) = 120 \times (1.04)^0 = 120$

$\therefore$ the settlement started with 120 people.

b $N(4) = 120(1.04)^4 \approx 140.4$

$\therefore$ there were 140 people after 4 years.

c If $N(t) = 240$

then $120 \times (1.04)^t = 240$

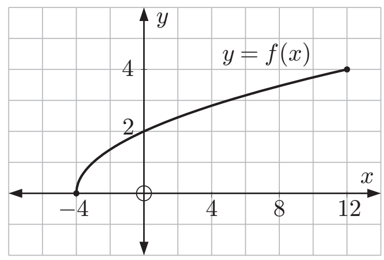
$$\therefore\ (1.04)^t = \tfrac{240}{120} = 2$$
$$\therefore\ t \approx 17.7$$

$\therefore$ it will take 18 years for the population to double.

11 **a** **i** $f(-4) = \sqrt{0} = 0$ **ii** $f(0) = \sqrt{4} = 2$

iii $f(12) = \sqrt{16} = 4$

b



c Range $= \{y \mid 0 \leqslant y \leqslant 4,\ y \in \mathbb{R}\}$

12 **a** When $d = 2$, $N = 810$

$$\therefore\ k(2)^{-3} = 810$$
$$\therefore\ \frac{k}{8} = 810$$
$$\therefore\ k = 6480$$

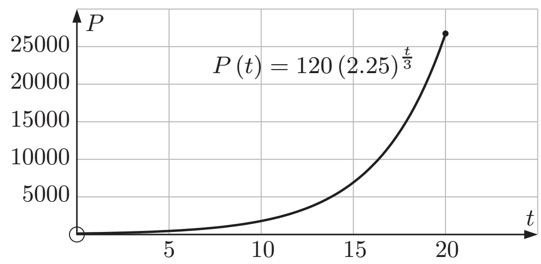
b $N = k(3)^{-3} = \frac{6480}{27} = 240$

So, 240 ball bearings can be made.

c If $N = 23$, $kd^{-3} = 23$

$$\therefore\ \frac{6480}{d^3} = 23$$
$$\therefore\ d^3 = \tfrac{6480}{23}$$
$$\therefore\ d \approx 6.56$$
$$\therefore\ r \approx \tfrac{6.56}{2} \approx 3.28 \text{ mm}$$

13 **a**



b $P(10) = 120(2.25)^{\frac{10}{3}} \approx 1790$ bees

c When $P = 5000$, $120(2.25)^{\frac{t}{3}} = 5000$

Using technology, $t \approx 13.8$

So, it will take about 13.8 weeks.

14 **a** The y-intercept $= 9$, so $c = 9$

b The axis of symmetry is $x = -\dfrac{b}{2a}$

$$\therefore\ -\frac{b}{2a} = 1$$
$$\therefore\ -b = 2a$$
$$\therefore\ 2a + b = 0 \quad .... (1)$$

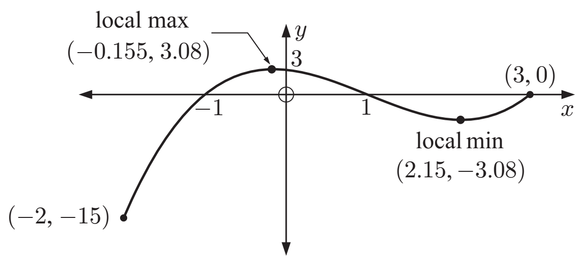
c The point $(1, 7)$ lies on the graph,

so $a(1)^2 + b(1) + 9 = 7$

$$\therefore\ a + b = -2 \quad .... (2)$$

d Solving (1) and (2) simultaneously, $a = 2$ and $b = -4$.

15 **a** Graphing $y = x^3 - 3x^2 - x + 3$ on $-2 \leqslant x \leqslant 3$:



b Range $= \{y \mid -15 \leqslant y \leqslant 3.08,\ y \in \mathbb{R}\}$

16 **a** $H(0) = 0$ m
The object begins at ground level.

b $H(t) = 19.6t - 4.9t^2$
$= 4.9t(4 - t)$
$\therefore$ $H(t) = 0$ when $t = 0$ and $t = 4$
The object returns to ground level after 4 seconds.

c Domain of $H(t) = \{t \mid 0 \leqslant t \leqslant 4, \ t \in \mathbb{R}\}$

d The maximum height occurs when
$$t = -\frac{b}{2a} = \frac{-19.6}{-9.8} = 2 \text{ seconds.}$$
$H(2) = 19.6$, so the maximum height is 19.6 m.

17 **a** $(x+7)(x-4) = x^2 + 7x - 4x - 28$
$= x^2 + 3x - 28$

b Using **a**, the zeros of $x^2 + 3x - 28$ are -7 and 4.
We hence find A$(-7, 0)$ and B$(4, 0)$.

c The axis of symmetry is midway between the x-intercepts.
$\therefore$ its equation is $x = \dfrac{-7+4}{2} = -\frac{3}{2}$.

d When $x = -\frac{3}{2}$, $y = \left(-\frac{3}{2}\right)^2 + 3\left(-\frac{3}{2}\right) - 28 = -30\frac{1}{4}$
So, C is $\left(-1\frac{1}{2}, -30\frac{1}{4}\right)$.

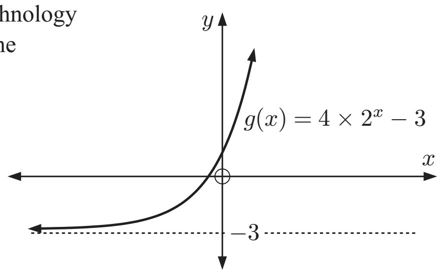
18 **a** $f(x) = 8x - 2x^2 = 2x(4 - x)$

b The x-intercepts are 0 and 4.

c The axis of symmetry is midway between the x-intercepts.
$\therefore$ its equation is $x = 2$.

d $f(2) = 8(2) - 2(2)^2 = 8$
$\therefore$ the vertex is (2, 8).

19 We use technology to sketch the function.



The range is $\{y \mid y > -3, \ y \in \mathbb{R}\}$.

20 **a** $N(0) = 30 - 3^3 = 3$
So, there were initially 3000 ants in the colony.

b $N(2) = 30 - 3^{3-2} = 27$
So, there were 27 000 ants after two months.

c $N(t) = 20$ when $30 - 3^{3-t} = 20$
$\therefore$ $3^{3-t} = 10$
$\therefore$ $t \approx 0.904$ months
So, the colony will reach a population of 20 000 in the first month.

d The horizontal asymptote is $N = 30$.

e The population should never reach the asymptote, which is at 30 000 ants.

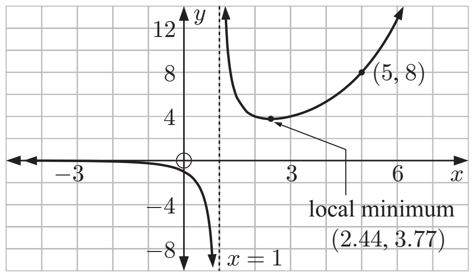
21 **a** $f(0) = \dfrac{2^0}{0-1} = -1$
$\therefore$ the y-intercept is -1

b Using technology, the minimum for $x > 1$ is at (2.44, 3.77). So, the minimum value is 3.77.

c The vertical asymptote is $x = 1$.

d $f(5) = \dfrac{2^5}{5-1} = \frac{32}{4} = 8$

e Graph of $y = \dfrac{2^x}{x-1}$

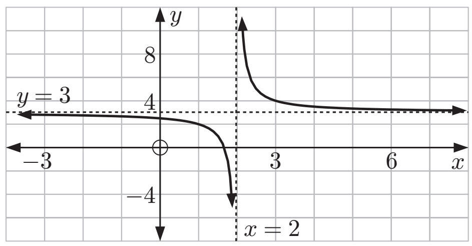


22 **a** The axis of symmetry has equation $x = -\dfrac{m}{2}$.
This axis passes through the vertex, so $-\dfrac{m}{2} = 1$
$\therefore$ $m = -2$
Since V(1, 3) lies on the quadratic, $1 - 2 + n = 3$
$\therefore$ $n = 4$

b $f(x) = x^2 - 2x + 4$
$\therefore$ $f(3) = 9 - 6 + 4 = 7$
$\therefore$ $k = 7$

c Domain $\{x \mid x \in \mathbb{R}\}$, Range $\{y \mid y \geqslant 3, \ y \in \mathbb{R}\}$

23 **a** Graph of $y = 3 + \dfrac{1}{x-2}$



b Vertical asymptote is $x = 2$
Horizontal asymptote is $y = 3$

24 **a** $T(t) = A \times B^{-t} + 3$
$T(0) = 27$
$\therefore$ $A \times B^0 + 3 = 27$
$\therefore$ $A = 24$

b $T(3) = 6$
$\therefore$ $24 \times B^{-3} + 3 = 6$
$\therefore$ $\dfrac{24}{B^3} = 3$
$\therefore$ $B^3 = 8$ and so $B = 2$

c $T(5) = 24 \times 2^{-5} + 3$
$\therefore$ $T(5) = 3.75\,^\circ$C

d $T(t) = 24 \times 2^{-t} + 3$ has an asymptote at $T = 3\,^\circ$C.
The temperature will approach 3°C, but will never reach it.

25

Graph	*Function*
a	**E**
b	**A**
c	**C**
d	**D**

26 **a** $f(x) = ax^2 + bx + 7$

$f(2) = 7$, then $4a + 2b + 7 = 7$

$\therefore\ 4a + 2b = 0$

$2a + b = 0$ .... (1)

$f(4) = 23$, then $16a + 4b + 7 = 23$

$\therefore\ 16a + 4b = 16$

$\therefore\ 4a + b = 4$ .... (2)

b Subtracting (1) from (2), $2a = 4$

$\therefore\ a = 2$

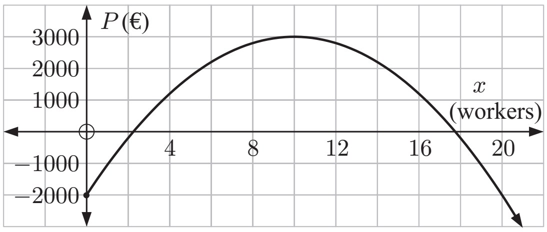
Using (1), $b = -4$.

c $f(x) = 2x^2 - 4x + 7$

$\therefore\ f(-1) = 2 + 4 + 7$

$= 13$

27 **a** Graph of $P(x) = -50x^2 + 1000x - 2000$



b 10 workers maximise the profit.

c The maximum profit is €3000.

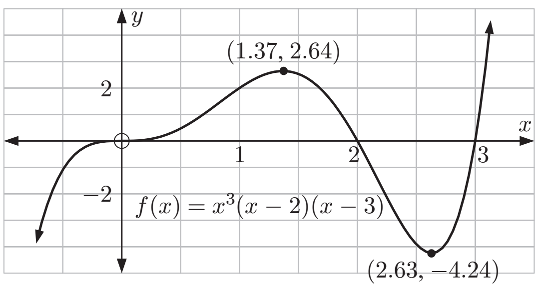
28

Function	*Graph*
a	**E**
b	**D**
c	**B**

29 **a** **i** The x-intercepts are 0, 2, and 3.

ii local maximum (1.37, 2.64), local minimum (2.63, −4.24)

b



30 **a** Since $y = a(x-1)(x-5)^2$, the graph cuts the x-axis at $x = 1$, and touches it at $x = 5$.

$\therefore\ b = 1$ and $c = 5$

b $y = a(x-1)(x-5)^2$

When $x = 0$, $y = -50$

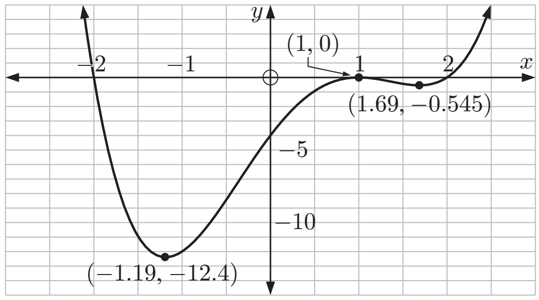
$\therefore\ a(-1)(-5)^2 = -50$

$\therefore\ -25a = -50$

$\therefore\ a = 2$

c Using technology, the local maximum P is $(2.33, 19.0)$.

31 **a** Graph of $y = x^4 - 2x^3 - 3x^2 + 8x - 4$



b Local minima are $(1.69, -0.545)$ and $(-1.19, -12.4)$. Local maximum is $(1, 0)$.

c $x^4 - 2x^3 - 3x^2 + 8x - 4 = 0$ when $x = 1$ or $x = \pm 2$

32 **a** $f(2) = 3 - 4^{-2} \approx 2.94$

$\therefore\ p \approx 2.94$

$f(-2) = 3 - 4^2 = -13$

$\therefore\ q = -13$

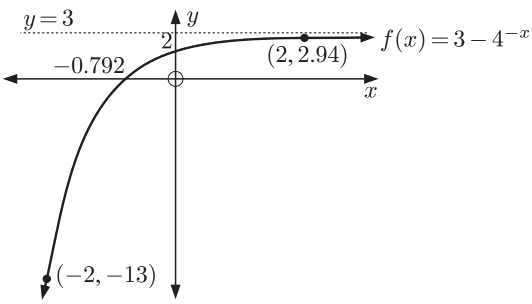
b **i** Using technology, the x-intercept ≈ -0.792.

$f(0) = 3 - 4^0 = 2$ $\therefore$ the y-intercept is 2.

ii As $x \to \infty$, $4^{-x} \to 0$ and so $y \to 3$

$\therefore$ $y = 3$ is the horizontal asymptote.

c



d Range $= \{y \mid y < 3,\ y \in \mathbb{R}\}$

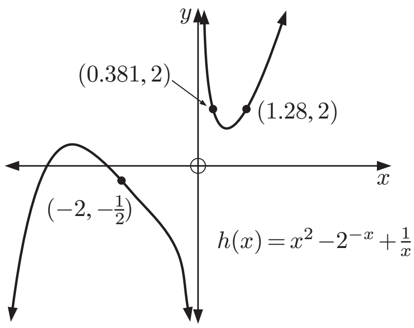
33 **a** $h(-2) = (-2)^2 - 2^{-(-2)} + \dfrac{1}{-2}$

$= 4 - 4 - \frac{1}{2} = -\frac{1}{2}$

b Using technology, $h(x) = 2$ when $x \approx 0.381$ or 1.28.

c The vertical asymptote is $x = 0$.

d



e Using technology, the range is $\{y \mid y < 0.741$ or $y > 1.30,\ y \in \mathbb{R}\}$.

LONG QUESTIONS

1 **a** **i** $C(20) = 20^2 + 400 = 800$

The weekly cost for producing 20 DVD players is \$800.

ii $I(20) = 50(20) = 1000$

The weekly income when 20 DVD players are sold is \$1000.

iii $P(20) = I(20) - C(20)$
$= 1000 - 800$
$= 200$

So, a profit of \$200 is made.

b $P(x) = I(x) - C(x)$
$\therefore \ P(x) = 50x - (x^2 + 400)$ dollars

c $P(x) = -x^2 + 50x - 400$

The vertex occurs when $x = -\dfrac{b}{2a} = \dfrac{-50}{2(-1)} = 25$

Hence, 25 DVD players must be made and sold to maximise the profit.

d $P(25) = -(25)^2 + 50(25) - 400 = \$225$

Profit **per** DVD player $= \dfrac{\$225}{25} = \$9$

e The function $P(x)$ has zeros at $x = 10$ and $x = 40$. In order to make positive (non-zero) profit, at most 39 DVD players can be made.

2 **a** x-intercepts occur when $y = 0$

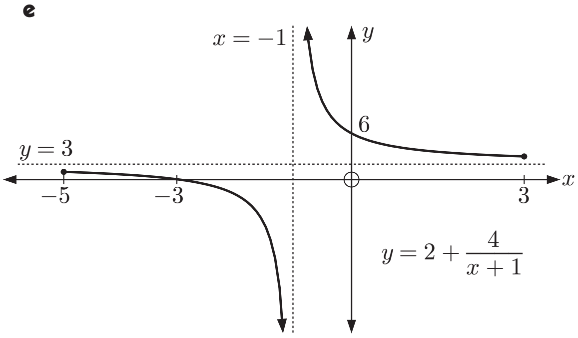
$\therefore \ 2 + \dfrac{4}{x+1} = 0$
$\therefore \ \dfrac{4}{x+1} = -2$
$\therefore \ 4 = -2(x+1)$
$\therefore \ 2x = -6$
$\therefore \ x = -3$

b y-intercept occurs when $x = 0$
$\therefore \ y = 2 + \frac{4}{1} = 6$

c $f(-2) = 2 + \dfrac{4}{-2+1} = -2$

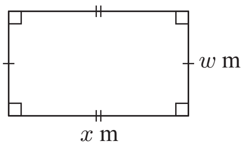
d **i** $y = 2$ **ii** $x = -1$

e



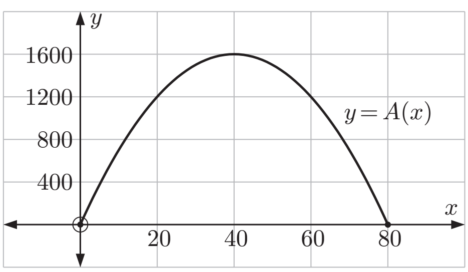
3 **a** $2x + 2w = 160$
$\therefore \ 2w = 160 - 2x$
$\therefore \ w = 80 - x$

The width is $(80 - x)$ m.



b Area $A(x) = x(80 - x)$ m^2

c



d The maximum area occurs when $x = 40$, which is when the field is a 40 m $\times$ 40 m square.

e **i** $A(x) = 1200$
$\therefore \ x(80 - x) = 1200$
$\therefore \ x^2 - 80x + 1200 = 0$
$\therefore \ (x - 60)(x - 20) = 0$
$\therefore \ x = 60$ or 20
$\therefore$ the field is 60 m $\times$ 20 m.

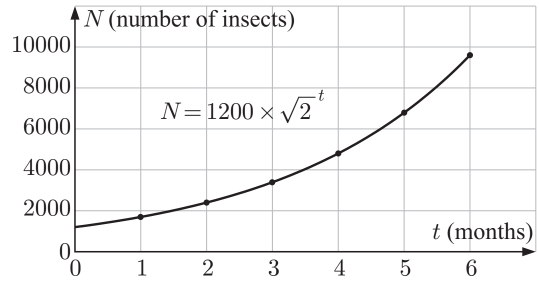
ii We lose $1600 - 1200 = 400$ m^2 of productive land
$\therefore$ the lost production $= 400 \times 6.5$
$= 2600$ kg

4 **a** $N = 1200 \times k^t$
$\therefore \ 1200 \times k^4 = 4800$
$\therefore \ k^4 = 4$
$\therefore \ k = \sqrt{2} \approx 1.41$

b

t	0	1	2	3	4	5	6
N	1200	1700	2400	3390	4800	6790	9600

c



d After $2\frac{1}{2}$ months there are about 2900 insects.

e $N = 20\,000$ when $1200 \times \left(\sqrt{2}\right)^t = 20\,000$
$\therefore \ \left(\sqrt{2}\right)^t = \dfrac{20\,000}{1200} = \frac{50}{3}$

Using technology, $t \approx 8.12$

So, it takes about 8.12 months for the population to reach 20 000 insects.

f The percentage change $= \dfrac{N(6) - N(5)}{N(5)} \times 100\%$
$= \dfrac{9600 - 6790}{6790} \times 100\%$
$\approx 41.4\%$

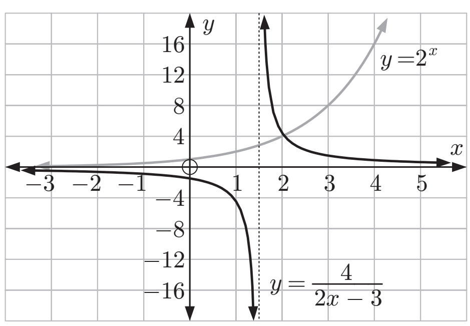
5 **a** **i** $x = \frac{3}{2}$ **ii** $y = 0$

b **i** When $x = 0$, $y = a^0 = 1$
$\therefore$ the y-intercept is 1.

ii The horizontal asymptote is $y = 0$.

c At $x = 2$, $a^2 = \dfrac{4}{2(2) - 3}$
$\therefore \ a^2 = 4$
$\therefore \ a = \pm 2$
$\therefore \ a = 2$ {as $a > 0$}

d Graphs of $y = \dfrac{4}{2x - 3}$ and $y = 2^x$:



6 **a** $f(x)$ is undefined when $x = 0$.

b Horizontal asymptote $y = -1$, vertical asymptote $x = 0$.

c $f(1) = \frac{5}{1} - 1 = 4 \quad \therefore \quad m = 4$

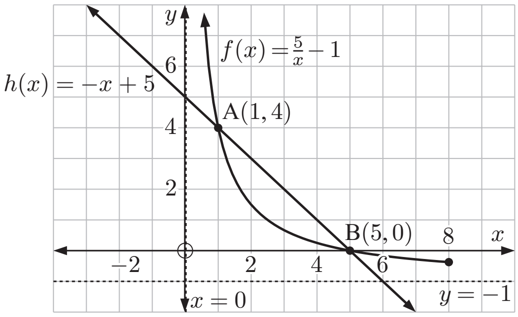
When $f(x) = 0$, $\frac{5}{x} - 1 = 0$

$\therefore \quad \frac{5}{x} = 1$

$\therefore \quad x = 5$

So, $n = 5$

d, f ii



e **i** $\frac{5}{x} - 1 = 5 - x$

$\therefore \quad x - 6 + \frac{5}{x} = 0$

$\therefore \quad x^2 - 6x + 5 = 0$

$\therefore \quad (x-1)(x-5) = 0$

ii Using the Null factor law, $x = 1$ or 5.

f **i** $y = h(x)$ passes through $(1, 4)$ and $(5, 0)$.

Its gradient $= \frac{0-4}{5-1} = -1$, so $c = -1$

$\therefore \quad h(x) = -x + d$

Now $h(1) = -1 + d = 4$

$\therefore \quad d = 5$

$\therefore \quad h(x) = -x + 5$

g From the graph, $h(x) \geqslant f(x)$ for positive x when $1 \leqslant x \leqslant 5$.

h The graph passes through the origin, so the y-intercept $r = 0$.

The graph passes through (1, 4), so $p + q = 4$ .... (1)

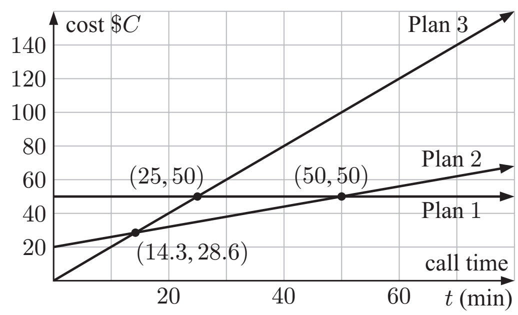
The graph passes through (5, 0), so

$25p + 5q = 0$ .... (2)

Solving (1) and (2) simultaneously, $p = -1$ and $q = 5$

So, $g(x) = -x^2 + 5x$.

7 **a, d**



b **i** *Plan 1*: \$50

Plan 2: $\$20 + 30 \times \$0.60 = \$38$

Plan 3: $30 \times \$2 = \$60$

ii *Plan 1*: \$50

Plan 2: $\$20 + 60 \times \$0.60 = \$56$

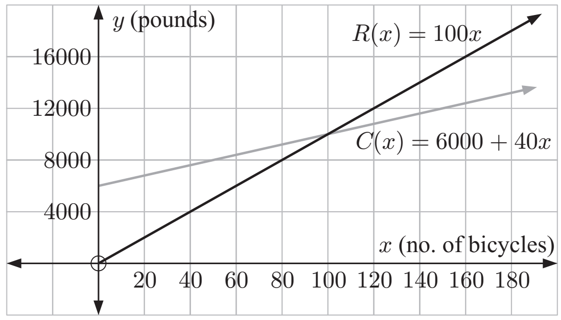
Plan 3: $60 \times \$2 = \$120$

c **i** $C = 50$ **ii** $C = 0.6t + 20$ **iii** $C = 2t$

e **i** more than 50 **ii** between 14.3 and 50

iii less than 14.3

8 **a**



b The initial setup cost $C(0) = \$6000$.

c Revenue = cost when $6000 + 40x = 100x$

$\therefore \quad 6000 = 60x$

$\therefore \quad x = 100$

The company breaks even when 100 bicycles are made.

d Total revenue from selling x bicycles $= \$100x$

$\therefore$ the revenue per bicycle $= \$100$

e $P(x) = R(x) - C(x)$

$= 100x - (6000 + 40x)$

$= 60x - 6000$ dollars

f The profit from the sale of 400 bicycles is

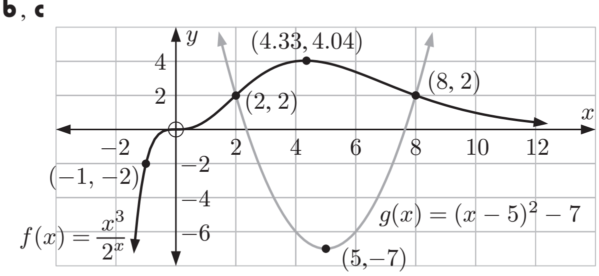
$P(400) = 60(400) - 6000 = \$18\,000$

9 **a** **i** y-intercept $= f(0) = 0$

ii The maximum is at (4.33, 4.04), so the maximum value of $f(x)$ is 4.04.

iii $f(2) = 2, \quad f(-1) = -2$

b, c



d If $\frac{x^3}{2^x} + 7 = (x-5)^2$

then $\frac{x^3}{2^x} = (x-5)^2 - 7$

$\therefore \quad f(x) = g(x)$

The graphs intersect at (2, 2) and (8, 2), so the solutions are $x = 2$ or 8.

10 **a** $V = l \times w \times h$

Given that $v = 2000\text{ cm}^3$ and $l = 2w$

$\therefore \quad 2000 = 2w \times w \times h$

$\therefore \quad \frac{2000}{2w^2} = h$

$\therefore \quad h = \frac{1000}{w^2}$

b $A = 2lw + 2lh + 2wh$

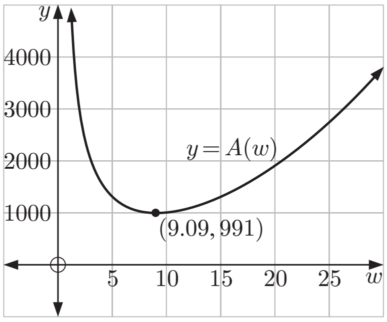
$= 2 \times 2w \times w + 2 \times 2w \times \frac{1000}{w^2} + 2w \times \frac{1000}{w^2}$

$= 4w^2 + \frac{4000}{w} + \frac{2000}{w}$

$= 4w^2 + \frac{6000}{w}\text{ cm}^2$

c w is a length, so it must be non-negative.
Hence, the domain is $D = \{w \mid w > 0,\ w \in \mathbb{R}\}$.

d



e **i** The minimum surface area is about 991 cm^2.

ii This occurs when $w \approx 9.09$ cm
$\therefore\ l \approx 18.2$ cm and $h \approx 12.1$ cm
$\therefore$ the dimensions
$l \times w \times h \approx 18.2 \text{ cm} \times 9.09 \text{ cm} \times 12.1 \text{ cm}$.

f The range is $\{A \mid A > 991,\ A \in \mathbb{R}\}$.

11 **a**

Graph	*Domain*
A	$\{x \mid x \geqslant -2,\ x \in \mathbb{R}\}$
B	$\{x \mid x \in \mathbb{R}\}$
C	$\{x \mid x \geqslant -4,\ x \in \mathbb{R}\}$
D	$\{x \mid x \in \mathbb{R},\ x \neq 0\}$

The domains of all of the graphs are different.

b

Graph	*Range*
A	$\{y \mid y \leqslant 5,\ y \in \mathbb{R}\}$
B	$\{y \mid y > -2,\ y \in \mathbb{R}\}$
C	$\{y \mid y \geqslant 0,\ y \in \mathbb{R}\}$
D	$\{y \mid y \neq 0,\ y \in \mathbb{R}\}$

c

Graph	*Equation*
A	$y = a(x-h)^2 + k$
B	$y = mx + c$
C	$y = p \times q^x + r$
D	$y = \dfrac{v}{x}$

d Graph **A** is a quadratic with vertex $(1, 5)$.
$\therefore\ h = 1$ and $k = 5$
When $x = 4,\ y = 0$
$$\therefore\ a(4-1)^2 + 5 = 0$$
$$\therefore\ 9a = -5$$
$$\therefore\ a = -\tfrac{5}{9}$$

Graph **B** is an exponential function with horizontal asymptote $y = -2$
$$\therefore\ r = -2$$
When $x = 0,\ y = 1$
$$\therefore\ p \times q^0 - 2 = 1$$
$$\therefore\ p = 3$$
When $x = 1,\ y = 4$
$$\therefore\ 3 \times q^1 - 2 = 4$$
$$\therefore\ 3q = 6$$
$$\therefore\ q = 2$$

Graph **C** is a linear function with gradient $\frac{2}{4} = \frac{1}{2}$ and y-intercept 2.
$\therefore\ m = \frac{1}{2}$ and $c = 2$

Graph **D** is a rectangular hyperbola passing through $(2, -2)$.
$$\therefore\ -2 = \frac{v}{2}$$
$$\therefore\ v = -4$$

SOLUTIONS TO TOPIC 7 (CALCULUS)

SHORT QUESTIONS

1 **a**
$$y = x^3 - 4.5x^2 - 6x + 13$$
$$\therefore\ \frac{dy}{dx} = 3x^2 - 2 \times 4.5x - 6$$
$$= 3x^2 - 9x - 6$$

b When the gradient of the tangent is 6,
$$\frac{dy}{dx} = 6$$
$$\therefore\ 3x^2 - 9x - 6 = 6$$
$$\therefore\ 3x^2 - 9x - 12 = 0$$
$$\therefore\ 3(x^2 - 3x - 4) = 0$$
$$\therefore\ 3(x-4)(x+1) = 0$$
$$\therefore\ x = 4 \text{ or } x = -1$$
So, the x-coordinates of the points are 4 and -1.

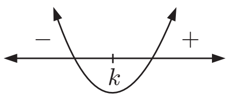
2 **a** $y = ax^2 + bx + c \quad \therefore\ \dfrac{dy}{dx} = 2ax + b$

b When $x = k,\ \dfrac{dy}{dx} = 0$
$$\therefore\ 2ak + b = 0$$
$$\therefore\ 2ak = -b$$
$$\therefore\ k = -\frac{b}{2a}$$

c **i** If $\dfrac{dy}{dx} < 0$, y is decreasing.

ii When $x < k,\ \dfrac{dy}{dx} < 0$, and when $x > k,\ \dfrac{dy}{dx} > 0$.

$\therefore\ \dfrac{dy}{dx}$ has sign diagram:



$\therefore\ x = k$ is a local minimum.

3 **a** $y = \dfrac{7}{x^3} = 7x^{-3}$

b
$$\frac{dy}{dx} = 7(-3)x^{-4}$$
$$= \frac{-21}{x^4}$$

4 **a**
$$y = \frac{2x^4 - 4x^2 - 3}{x}$$
$$= \frac{2x^4}{x} - \frac{4x^2}{x} - \frac{3}{x}$$
$$= 2x^3 - 4x - 3x^{-1}$$

b
$$\frac{dy}{dx} = 2(3)x^2 - 4 - 3(-1)x^{-2}$$
$$= 6x^2 - 4 + \frac{3}{x^2}$$

c At $x = -1$,
$$\frac{dy}{dx} = 6(-1)^2 - 4 + \frac{3}{(-1)^2}$$
$$= 6 - 4 + 3$$
$$= 5$$
So, at $x = -1$, the gradient is 5.