CHAPTER 19: EXPONENTIAL FUNCTIONS

19A EXPONENTIAL FUNCTIONS

REMINDER

An **exponential function** is a function in which the variable occurs as part of the index or exponent.

1 Determine whether the following are exponential functions:

a $f(x) = 3^x$ **b** $f(x) = x^3$

c $f(x) = 2 - 2^x$ **d** $f(x) = 7 + 4^{x-1}$

2 For the function $f(x) = 3^x$, find:

a $f(2)$ **b** $f(-1)$

3 For the function $g(x) = 2 - 2^{-x}$, find:

a $g(0)$ **b** $g(-x)$

4 Suppose $f(x) = 2 - 5^{-x-1}$ and $g(x) = 5^{x+2} - 4$.

a Find $f(0)$ and $g(0)$.

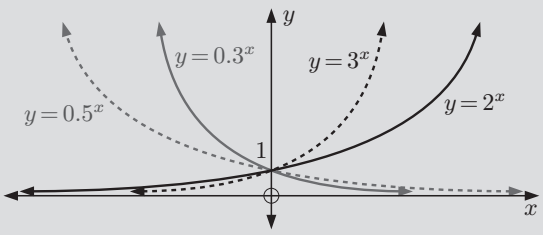
b Show that $f(-1) = g(-1)$.

19B GRAPHS OF EXPONENTIAL FUNCTIONS

REMINDER

For all exponential functions of the form $f(x) = a^x$, $a > 0$, $a \neq 1$:

- The y-intercept is 1, since $f(0) = a^0 = 1$.
- The graph has the horizontal asymptote $y = 0$.
- If $a > 1$, the graph is **increasing**.
  If $0 < a < 1$, the graph is **decreasing**.



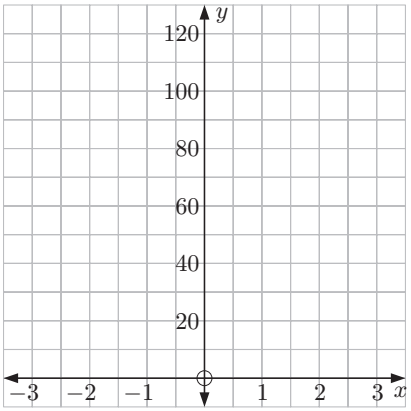
- The graph of $f(x) = a^x$ becomes steeper as a moves further away from 1.

1 Consider the exponential functions $f(x) = 5^x$ and $g(x) = \left(\frac{1}{4}\right)^x$.

a Fill the values in the table below.

x	-3	-2	-1	0	1	2	3
$f(x) = 5^x$	$\frac{1}{125}$			1	5		125
$g(x) = \left(\frac{1}{4}\right)^x$		16					$\frac{1}{64}$

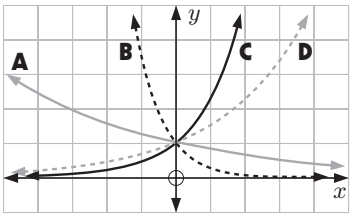
b Sketch the functions on the grid below.



2 Match each function with the correct graph:

a $y = 2.3^x$ **b** $y = \left(\frac{3}{2}\right)^x$

c $y = \left(\frac{2}{7}\right)^x$ **d** $y = 0.8^x$



REMINDER

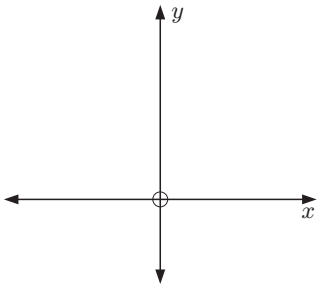
- $y = a^x + b$ translates $y = a^x$ upwards by b units.
- $y = a^x - b$ translates $y = a^x$ downwards by b units.
- $y = a^{x+b}$ translates $y = a^x$ to the left by b units.
- $y = a^{x-b}$ translates $y = a^x$ to the right by b units.

3 For each of the following functions:

i sketch the graph on the axes provided
ii state the equation of the horizontal asymptote
iii find the range
iv find the y-intercept.

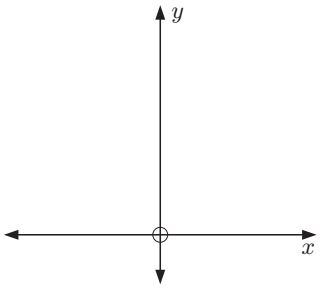
a $y = 3^x - 5$

i **ii** **iii** **iv**



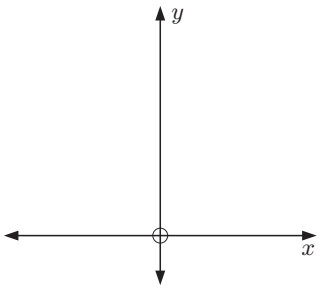
b $y = \left(\frac{1}{4}\right)^x + 3$

i **ii** **iii** **iv**



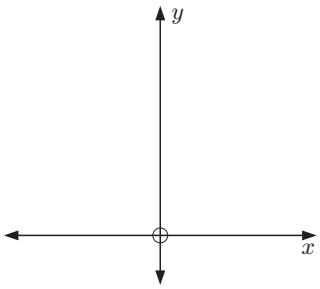
c $y = 2^{x+3}$

i **ii** **iii** **iv**



d $y = \left(\frac{1}{2}\right)^{x-4}$

i **ii** **iii** **iv**

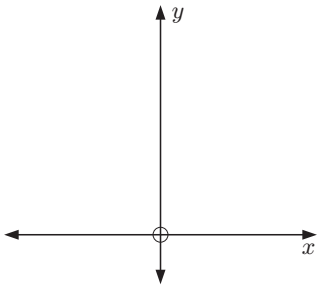


REMINDER

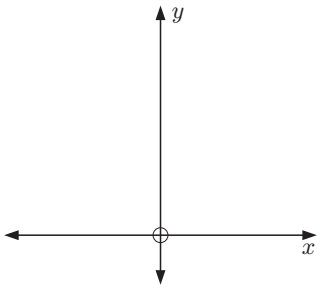
- $y = b \times a^x$ is found by vertically stretching $y = a^x$ with scale factor b.
- $y = a^{\frac{x}{b}}$ is found by horizontally stretching $y = a^x$ with scale factor b.

4 Sketch the graph of:

a $y = \frac{1}{2} \times 4^x$



b $y = 2^{\frac{x}{3}}$

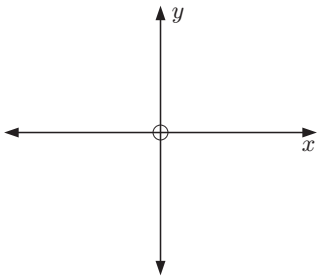


REMINDER

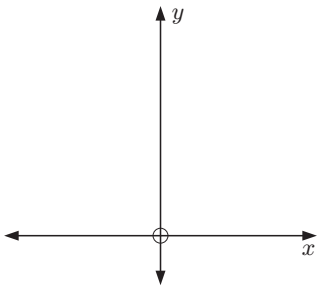
- $y = -a^x$ is found by reflecting $y = a^x$ in the x-axis.
- $y = a^{-x}$ is found by reflecting $y = a^x$ in the y-axis.

5 Sketch the graph of:

a $y = -\left(\frac{3}{2}\right)^x$



b $y = 7^{-x}$



19C GROWTH AND DECAY

REMINDER

An exponential function of the form $f(x) = k \times a^n$ exhibits:

- **growth** if $a > 1$
- **decay** if $0 < a < 1$.

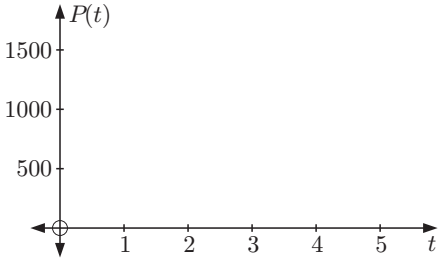
1 The population of pigeons in a town is growing out of control. The number of pigeons after t years is given by $P(t) = 200 \times 1.47^t$.

a Find the initial number of pigeons in the town.

b Find the number of pigeons after:

i 1 year **ii** 5 years.

c Use **a** and **b** to sketch the graph of $P(t)$ against t on the axes below.



d Find the percentage increase in the number of pigeons during the first 3 years.

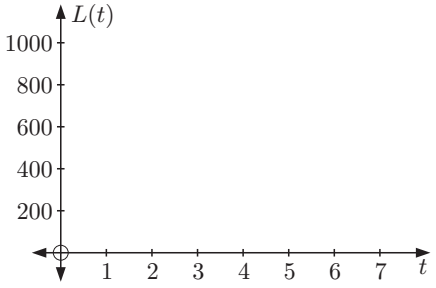
2 The number of leaves on a certain tree t years after it was bought is modelled by $L(t) = L_0 \times 1.9^t$.

a When the tree was bought it had 10 leaves. Find L_0, the initial number of leaves on the tree when it was bought.

b Predict the number of leaves on the tree after:

i 3 years **ii** 7 years.

c Sketch $L(t)$ against t using **a** and **b**.



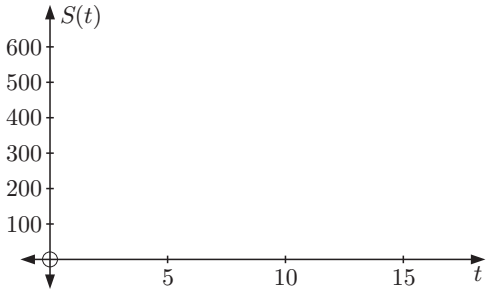
3 The value of a small family business after t years is given by $S(t) = 250 \times 1.05^t$ thousand dollars.

a Find the initial value of the family business.

b Find the value after:

i 5 years **ii** 15 years.

c Use **a** and **b** to sketch the graph of $S(t)$ against t.



4 The number of bubbles in a soft drink t minutes after it is poured, can be modelled by $B = 1000 \times 0.8^t$.

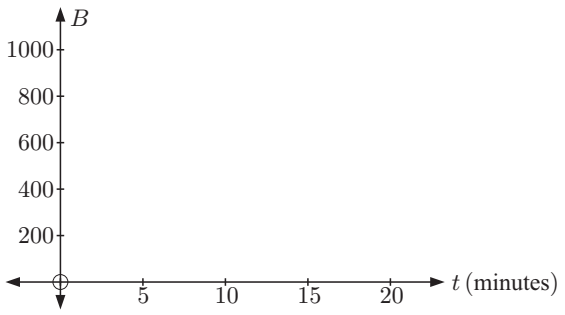
a Find the initial number of bubbles in the soft drink.

b Find the number of bubbles in the drink after:

i 2 minutes **ii** 5 minutes

iii 10 minutes **iv** 20 minutes.

c Graph B against t using **a** and **b**.



5 A wombat population is in danger due to damage to its habitat. The current population of 200 is expected to fall by 5% each year.

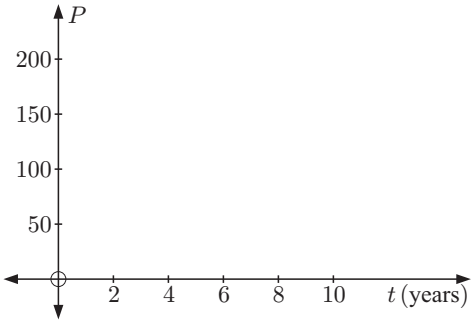
a Write an exponential function for the expected wombat population P after t years.

b Find the expected wombat population after:

i 1 year **ii** 3 years

iii 5 years **iv** 10 years.

c Sketch the graph of P against t.



REVIEW OF CHAPTER 19

1 Determine whether the following are exponential functions:

a $f(x) = 7^x + 9$ **b** $f(x) = 6x + x^5$

2 For the function $f(x) = 5^{x-2} + 3$, find:

a $f(2)$ **b** $f(1)$

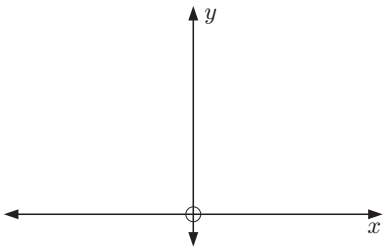
3 Suppose $f(x) = 8 - 4^x$ and $g(x) = 3 + 2^{x+2}$.

a Find $f(2)$ and $g(2)$.

b Show that $f(1) = g(-2)$.

c Show that $f(x) = g(x)$ when $x = 0$.

4 Use transformations to sketch the graph of $y = 2^{x-2}$.



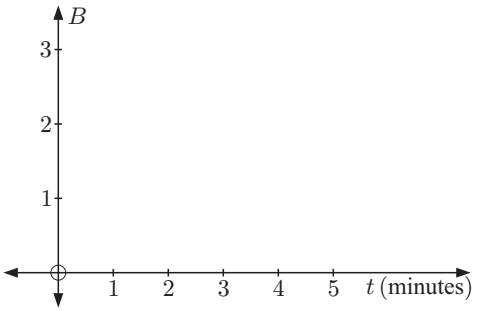
5 The weight of bacteria in a pot of boiling water is given by $B = 3 \times 0.7^t$ grams, where t is in minutes.

a Find the initial weight of the bacteria.

b Find the weight after:

i 1 minute **ii** 5 minutes.

c Use **a** and **b** to graph B against t.



6 The area of land affected by a fire is given by $F(t) = 10 \times 1.8^t$ m^2, where t is in minutes.

a Find the area of land affected:

i initially **ii** after 2 minutes

iii after 6 minutes **iv** after 10 minutes.

b Use **a** to sketch the graph of $F(t)$ against t.