Chapter 8
LOGIC

EXERCISE 8A.1

1 A proposition is a statement which may be true or false. A proposition is indeterminate if it does not have the same answer for all people.

a proposition, false **b** proposition, false **c** proposition, true
d proposition, false **e** proposition, true **f** proposition, true
g a question, so not a proposition **h** proposition, true
i a question, so not a proposition **j** proposition, false
k proposition, indeterminate **l** a question, so not a proposition
m proposition, indeterminate **n** proposition, indeterminate
o proposition, indeterminate (only true when a transversal crosses two parallel lines)
p proposition, false

2 **a** **i** $\neg p$: not all rectangles are parallelograms. **ii** p is true.
b **i** $\neg m$: $\sqrt{5}$ is a rational number. **ii** m is true.
c **i** $\neg r$: 7 is an irrational number. **ii** r is true.
d **i** $\neg q$: $23 - 14 \neq 12$ **ii** $\neg q$ is true.
e **i** $\neg r$: $52 \div 4 \neq 13$ **ii** r is true.
f **i** $\neg s$: The difference between two odd numbers is not always even. **ii** s is true.
g **i** $\neg t$: The product of consecutive integers is not always even. **ii** t is true.
h **i** $\neg u$: Not all obtuse angles are equal. **ii** $\neg u$ is true.
i **i** $\neg p$: Not all trapeziums are parallelograms. **ii** $\neg p$ is true.
j **i** $\neg q$: Not all triangles with two equal angles are isosceles. **ii** q is true.

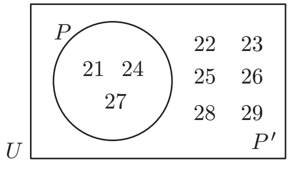
3 **a** $x \geqslant 5$ **b** $x < 3$ **c** $y \geqslant 8$ **d** $y > 10$

4 **a** **i** No **ii** $\neg r$: Kania scored 60% or less. **b** **i** Yes
c **i** No **ii** $\neg r$: Fari is not at soccer practice. **d** **i** Yes
e **i** No **ii** $\neg r$: I did not drink black tea today.

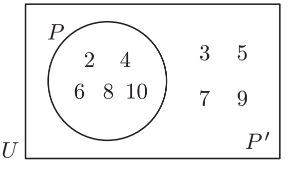
5 **a** $x \in \{1, 2, 3, 4\}$ **b** $x < 0,\ x \in \mathbb{Z}$ **c** $x \in$ {horses, sheep, goats, deer}
d x is a female student **e** x is a female non-student

EXERCISE 8A.2

1 **a** $P = \{21, 24, 27\}$

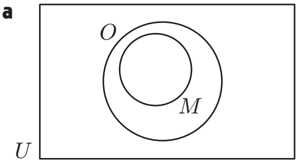


b $P = \{2, 4, 6, 8, 10\}$

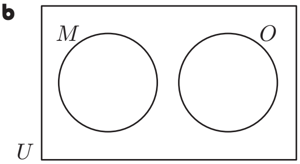


c $P = \{1, 2, 3, 6, 7, 14, 21, 42\}$

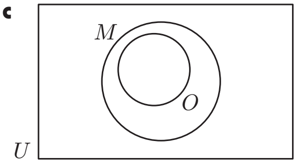
2 **a**



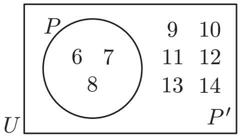
b



c

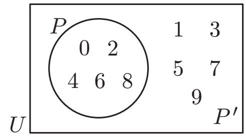


3 **a** If P is the truth set of p then $P = \{6, 7, 8\}$.



b The truth set of $\neg p$ is $P' = \{9, 10, 11, 12, 13, 14\}$.

4 **a** If P is the truth set of p then $P = \{0, 2, 4, 6, 8\}$.



b The truth set of $\neg p$ is $P' = \{1, 3, 5, 7, 9\}$.

EXERCISE 8B.1

1 **a** $p \wedge q$: Ted is a doctor and Shelly is a dentist.

b $p \wedge q$: x is greater than 15 and less than 30.

c $p \wedge q$: It is windy and it is raining.

d $p \wedge q$: Kim has brown hair and blue eyes.

2 **a** p is true and q is true, so $p \wedge q$ is true.

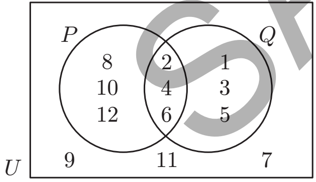
b p is true but q is false (as triangles have three sides), so $p \wedge q$ is false.

c p is false (as $39 > 27$) and q is false (as $16 < 23$), so $p \wedge q$ is false.

d p is true and q is true, so $p \wedge q$ is true.

e p is false (as $5 + 8 = 13$) and q is true, so $p \wedge q$ is false.

3 **a**



b The truth set of $p \wedge q$ is $P \cap Q = \{2, 4, 6\}$.

EXERCISE 8B.2

1 **a** $p \vee q$: Tim owns a bicycle or a scooter.

b $p \vee q$: x is a multiple of 2 or a multiple of 5.

c $p \vee q$: Dana studies Physics or Chemistry.

2 **a** p is true and q is true, so $p \vee q$ is true.

b p is false (as a right angle has $90°$) but q is true, so $p \vee q$ is true.

c p is false (as $-8 < -5$) and q is false (as $5 > 0$), so $p \vee q$ is false.

d p is true but q is false (as the mean of 8 and 14 is 11), so $p \vee q$ is true.

3 **a** $p \veebar q$: Meryn will visit Japan or Singapore, but not both, next year.

b $p \veebar q$: Ann will invite Kate or Tracy, but not both, to her party.

c $p \veebar q$: x is a factor of 56 or 40, but not both.

4 **a** a is true and b is true, so $a \veebar b$ is false.

b a is false (as 15 is odd) and b is true, so $a \veebar b$ is true.

c a is false (as 4.5 is not an integer) and b is false ($\mathbb{N}$ does not include negative numbers), so $a \veebar b$ is false.

d a is true but b is false (as $(2^8)^6 = 2^{8\times 6} = 2^{48}$), so $a \veebar b$ is true.

5 r: Kelly is a good driver and s: Kelly has a good car.

a $\neg r$ **b** $r \wedge s$ **c** $\neg s \wedge \neg r$ **d** $r \vee s$

6 x: Sergio would like to go swimming tomorrow and y: Sergio would like to go bowling tomorrow.

a $\neg x$ **b** $x \wedge y$ **c** $x \vee y$ **d** $\neg(x \wedge y)$ **e** $x \veebar y$

7 **a** p: Phillip likes icecream. q: Phillip likes jelly.
$p \wedge q$: Phillip likes icecream and jelly.

b p: Phillip likes icecream. q: Phillip likes jelly.
$p \vee \neg q$: Phillip likes ice cream or Phillip does not like jelly.

c p: x is greater than 10. q: x is a prime number.
$p \wedge q$: x is both greater than 10 and a prime number.

d p: Tuan can go to the mountains. q: Tuan can go to the beach.
$p \veebar q$: Tuan can go to the mountains or to the beach, but not both.

e p: The computer is on. $\neg p$: The computer is not on.

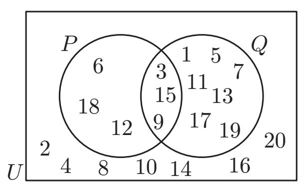
f p: Angela has a watch. q: Angela has a mobile phone.
$\neg p \wedge q$: Angela does not have a watch but does have a mobile phone.

g p: Maya studied Spanish. q: Maya studied French.
$p \veebar q$: Maya studied one of Spanish or French.

h p: I can hear thunder. q: I can hear an aeroplane.
$p \vee q$: I can hear thunder or an aeroplane.

8 Since $p \veebar q$ is false, p and q are either both true or both false.
If p and q were both false, then $p \vee q$ would be false.
But $p \vee q$ is true, so p is true and q is true.

9 **a** If P is the truth set of p, then
$P = \{3, 6, 9, 12, 15, 18\}$.
If Q is the truth set of q, then
$Q = \{1, 3, 5, 7, 9, 11, 13, 15, 17, 19\}$.



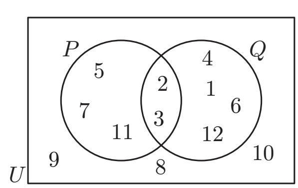
b **i** The truth set of $\neg q$ is $Q' = \{2, 4, 6, 8, 10, 12, 14, 16, 18, 20\}$.

ii The truth set of $p \vee q$ is $P \cup Q = \{1, 3, 5, 6, 7, 9, 11, 12, 13, 15, 17, 18, 19\}$.

iii The truth set of $p \wedge q$ is $P \cap Q = \{3, 9, 15\}$.

iv The truth set of $p \veebar q$ is $\{1, 5, 6, 7, 11, 12, 13, 17, 18, 19\}$.

10 **a** If P is the truth set of p, then
$P = \{2, 3, 5, 7, 11\}$.
If Q is the truth set of q, then
$Q = \{1, 2, 3, 4, 6, 12\}$.



b **i** $p \wedge q$: x is both prime and a factor of 12.

ii $p \vee q$: x is prime or a factor of 12.

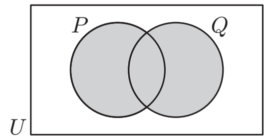
iii $p \veebar q$: x is prime or a factor of 12, but not both.

c **i** $p \wedge q$ has the truth set $P \cap Q = \{2, 3\}$.
ii $p \vee q$ has the truth set $P \cup Q = \{1, 2, 3, 4, 5, 6, 7, 11, 12\}$.
iii $p \veebar q$ has the truth set $\{1, 4, 5, 6, 7, 11, 12\}$.

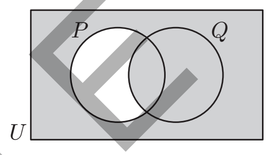
11 **a** p is false.
b s is true.
c q is true and u is true, so $q \wedge u$ is true.
d p is false but w is true, so $p \vee w$ is true.
e r is false but s is true, so $r \vee s$ is true.
f r is false and s is true, so $r \wedge s$ is false.
g r is false and s is true, so $r \veebar s$ is true.
h t is false and v is false, so $t \vee v$ is false.

12 Let P be the truth set of p and Q be the truth set of q.

a $p \vee q$ means p or q or both are true. So we want the region in P or Q or both, which is $P \cup Q$.



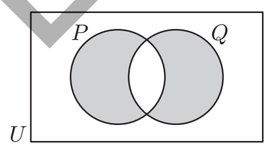
b $\neg p \vee q$ means $\neg p$ or q or both are true. So, p is false, or q is true, or both. So we want the region in P' or Q or both, which is $P' \cup Q$.



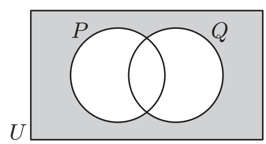
c $p \veebar q$ means p or q are true but not both. So, p is true and q is false, or p is false and q is true. We therefore want the region in P but not Q, or the region in Q but not in P.
So, $(P \cap Q') \cup (Q \cap P')$.

or

p is true, q is true, but exclude the region where p and q are true.
So, $(P \cup Q) \cap ((P \cap Q)')$



d $\neg p \wedge \neg q$ means $\neg p$ is true and $\neg q$ is true. So p is false and q is false. We want the region not in P and not in Q, which is $P' \cap Q'$.



13 **a** The shaded region is $P \cap Q$, which is the region in P and Q.
So, both p and q are true, which is $p \wedge q$.
b The shaded region is the region in P or Q, but not both.
So, p or q is true, but not both, which is $p \veebar q$.
c The shaded region is P', which is the region not in P.
So, p is not true, which is $\neg p$.

14 **a** The captain is old, but not male.
b The captain is old or male.
c The captain is old.

EXERCISE 8C.1

1 **a**

p	q	$\neg p$	$\neg p \wedge q$
T	T	F	F
T	F	F	F
F	T	T	T
F	F	T	F

b

p	q	$p \veebar q$	$\neg(p \veebar q)$
T	T	F	T
T	F	T	F
F	T	T	F
F	F	F	T

c

p	q	$\neg p$	$\neg q$	$\neg p \vee \neg q$
T	T	F	F	F
T	F	F	T	T
F	T	T	F	T
F	F	T	T	T

d

p	$p \vee p$
T	T
F	F

2 **a** **i**

p	q	$\neg p$	$\neg q$	$\neg p \wedge \neg q$
T	T	F	F	F
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

ii neither
(values in the $\neg p \wedge \neg q$ column are neither all true nor all false).

b **i**

p	q	$p \vee q$	$\neg p$	$(p \vee q) \vee \neg p$
T	T	T	F	T
T	F	T	F	T
F	T	T	T	T
F	F	F	T	T

ii tautology
(values in the $(p \vee q) \vee \neg p$ column are all true).

c **i**

p	q	$p \veebar q$	$p \wedge (p \veebar q)$
T	T	F	F
T	F	T	T
F	T	T	F
F	F	F	F

ii neither
(values in the $p \wedge (p \veebar q)$ column are neither all true nor all false).

d **i**

p	q	$p \wedge q$	$p \veebar q$	$(p \wedge q) \wedge (p \veebar q)$
T	T	T	F	F
T	F	F	T	F
F	T	F	T	F
F	F	F	F	F

ii logical contradiction
(values in the $(p \wedge q) \wedge (p \veebar q)$ column are all false).

3 **a** $p \wedge \neg p$ is only true if both p and $\neg p$ are true at the same time, which cannot occur.

b

p	$\neg p$	$p \wedge \neg p$
T	F	F
F	T	F

Since the values in the $p \wedge \neg p$ column are all false, $p \wedge \neg p$ is a logical contradiction.

4 **a**

p	$\neg p$	$\neg(\neg p)$
T	F	T
F	T	F

Since the truth table columns for p and $\neg(\neg p)$ are identical, p and $\neg(\neg p)$ are logically equivalent.
So, $\neg(\neg p) = p$.

b

p	$p \wedge p$
T	T
F	F

Since the truth table columns for p and $p \wedge p$ are identical, p and $p \wedge p$ are logically equivalent.
So, $p \wedge p = p$.

c

p	q	$\neg p$	$\neg p \wedge q$	$p \vee (\neg p \wedge q)$	$p \vee q$
T	T	F	F	T	T
T	F	F	F	T	T
F	T	T	T	T	T
F	F	T	F	F	F

Since the truth table columns for $p \vee (\neg p \wedge q)$ and $p \vee q$ are identical, $p \vee (\neg p \wedge q)$ and $p \vee q$ are logically equivalent.
So, $p \vee (\neg p \wedge q) = p \vee q$.

d

p	q	$p \veebar q$	$\neg(p \veebar q)$	$\neg q$	$p \veebar \neg q$
T	T	F	T	F	T
T	F	T	F	T	F
F	T	T	F	F	F
F	F	F	T	T	T

Since the truth table columns for $\neg(p \veebar q)$ and $p \veebar \neg q$ are identical, $\neg(p \veebar q)$ and $p \veebar \neg q$ are logically equivalent.
So, $\neg(p \veebar q) = p \veebar \neg q$.

e

p	q	$\neg p$	$q \vee \neg p$	$\neg(q \vee \neg p)$	$\neg q$	$p \vee q$	$\neg q \wedge (p \vee q)$
T	T	F	T	F	F	T	F
T	F	F	F	T	T	T	T
F	T	T	T	F	F	T	F
F	F	T	T	F	T	F	F

Since the truth table columns for $\neg(q \vee \neg p)$ and $\neg q \wedge (p \vee q)$ are identical, $\neg(q \vee \neg p)$ and $\neg q \wedge (p \vee q)$ are logically equivalent.
So, $\neg(q \vee \neg p) = \neg q \wedge (p \vee q)$.

f

p	q	$\neg p$	$p \vee q$	$\neg p \veebar (p \vee q)$	$\neg q$	$p \vee \neg q$
T	T	F	T	T	F	T
T	F	F	T	T	T	T
F	T	T	T	F	F	F
F	F	T	F	T	T	T

Since the truth table columns for $\neg p \veebar (p \vee q)$ and $p \vee \neg q$ are identical, $\neg p \veebar (p \vee q)$ and $p \vee \neg q$ are logically equivalent.
So, $\neg p \veebar (p \vee q) = p \vee \neg q$.

5 **a**

p	q	$\neg p$	$\neg q$	$\neg p \wedge q$	$p \wedge \neg q$	$(\neg p \wedge q) \vee (p \wedge \neg q)$
T	T	F	F	F	F	F
T	F	F	T	F	T	T
F	T	T	F	T	F	T
F	F	T	T	F	F	F

b $p \veebar q$ is logically equivalent to $(\neg p \wedge q) \vee (p \wedge \neg q)$.

6 **a** **i** $p \vee q$: I like apples or bananas.
ii $\neg(p \vee q)$: I do not like apples or bananas.
iii $\neg p$: I do not like apples.
iv $\neg p \wedge \neg q$: I do not like apples and I do not like bananas.

b

p	q	$p \vee q$	$\neg(p \vee q)$	$\neg p$	$\neg q$	$\neg p \wedge \neg q$
T	T	T	F	F	F	F
T	F	T	F	F	T	F
F	T	T	F	T	F	F
F	F	F	T	T	T	T

Since the truth table columns for $\neg(p \vee q)$ and $\neg p \wedge \neg q$ are identical, $\neg(p \vee q)$ and $\neg p \wedge \neg q$ are logically equivalent.
So, $\neg(p \vee q) = \neg p \wedge \neg q$.

7 **a**

p	q	$p \veebar q$	$q \wedge (p \veebar q)$	$(p \veebar q) \vee p$
T	T	F	F	T
T	F	T	F	T
F	T	T	T	T
F	F	F	F	F

b **i** $p \veebar q$ needs only one of p or q true, and not both.
So, $-3 \leqslant x \leqslant 7$ or $x \geqslant 2$, but not both.
$\therefore$ $-3 \leqslant x < 2$ or $x > 7$.

ii From the table, $q \wedge (p \veebar q)$ needs p false and q true.
So, $-3 \leqslant x \leqslant 7$ is not true, and $x \geqslant 2$ is true.
$\therefore$ $x > 7$

iii From the table, $(p \veebar q) \vee p$ is true when p or q or both are true.
So, $-3 \leqslant x \leqslant 7$ or $x \geqslant 2$ or both.
$\therefore$ $x \geqslant -3$

8 **a** Any tautology has all the values in its truth table column as true, so any two tautologies will have matching (all true) truth table columns.

b Any logical contradiction has all the values in its truth table column as false, so any two logical contradictions will have matching (all false) truth table columns.

9 **a** A logical contradiction has a truth table column of all Fs, so its negation will have all Ts.
$\therefore$ the negation of a logical contradiction is a tautology.