SOLUTIONS TO TRIAL EXAMINATION 1

PAPER 1

1 **a** $5, -5, \sqrt{16}$ A2 **C2**

b $\frac{1}{3}, 5, -5, \sqrt{16}, 0.\overline{6}$ A2 **C2**

c $5, \sqrt{16}$ A2 **C2**

[6 marks]

2 **a** **i** $P(\text{scores with both attempts}) = \frac{3}{5} \times \frac{3}{5}$ M1

$= \frac{9}{25}$ or 0.36 A1 **C2**

ii $P(\text{misses at least once}) = 1 - P(\text{scores with both attempts})$

$= 1 - \frac{9}{25}$ M1

$= \frac{16}{25}$ or 0.64 A1ft **C2**

b Out of 30 shots, we expect the player to miss $30 \times \frac{2}{5}$ M1

$= 12$ times A1 **C2**

[6 marks]

3 **a** $\text{Surface} = 9.85 \times 5.90$ M1

$= 58.115 \text{ m}^2$

$\approx 58.1 \text{ m}^2$ A1 **C2**

b Rounded measurements are 10 m by 6 m.

$\text{Surface area} = 10 \times 6$

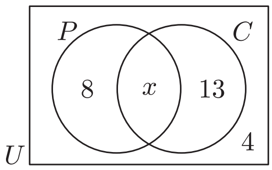
$= 60 \text{ m}^2$ A1

$\text{Percentage error} = \frac{60 - 58.115}{58.115} \times 100\%$ M1 A1ft

$\approx 3.24\%$ A1ft **C4**

[6 marks]

4 **a**



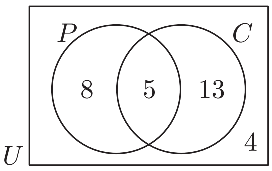
Total number of students $= 8 + 13 + 4 + x$

$\therefore \quad 30 = 25 + x$

$\therefore \quad x = 5$

$\therefore$ 5 students like both plain and chocolate milk. A1 **C1**

b



A1 A1 A1ft **C3**

c $P(\text{likes only one type of milk}) = \frac{8 + 13}{30}$

$= \frac{21}{30}$

$= \frac{7}{10}$ or 0.7 A1 A1ft **C2**

[6 marks]

5 **a** Total number of events $= 12 + 15 + 11 + 7 + 5$

$= 50$

$P(\text{ticket costs more than \$60}) = \frac{11 + 7 + 5}{50}$

$= \frac{23}{50}$ or 0.46 A1 **C1**

b

Cost (\$)	*Number of events*	*Midpoint* (x)
20 - 39	12	29.5
40 - 59	15	49.5
60 - 79	11	69.5
80 - 99	7	89.5
100 - 119	5	109.5
Total	50	

Using technology, estimates are: mean $\approx$ \$60.70 — A1

standard deviation $\approx$ \$25.35 — A1 **C2**

c 0.722 standard deviations above the mean is $\$60.70 + 0.722 \times \$25.35 = \$79.00$ — A1ft

The percentage of events less than \$79 $= \dfrac{12+15+11}{50}$

$= \frac{38}{50} \times 100\%$ — M1

$= 76\%$ — A1ft **C3**

[6 marks]

6 **a** $2000 \text{ USD} = 2000 \times 0.64 \text{ GBP}$ — M1

$= 1280 \text{ GBP}$ — A1 **C2**

b Amount remaining $= 1280 - 1100$ — M1

$= 180 \text{ GBP}$ — A1ft **C2**

$180 \text{ GBP} = 180 \times \dfrac{1}{0.68} \text{ USD}$ — M1

$\approx 265 \text{ USD}$ — A1ft **C2**

[6 marks]

7 **a** $u_1 r^6 = 320$ and $u_1 r^9 = 2560$ — M1

$\therefore \dfrac{u_1 r^9}{u_1 r^6} = \dfrac{2560}{320}$

$\therefore r^3 = 8$

$\therefore r = 2$ — A1 **C2**

b $u_1 2^6 = 320$ — M1

$\therefore 64 u_1 = 320$

$\therefore u_1 = 5$ — A1ft **C2**

c $u_{20} = u_1 r^{19}$ — M1

$= 5 \times 2^{19}$

$= 2\,621\,440$ — A1ft **C2**

[6 marks]

8 **a** The interest compounds monthly over 3 years.

$\therefore$ the total amount Ali needs to repay is $12\,000 \times \left(1 + \frac{8.5}{1200}\right)^{3\times 12}$ euros — M1 A1

$\approx$ €15 471.63 — A1ft **C3**

b The car depreciates at 22.5% p.a. for 4 years.

$\therefore$ the value at the end of 2013 is $12\,000 \times \left(1 - \frac{22.5}{100}\right)^4$ euros — M1 A1

$\approx$ €4329.00 — A1 **C3**

[6 marks]

9 **a** $Q_3 = 77$, $Q_1 = 65$ — A1 A1 **C2**

b **i** $\text{IQR} = Q_3 - Q_1$

$= 77 - 65$

$= 12$ — A1ft

ii Range = maximum − minimum

$= 85 - 45$ — A1

$= 40$ — A1 **C3**

c 70 is the median value.

$\therefore$ 50% of the values are less than 70. — A1 **C1**

[6 marks]

10 **a** Profit = total sales − total cost

$\therefore$ for 100 boxes, profit $= 12.50(100) - (9.5(100) + 45)$ M1

$= \$255$ A1 **C2**

b The firm breaks even when $12.5x = 9.5x + 45$ M1

$\therefore\ 3x = 45$

$\therefore\ x = 15$

So, 15 boxes must be produced and sold to break even. A1 **C2**

c For the profit to be greater than \$1000, $\therefore\ 12.50x - (9.5x + 45) > 1000$ M1

$\therefore\ 3x - 45 > 1000$

$\therefore\ x > 348.3$

So, 349 boxes must be produced and sold. A1 **C2**

[6 marks]

11 **a**

$p \wedge q$	$\neg(p \wedge q)$	$p \veebar q$	$\neg(p \wedge q) \vee q$	$(p \veebar q) \Rightarrow \neg(p \wedge q) \vee q$	
T	F	F	T	T	A1
F	T	T	T	T	A1
F	T	T	T	T	A1
F	T	F	T	T	A1 **C4**

b If Bozo does not have a red nose then Bozo is not a clown. A1 A1 **C2**

[6 marks]

12 A is $(-2, -3)$, B is $(1, 3)$

a The gradient of AB $= \dfrac{3-(-3)}{1-(-2)} = 2$ A1

$\therefore$ the equation of AB is $y = 2x + c$

Substituting $(1, 3)$ gives $3 = 2(1) + c$

$\therefore\ c = 1$ M1

The equation of AB is $y = 2x + 1$

or $2x - y + 1 = 0$ A1ft **C3**

b Midpoint of AB is $\left(\dfrac{-2+1}{2}, \dfrac{-3+3}{2}\right)$, or $\left(-\frac{1}{2}, 0\right)$ A1

The gradient of the perpendicular bisector is $-\frac{1}{2}$ $\{$as $2 \times -\frac{1}{2} = -1\}$ A1ft

$\therefore$ its equation is $y = -\frac{1}{2}x + c$

Substituting $\left(-\frac{1}{2}, 0\right)$ gives $0 = -\frac{1}{2}\left(-\frac{1}{2}\right) + c$

$\therefore\ c = -\frac{1}{4}$

$\therefore$ the equation of perpendicular bisector is $y = -\frac{1}{2}x - \frac{1}{4}$

or $4y = -2x - 1$

or $2x + 4y + 1 = 0$ A1ft **C3**

[6 marks]

13 **a** $f(x) = ax^2 + bx + d$ $\therefore$ $f'(x) = 2ax + b$ A1 **C1**

b $f'(x) = 5x - 10$

Equating coefficients gives $2a = 5$ and $b = -10$

$\therefore\ a = 2.5$ and $b = -10$ A1ft A1ft **C2**

c $f'(x) = 0$ when $x = 2$ A1ft

Now $f(2) = 2.5 \times 2^2 - 10 \times 2 + d$

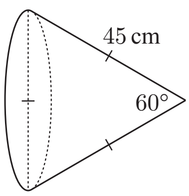
$= d - 10$

$\therefore\ d - 10 = -4$ M1

$\therefore\ d = 6$ A1ft **C3**

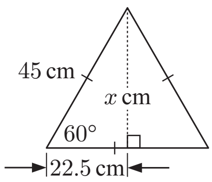
[6 marks]

14 **a**



The triangle which bisects the cone is equilateral with sides 45 cm.

$\therefore$ the diameter of the megaphone is 45 cm. A1 **C1**

b 

Let the height be x cm.

Now $\tan 60^\circ = \dfrac{x}{22.5}$ — M1

$\therefore \quad x \approx 39.0$ — A1ft

$\therefore$ height of cone is 39 cm.

Volume of cone $= \frac{1}{3}\pi r^2 h$

$\approx \frac{1}{3}\pi \times 22.5^2 \times 39$ — M1 A1ft

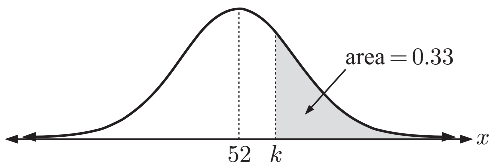
$\approx 20\,700 \text{ cm}^3$ — A1ft — **C5**

[6 marks]

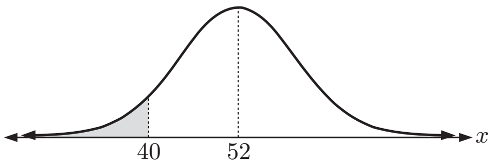
15 **a** The top 33% of candidates scored a B or better.

If $\quad P(X > k) = 0.33$

then $\quad k \approx 57.3$ — M1 A1



$\therefore$ the minimum mark required for a B is 58. — A1 — **C3**

b 

$P(X < 40) \approx 0.159$ — M1 A1

$\therefore$ the expected number of students scoring less than 40 is

300×0.159

$= 48$ students — A1 — **C3**

[6 marks]

PAPER 2

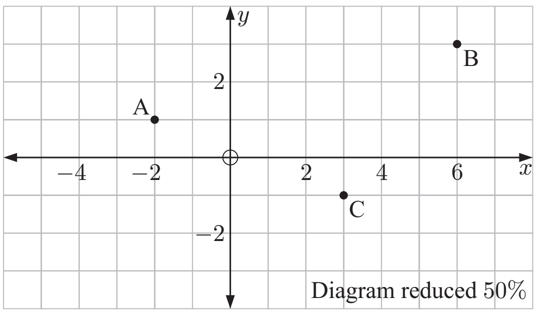
1 **a** 

Diagram reduced 50%

A1 A1 — **2**

b **i** Gradient of BC $= \dfrac{-1-3}{3-6} = \dfrac{-4}{-3} = \frac{4}{3}$ — M1 A1 (G2)

ii The opposite sides of a parallelogram are parallel, and parallel lines have the same gradient.

$\therefore$ gradient of AD = gradient of BC — A1

iii Gradient of AD $= \dfrac{d-1}{-5--2} = \dfrac{4}{3}$ — M1

$\therefore \quad \dfrac{d-1}{-3} = \dfrac{4}{3}$

$\therefore \quad d - 1 = \frac{4}{3} \times -3$

$\therefore \quad d - 1 = -4$

$\therefore \quad d = -3$ — A1ft (G2) — **5**

c **i** Length AB $= \sqrt{(6--2)^2 + (3-1)^2}$ — M1 A1

$= \sqrt{68}$ units — A1 (G2)

ii $\cos A\widehat{B}C = \dfrac{\sqrt{68}^2 + 5^2 - \sqrt{29}^2}{2 \times \sqrt{68} \times 5}$ — M1 A1ft

$\therefore \quad A\widehat{B}C \approx 39.1^\circ$ — A1ft (G2) — **6**

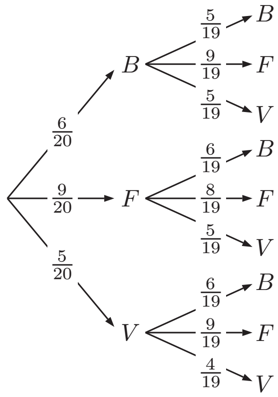
d Area ABCD = 2 × area triangle ABC

$= 2 \times \frac{1}{2} \times 5 \times \sqrt{68} \sin A\widehat{B}C$ — M1 A1ft

$= 26 \text{ units}^2$ — A1ft (G2) — **3**

[16 marks]

2 **a** Let B represent a basketball being chosen, F represent a football being chosen, and V represent a volleyball being chosen.



A4 **4**

b **i** P(two basketballs) $= \text{P}(B \text{ then } B)$

$= \frac{6}{20} \times \frac{5}{19}$ M1

$= \frac{30}{380} \quad (\approx 0.0789)$ A1ft (G2)

ii P(a basketball and a football) $= \text{P}(B \text{ then } F) + \text{P}(F \text{ then } B)$

$= \frac{6}{20} \times \frac{9}{19} + \frac{9}{20} \times \frac{6}{19}$ M1 A1ft

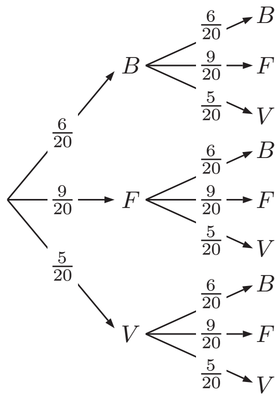
$= \frac{108}{380} \quad (\approx 0.284)$ A1ft (G2)

iii P(both balls are the same) $= \text{P}(B \text{ then } B) + \text{P}(F \text{ then } F) + \text{P}(V \text{ then } V)$

$= \frac{6}{20} \times \frac{5}{19} + \frac{9}{20} \times \frac{8}{19} + \frac{5}{20} \times \frac{4}{19}$ M1 A1ft

$= \frac{122}{380} \quad (\approx 0.321)$ A1ft (G2) **8**

c With replacement:



i P(two volleyballs) $= \text{P}(V \text{ and } V)$

$= \frac{5}{20} \times \frac{5}{20}$ M1 A1ft

$= \frac{1}{4} \times \frac{1}{4}$

$= \frac{1}{16}$ A1ft (G2) **3**

ii P(both B | the two balls are the same)

$= \dfrac{\text{P}(B \text{ then } B)}{\text{P}(B \text{ then } B) + \text{P}(F \text{ then } F) + \text{P}(V \text{ then } V)}$

$= \dfrac{\frac{6}{20} \times \frac{6}{20}}{\frac{6}{20} \times \frac{6}{20} + \frac{9}{20} \times \frac{9}{20} + \frac{5}{20} \times \frac{5}{20}}$ M1 A1ft

$= \frac{18}{71}$

≈ 0.254 A1ft (G2) **3**

[18 marks]

3 **a** **i** $T_P(0) = 61 \times (0.95)^0 + 18$

$= 61 + 18$

$= 79°\text{C} \qquad \therefore \; a = 79$ A1

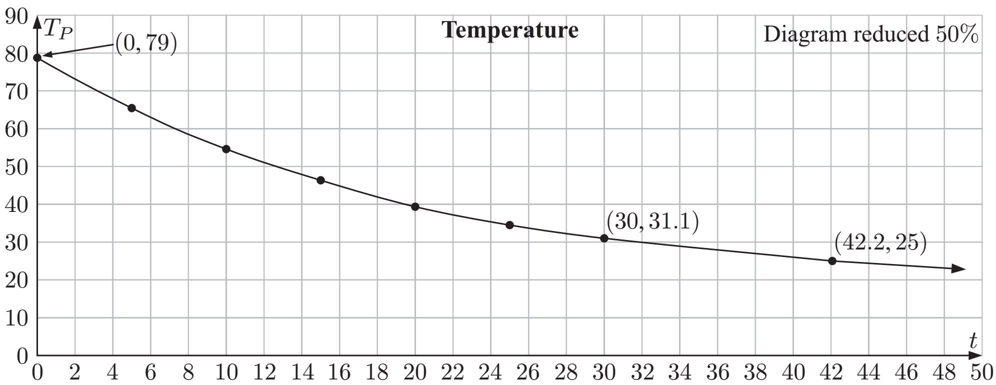
$T_P(30) = 61 \times (0.95)^{30} + 18$

$\approx 31.1°\text{C} \qquad \therefore \; b \approx 31.1$ A1 **2**

ii We need to solve $61 \times (0.95)^t + 18 = 25$ M1

So, $t \approx 42.2$ min {using technology} A1 (G2) **2**

b



(A1 for scale and labels, A2 for points, A1 for curve)

A1 A2ft A1 — **4**

c **i** $T_F(0) = 53 \times (0.98)^0 + 18$ — M1

$= 53 + 18$

$= 71°\text{C}$ — A1

ii As $0.95 < 0.98$, the T_P function decreases at a faster rate than the T_F function.
$\therefore$ heat is lost faster in the plastic cup. — A1ft

iii We need to solve $53 \times (0.98)^t + 18 = 61 \times (0.95)^t + 18$ — M1

$\therefore \quad 53 \times (0.98)^t = 61 \times (0.95)^t$

Using technology, $t \approx 4.52$ — A1

So it takes about 4.5 minutes for the temperatures in each cup to be equal. — A1 (G3) — **6**

d In the long term, $(0.95)^t$ and $(0.98)^t$ both decrease to almost zero.
So, $T_P(t)$ and $T_F(t)$ both approach 18°C. — A2 — **2**

[16 marks]

4 **a** Using technology, $r \approx 0.849$ — G2

b A moderate positive relationship may exist between the forecast temperature and the number of people attending the swimming pool. — A1ft A1ft — **4**

c **i** Using technology, $N = 4.42T - 35.6$ — G1 G1

ii When $T = 20°\text{C}$, $N = 4.42 \times 20 - 35.6 = 53$ people. — M1 A1 (G2)
When $T = 40°\text{C}$, $N = 4.42 \times 40 - 35.6 = 141$ people. — M1 A1 (G2)

iii The estimate when $T = 20°\text{C}$ is more reliable, since it is an interpolated value between known values. — A1 R1ft — **8**

d Since the correlation is moderate, the manager's plan seems sensible. — A1 R1ft — **2**

[14 marks]

5 **a** H_0: Attendance by gender is independent of temperature. — A1
H_1: Attendance by gender is not independent of temperature. — A1 — **2**

b The number of degrees of freedom is $(2-1)(2-1) = 1$. — M1 AG — **1**

c p-value $= 0.0950$ — G2 — **2**

d Since $p\text{-value} > \text{P}_{calc} = 0.05$, we do not reject H_0.
We conclude that attendance by gender and temperature are independent. — A1ft R1ft — **2**

[7 marks]

6 **a** $f(x) = 3x^3 - 4x + 5$

i $f(1) = 3(1)^3 - 4(1) + 5$ — M1

$= 4$ — A1 (G2) — **2**

ii $f'(x) = 9x^2 - 4$ — A1 A1 — **2**

iii $f'(1) = 9 - 4 = 5 \quad \therefore$ the gradient at $x = 1$ is 5 — M1 A1ft (G2) — **2**

iv At (1, 4) the tangent has gradient 5.

$\therefore$ its equation is $y - 4 = 5(x - 1)$ — M1

or $y - 4 = 5x - 5$

or $y = 5x - 1$ — A1ft (G1) — **2**

v The tangent $y = 5x - 1$ meets $y = f(x)$ where $3x^3 - 4x + 5 = 5x - 1$
Using technology, $x = 1$ or -2
$\therefore$ they meet again at $(-2, -11)$. — A1 A1ft — **2**

b **i** Volume = length × width × height

$\therefore \quad V = x \times x \times y$

$= x^2 y$ — A2 — **2**

ii $V = x^2\left(\dfrac{30\,000 - x^2}{2x}\right)$ M1

$= \frac{1}{2}x(30\,000 - x^2)$

$= 15\,000x - \frac{1}{2}x^3$ A1ft (G2) **2**

iii $\dfrac{dV}{dx} = 15\,000 - \frac{3}{2}x^2$ A1 A1ft **2**

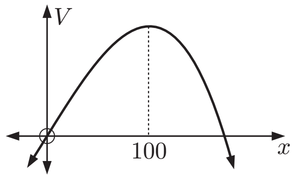
iv The maximum value occurs when $\dfrac{dV}{dx} = 0$ M1

$\therefore\ \frac{3}{2}x^2 = 15\,000$

$\therefore\ 3x^2 = 30\,000$

$\therefore\ x^2 = 10\,000$

$\therefore\ x = 100 \quad \{x > 0\}$ A1



$\therefore$ the maximum value of V is when $x = 100$. A1ft (G2) **3**

[19 marks]

SOLUTIONS TO TRIAL EXAMINATION 2

PAPER 1

1 **a** **C** and **D** A1 A1 **C2**

b $0.0518 = 5.18 \times 10^{-2}$ A1 A1 **C2**

c Percentage error in rounding is

$\dfrac{0.0518 - 0.051\,762}{0.051\,762} \times 100\%$ M1

$\approx 0.0734\%$ A1 **C2**

[6 marks]

2 **a** $2 + 5 + 8 + 12 + 5 + 6 + 2 = 40$ sheep were weighed A1 **C1**

b Mean weight

$= \dfrac{2 \times 10 + 5 \times 20 + 8 \times 30 + ... + 2 \times 70}{40}$ M1

$= \dfrac{1590}{40}$

$= 39.75$ kg A1ft **C2**

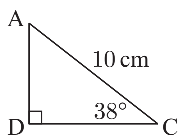
c 150% of the mean weight is $39.75 \times 1.50 \approx 59.625$ kg

Percentage of sheep going to market is $\dfrac{6+2}{40} \times 100\% = \frac{8}{40} \times 100\%$ M1

$= 20\%$ A1 A1ft **C3**

[6 marks]

3 **a**



$\cos 38° = \dfrac{\text{DC}}{10}$ M1

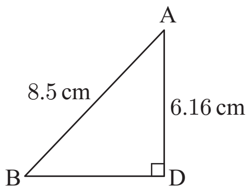
$10 \times \cos 38° = \text{DC}$

$\therefore\ \text{DC} \approx 7.88$ cm A1 **C2**

b $\sin 38° = \dfrac{\text{AD}}{10}$ M1

$10 \times \sin 38° = \text{AD}$

$\therefore\ \text{AD} \approx 6.16$ cm A1



Now $\text{AB}^2 = \text{AD}^2 + \text{BD}^2$

$\therefore\ \text{BD}^2 = \text{AB}^2 - \text{AD}^2$

$\therefore\ \text{BD}^2 = 8.5^2 - 6.16^2$ M1

$\therefore\ \text{BD} = \sqrt{8.5^2 - 6.16^2}$

$\therefore\ \text{BD} \approx 5.86$ cm A1ft **C4**

[6 marks]

4 **a** The total revenue is $22 \times €42.50 = €935$. A1 **C1**

b The cost of production is $C(22) = 15.6 \times 22 + 245$ M1

$= €588.20$ A1 **C2**