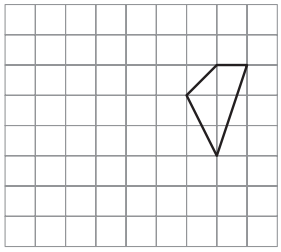
CHAPTER 17: CONGRUENCE AND TRANSFORMATIONS

17A TRANSFORMATIONS

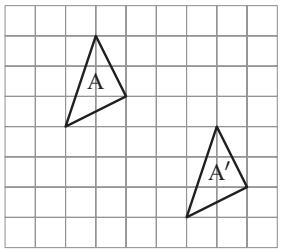
REMINDER

Under a translation, every point on the figure moves a fixed distance in a given direction.

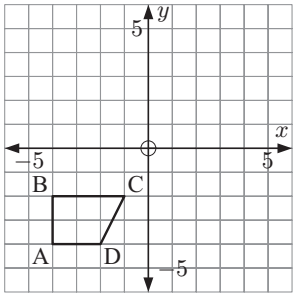
1 Translate the given figure 5 units left and 2 units down.



2 Determine the translation from A to A′ in the following:

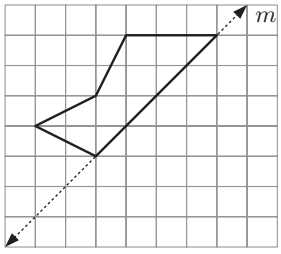


3 **a** Translate ABCD 6 units right and 4 units up.



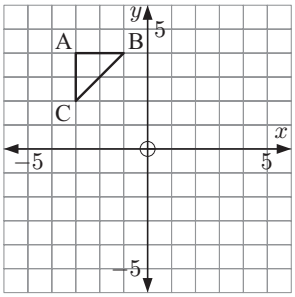
b State the vertex coordinates of the image.

4 Reflect the following figure in the mirror line given:



5 **a** Reflect ABC in the:

i x-axis **ii** y-axis.



b For each reflection in **a**, state the vertex coordinates of the image.

i

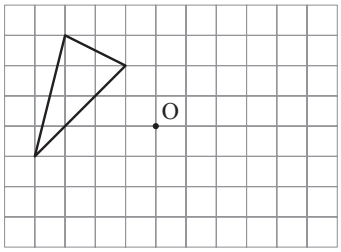
ii

REMINDER

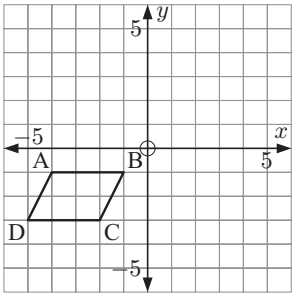
A **rotation** is a transformation in which every point on the figure is turned through a given angle about a fixed point.

The fixed point is called the **centre of rotation** and is usually labelled O.

6 Rotate the given figure about O through 180°.



7



a State the vertex coordinates of ABCD.

b Rotate the figure 270° clockwise about O.

c State the vertex coordinates of the image.

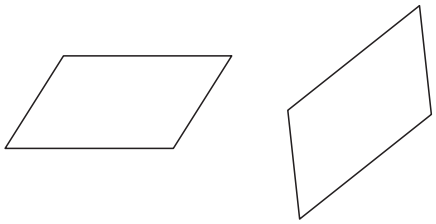
17B CONGRUENT FIGURES

REMINDER

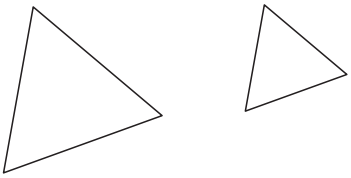
Two figures are **congruent** if one figure lies exactly on top of the other after a combination of translations, rotations, and reflections.

1 Are the following pairs of figures congruent?

a

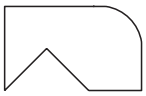


b

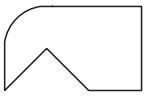


2 Which two of these figures are congruent?

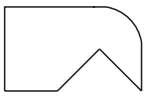
A



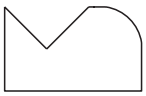
B



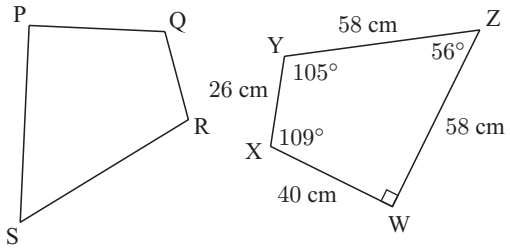
C



D



3 Quadrilaterals PQRS and WXYZ are congruent.



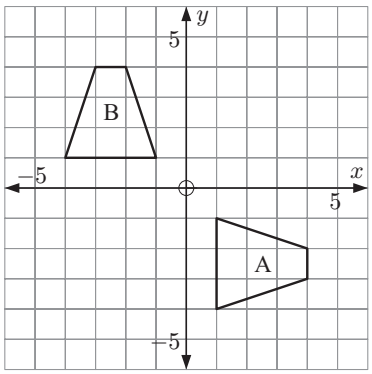
Determine the:

a length of [QR]

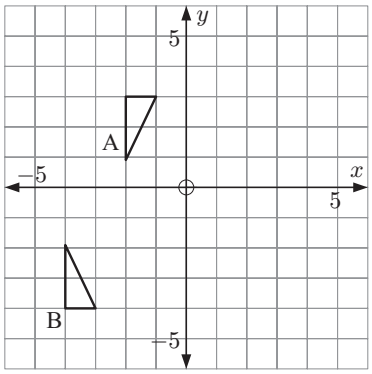
b size of $\widehat{QPS}$

c perimeter of PQRS.

4 Show that A is congruent to B by transforming A to B in a single transformation.

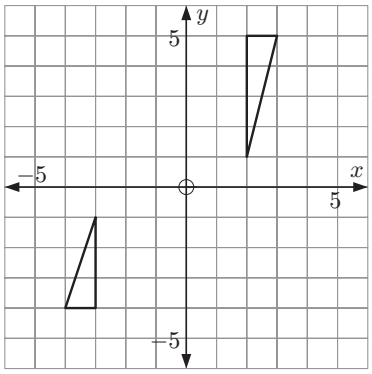


5 Show that A is congruent to B by transforming A to B in a combination of transformations.

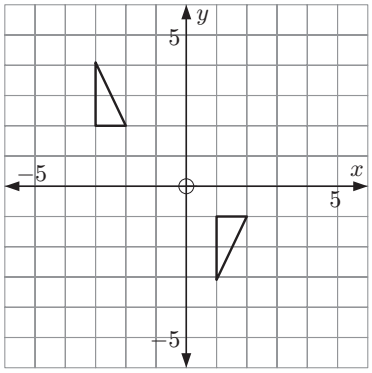


6 Are the following pairs of figures congruent?

a



b

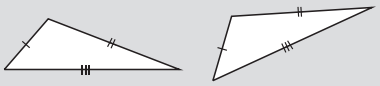


17C CONGRUENT TRIANGLES

REMINDER

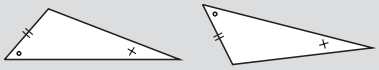
Two triangles are **congruent** if any one of the following is true:

- All corresponding sides are equal in length. **(SSS)**



- Two sides and the **included angle** are equal. **(SAS)**

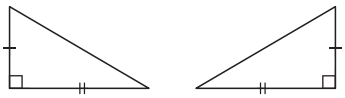
- Two angles and a pair of **corresponding sides** are equal. **(AAcorS)**



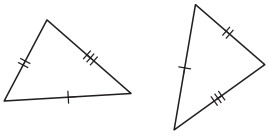
- For right angled triangles, the hypotenuses and one pair of sides are equal. **(RHS)**

1 State whether these pairs of triangles are congruent, giving reasons for your answers:

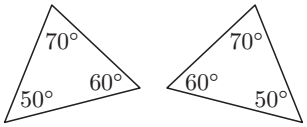
a



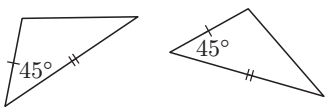
b



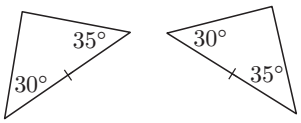
c



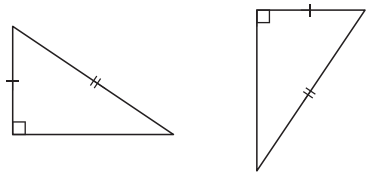
d



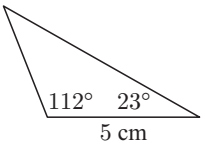
e



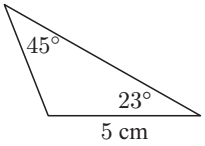
f



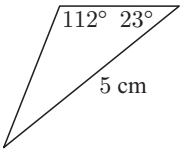
2 Which of the following triangles is congruent to the one alongside?



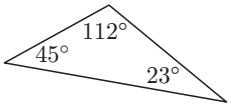
A



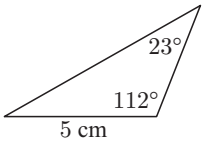
B



C

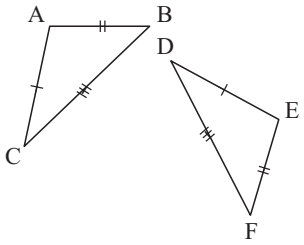


D

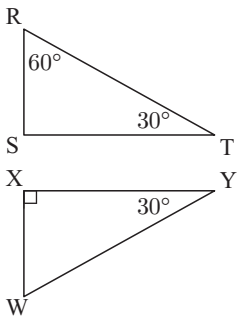


3 The following pairs of triangles are not drawn to scale. Determine whether each pair is congruent. If it is, what else can we deduce about them?

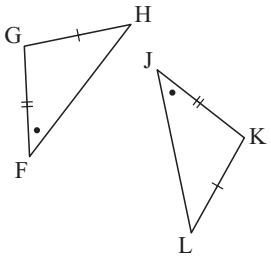
a



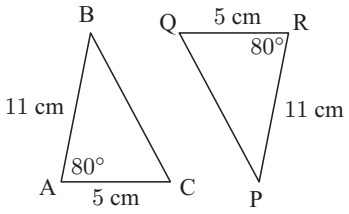
b



c

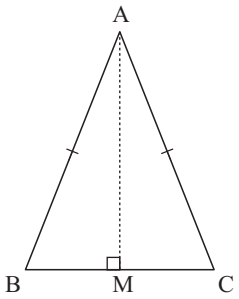


d



17D PROOF USING CONGRUENCE

1 Consider the isosceles triangle shown alongside.



a Complete the following:
In triangles ABM and ACM:

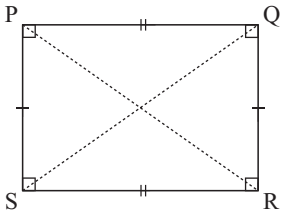
- AB = ... {given}
- $A\widehat{M}B$ = ... = ... °
  {.....}
- [AM] is ..... to both triangles.

$\therefore$ $\triangle ABM \cong \triangle ACM$ {.....}

Equating corresponding angles, $B\widehat{A}M$ = ...

b What property of isosceles triangles has been proven in **a**?

2 Consider the rectangle PQRS.

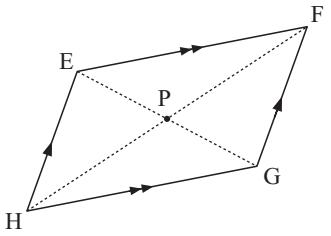


a Show that $\triangle PQR \cong \triangle SRQ$.

b Explain why PR = SQ.

c What property of rectangles has been proven?

3 The diagonals of parallelogram EFGH meet at P, as shown.



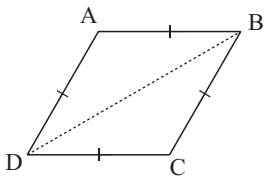
a Show that $\triangle EPF \cong \triangle GPH$.

b Explain why EP = GP.

c Explain why HP = FP.

d What property of parallelogram has been proven in **c** and **d**?

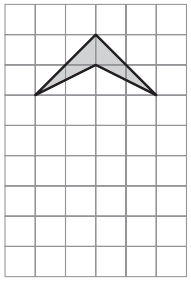
4 Use congruence to show that the opposite angles of a rhombus are equal in size.



REVIEW OF CHAPTER 17

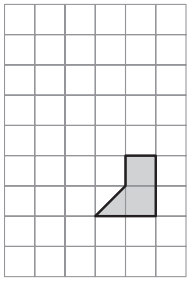
1 Translate the given figures in the direction indicated.

a



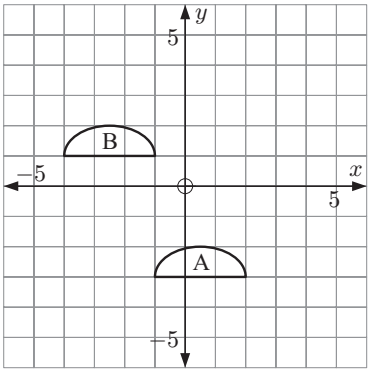
5 units down

b

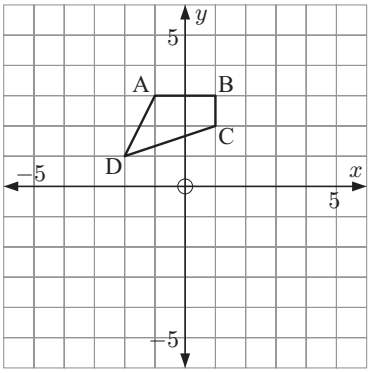


1 unit left and
4 units up

2 State the translation that maps A onto B.

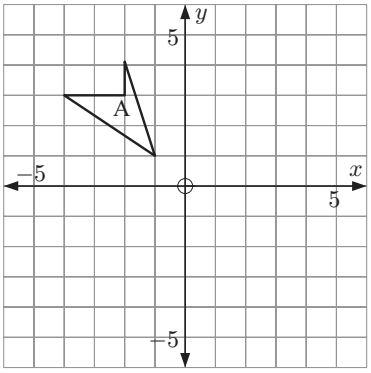


3 **a** Reflect ABCD in the x-axis.

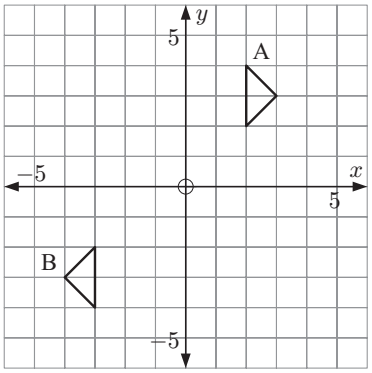


b State the coordinates of the image vertices.

4 Rotate A 90° anticlockwise about O.

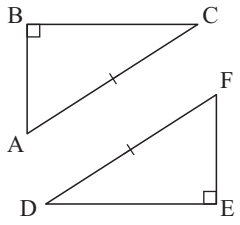


5 Show that A is congruent to B by transforming A to B in a series of transformations:

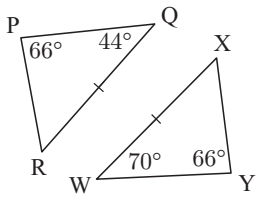


6 The following pairs of triangles are not drawn to scale. Determine whether each pair is congruent. If it is, what else can we deduce about them?

a



b



7 Use congruence to show that the diagonals of a rectangle bisect each other.