b $f'(x) = 0$ when $2x - 1 = 0$ or $\cos x = 0$

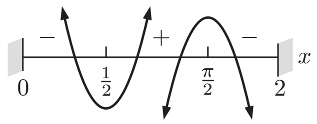
$\therefore$ for $0 \leqslant x \leqslant 2$, $x = \frac{1}{2}$ or $x = \frac{\pi}{2}$

$f'(\frac{1}{4}) = -\frac{1}{2}\cos\frac{1}{4} < 0$

$f'(1) = \cos 1 > 0$

$f'(1.9) = 2.8 \times \cos 1.9 \approx -0.9052 < 0$

Sign diagram for $f'(x)$:



Now $f(\frac{1}{2}) \approx 1.755$

and $f(2) \approx 1.896$

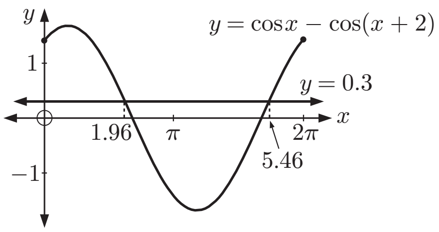
$\therefore$ the smallest value is ≈ 1.76 when $x = \frac{1}{2}$.

34 $\int_a^{a+2} \sin x\, dx = 0.3$

$\therefore [-\cos x]_a^{a+2} = 0.3$

$\therefore -\cos(a+2) + \cos a = 0.3$

$\therefore \cos a - \cos(a+2) = 0.3$



So, $a \approx 1.96$ or 5.46 {using technology}

SOLUTIONS TO TRIAL EXAMINATION 1

Paper 1 - No calculators

Section A

1 a $\dfrac{x}{24} = \dfrac{6}{x}$

$\therefore x^2 = 144$

$\therefore x = \pm 12$

But $x > 0$, $\therefore x = 12$

b $r = \frac{12}{24} = \frac{1}{2}$

c $u_5 = u_1 r^4$

$= 24 \times \left(\frac{1}{2}\right)^4$

$= \frac{24}{16}$

$= \frac{3}{2}$

d $S_\infty = \dfrac{u_1}{1-r}$

$= \dfrac{24}{1-\frac{1}{2}}$

$= \dfrac{24}{\frac{1}{2}}$

$= 48$

2 a $h(x) = e^{-x}\cos x$

$\therefore h'(x) = e^{-x}(-1)\cos x + e^{-x}(-\sin x)$

$= -e^{-x}(\cos x + \sin x)$

b $h'(\frac{\pi}{2}) = -e^{-\frac{\pi}{2}}\left(\cos\frac{\pi}{2} + \sin\frac{\pi}{2}\right)$

$= -e^{-\frac{\pi}{2}}(0+1)$

$= -e^{-\frac{\pi}{2}}$

c $h(\frac{\pi}{2}) = e^{-\frac{\pi}{2}}\cos\frac{\pi}{2}$

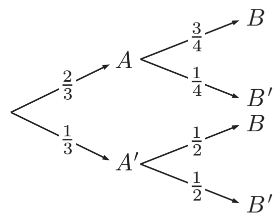
$= 0$

$\therefore$ the point of contact is $(\frac{\pi}{2}, 0)$.

$\therefore$ the tangent has equation $\dfrac{y-0}{x-\frac{\pi}{2}} = -e^{-\frac{\pi}{2}}$

$\therefore e^{\frac{\pi}{2}}y = -x + \frac{\pi}{2}$

3 a



$\therefore x = \frac{2}{3}$, $y = \frac{1}{4}$, and $z = \frac{1}{2}$.

b $P(B) = P(A \cap B) + P(A' \cap B)$

$= \left(\frac{2}{3}\right)\left(\frac{3}{4}\right) + \left(\frac{1}{3}\right)\left(\frac{1}{2}\right)$

$= \frac{1}{2} + \frac{1}{6}$

$= \frac{2}{3}$

c $P(A' \mid B) = \dfrac{P(A' \cap B)}{P(B)}$

$= \dfrac{\left(\frac{1}{3}\right)\left(\frac{1}{2}\right)}{\frac{2}{3}}$

$= \frac{1}{4}$

4 a If $\sin 2\theta = \tan\theta$, then

$2\sin\theta\cos\theta = \dfrac{\sin\theta}{\cos\theta}$

$\therefore 2\sin\theta\cos^2\theta = \sin\theta$

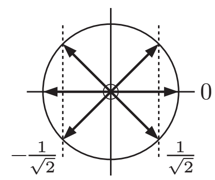
$\therefore 2\sin\theta\cos^2\theta - \sin\theta = 0$

$\therefore \sin\theta(2\cos^2\theta - 1) = 0$

$\therefore \sin\theta = 0$ or $\cos^2\theta = \frac{1}{2}$

$\therefore \sin\theta = 0$ or $\cos\theta = \pm\frac{1}{\sqrt{2}}$

b $\theta = 0, \pm\frac{\pi}{4}, \pm\frac{3\pi}{4}, \pm\pi$



5 a The vertex is at (3, 7), so

$f(x) = a(x-3)^2 + 7$

$\therefore h = 3$, $k = 7$

b $f(0) = 2.5$ so $a(-3)^2 + 7 = 2.5$

$\therefore 9a = -4.5$

$\therefore a = -\frac{1}{2}$

c $f(x) = -\frac{1}{2}(x-3)^2 + 7$

$\therefore f(x) = 0$ when $\frac{1}{2}(x-3)^2 = 7$

$\therefore (x-3)^2 = 14$

$\therefore x - 3 = \pm\sqrt{14}$

$\therefore x = 3 \pm \sqrt{14}$

But $x > 0$ at A $\therefore$ A is $(3+\sqrt{14}, 0)$.

6 $f(x) = \int \left(2x - 3x^{-\frac{1}{2}}\right) dx$

$= \dfrac{2x^2}{2} - \dfrac{3x^{\frac{1}{2}}}{\frac{1}{2}} + c$

$= x^2 - 6\sqrt{x} + c$

But $f(4) = 3$

$\therefore 16 - 12 + c = 3$

$\therefore c = -1$

$\therefore f(x) = x^2 - 6\sqrt{x} - 1$

7 **a** $m + 0.15 + 2m + n = 1$

$\therefore\ 3m + n = 0.85$

b $\text{E}(X) = m + 0.3 + 6m + 4n$

$= 7m + 4n + 0.3$

$= 7m + 4(0.85 - 3m) + 0.3 \quad$ {using **a**}

$= -5m + 3.7$

c If $\ \text{E}(X) = 2.7, \quad -5m + 3.7 = 2.7$

$\therefore\ -5m = -1$

$\therefore\ m = 0.2$

Section B

8 **a** $\overrightarrow{\text{BA}} = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \\ -4 \end{pmatrix} = \begin{pmatrix} 1 \\ -3 \\ 7 \end{pmatrix}$

b Suppose R lies on line L_1.

$\overrightarrow{\text{OR}} = \overrightarrow{\text{OB}} + t\,\overrightarrow{\text{BA}}$

$\therefore\ \mathbf{r} = \begin{pmatrix} 1 \\ 2 \\ -4 \end{pmatrix} + t \begin{pmatrix} 1 \\ -3 \\ 7 \end{pmatrix}, \quad t \in \mathbb{R}$

c L_2 has direction vector $\begin{pmatrix} 4 \\ 2m \\ m \end{pmatrix}$ and $L_1 \perp L_2$.

$\therefore\ \begin{pmatrix} 1 \\ -3 \\ 7 \end{pmatrix} \bullet \begin{pmatrix} 4 \\ 2m \\ m \end{pmatrix} = 0$

$\therefore\ 4 - 6m + 7m = 0$

$\therefore\ m = -4$

d L_2 has direction vector $\begin{pmatrix} 4 \\ -8 \\ -4 \end{pmatrix} = 4 \begin{pmatrix} 1 \\ -2 \\ -1 \end{pmatrix}$

$\therefore$ a vector equation for L_2 is

$\mathbf{r} = \begin{pmatrix} 2 \\ 3 \\ k \end{pmatrix} + s \begin{pmatrix} 1 \\ -2 \\ -1 \end{pmatrix}, \quad s \in \mathbb{R}$

e L_1 meets L_2 where

$\begin{pmatrix} 2 \\ 3 \\ k \end{pmatrix} + s \begin{pmatrix} 1 \\ -2 \\ -1 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ -4 \end{pmatrix} + t \begin{pmatrix} 1 \\ -3 \\ 7 \end{pmatrix}$

$\therefore\ 2 + s = 1 + t \quad .... (1)$

$3 - 2s = 2 - 3t \quad .... (2)$

$k - s = -4 + 7t \quad .... (3)$

$4 + 2s = 2 + 2t \quad \{2 \times (1)\}$

$3 - 2s = 2 - 3t \quad \{(2)\}$

Adding, $\quad 7 = 4 - t$

$\therefore\ t = -3$

$\therefore\ 2 + s = -2$

$\therefore\ s = -4$

So, in (3), $\ k + 4 = -4 - 21$

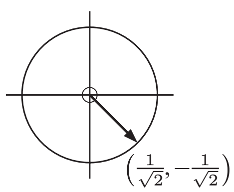
$\therefore\ k = -29$

9 **a** $f(0) = -4\cos\left(-\frac{\pi}{4}\right) + 2$

$= -4\left(\frac{1}{\sqrt{2}}\right) + 2$

$= 2 - 2\sqrt{2}$

So, the y-intercept is $\ 2 - 2\sqrt{2}$.



b $f(x) = 0 \quad$ when

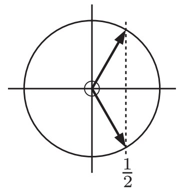
$-4\cos\left(\frac{\pi}{4}(x-1)\right) + 2 = 0$

$\therefore\ \cos\left(\frac{\pi}{4}(x-1)\right) = \frac{1}{2}$

$\therefore\ \frac{\pi}{4}(x-1) = \frac{\pi}{3}$

$\therefore\ x - 1 = \frac{4}{3}$

$\therefore\ x = 2\frac{1}{3}$



So, the x-intercept is $2\frac{1}{3}$.

c $f'(x) = -4\left[-\sin\left(\frac{\pi}{4}(x-1)\right)\right]\frac{\pi}{4} + 0$

$= \pi \sin\left(\frac{\pi}{4}(x-1)\right)$

d $f'(x) = 0 \quad$ when $\quad \sin\left(\frac{\pi}{4}(x-1)\right) = 0$

$\therefore\ \frac{\pi}{4}(x-1)$ is a multiple of π

$\therefore$ for the domain $\ 0 \leqslant x \leqslant 6, \ x = 1$ or 5

$\therefore$ the x-coordinate of A is 1 and the x-coordinate of B is 5.

e Area $= \int_3^5 \left(-4\cos\left(\frac{\pi}{4}(x-1)\right) + 2\right)\,dx$

$= \left[-4\left(\frac{4}{\pi}\right)\sin\left(\frac{\pi}{4}(x-1)\right) + 2x\right]_3^5$

$= \left(-\frac{16}{\pi}\right)\sin\pi + 10 - \left[-\frac{16}{\pi}\sin\left(\frac{\pi}{2}\right) + 6\right]$

$= 10 + \frac{16}{\pi} - 6$

$= 4 + \frac{16}{\pi} \ \text{units}^2$

10 **a** $(g \circ f)(x) = g(f(x))$

$= 3\,[f(x)]^2 - 1$

$= 3(2x-1)^2 - 1$

$= 3(4x^2 - 4x + 1) - 1$

$= 12x^2 - 12x + 2$

b $y = 12x^2 - 12x + 2$ translated through $\begin{pmatrix} -1 \\ 4 \end{pmatrix}$ becomes

$y - 4 = 12(x+1)^2 - 12(x+1) + 2$

$\therefore\ y - 4 = 12x^2 + 24x + 12 - 12x - 12 + 2$

$\therefore\ y = 12x^2 + 12x + 6$

So, $\ h(x) = 12x^2 + 12x + 6$

c $h(x) = 12(x^2 + x) + 6$

$= 12\left(x^2 + x + \left(\frac{1}{2}\right)^2\right) + 6 - 12\left(\frac{1}{2}\right)^2$

$= 12\left(x + \frac{1}{2}\right)^2 + 3$

$= 12\left(x - \left(-\frac{1}{2}\right)\right)^2 + 3$

d **i** $g(x)$ has vertex $(0, -1)$.

ii $h(x)$ has vertex $\left(-\frac{1}{2}, 3\right)$.

e $y = 12x^2 - 12x + 6$ meets $\ y = 2x + c$

where $\ 12x^2 - 12x + 6 = 2x + c$

$\therefore\ 12x^2 - 14x + (6 - c) = 0$

In the case of a tangent, this equation has a repeated root.

$\therefore\ \Delta = 0$

$\therefore\ (-14)^2 - 4(12)(6 - c) = 0$

$\therefore\ 196 - 288 + 48c = 0$

$\therefore\ 48c = 92$

$\therefore\ c = \frac{23}{12}$

Paper 2 - Calculators

Section A

1 **a** The reflex angle is $(2\pi - \theta)$

$\therefore$ arc length AXB $= r(2\pi - \theta)$

$\therefore\ 14.3 = 3.8(2\pi - \theta)$

$\therefore\ 2\pi - \theta = \dfrac{14.3}{3.8}$

$\therefore\ \theta = 2\pi - \dfrac{14.3}{3.8}$

$\therefore\ \theta \approx 2.52^c$

b Area $= \frac{1}{2}r^2\theta$

$\approx \frac{1}{2} \times 3.8^2 \times (2\pi - 2.520\,03)$

$\approx 27.2\ \text{m}^2$

2 **a** The $(r+1)$th term is $T_{r+1} = \binom{n}{r} a^{n-r}b^r$

where $a = 2x^2,\ b = (-1),\ n = 12$

$\therefore\ T_{r+1} = \binom{12}{r}\left(2x^2\right)^{12-r}(-1)^r$

$= \binom{12}{r} 2^{12-r}x^{24-2r}(-1)^r$

b If $24 - 2r = 10$ then $2r = 14$

$\therefore\ r = 7$

Thus $T_8 = \binom{12}{7} 2^5x^{10}(-1)^7$

$\therefore$ the coefficient of $x^{10} = -\binom{12}{7}2^5 = -25\,344$.

3 **a** mean $\overline{x} \approx 2.94$, median $= 3$

b standard deviation ≈ 1.43

c IQR $= Q_3 - Q_1$

$= 4 - 2$

$= 2$

4 **a** $u_1 = 17,\ \ u_2 = 15,\ \ u_3 = 13,\ \ u_4 = 11,\ \ ....$
The terms are decreasing by 2 each time. This pattern will continue because the general term has the form $u_n = u_1 - 2n$.
$\therefore$ the sequence is arithmetic.

b $d = -2$

c If $u_n = -55$, then $19 - 2n = -55$

$\therefore\ 2n = 74$

$\therefore\ n = 37$

$\therefore$ -55 is the 37th term of the sequence.

d $S_n = \dfrac{n}{2}\left[2u_1 + (n-1)d\right]$

$= \dfrac{n}{2}\left[34 + (n-1)(-2)\right]$

$= \dfrac{n}{2}\left[34 - 2n + 2\right]$

$= \dfrac{n}{2}\left[36 - 2n\right]$

$\therefore\ S_n = n(18 - n)$

5 Using $\mathbf{a} \bullet \mathbf{b} = |\,\mathbf{a}\,|\,|\,\mathbf{b}\,|\cos\theta$,

$$\begin{pmatrix}1\\-1\\2\end{pmatrix} \bullet \begin{pmatrix}3\\1\\t\end{pmatrix} = \sqrt{1+1+4}\sqrt{9+1+t^2}\cos 60^\circ$$

$\therefore\ 3 - 1 + 2t = \sqrt{6}\sqrt{t^2+10}\left(\frac{1}{2}\right)$

$\therefore\ 4t + 4 = \sqrt{6t^2 + 60}$

$\therefore\ 16t^2 + 32t + 16 = 6t^2 + 60$

$\therefore\ 10t^2 + 32t - 44 = 0$

$\therefore\ t \approx 1.038\quad \{\text{as } t > 0\}$

6 **a** Using A, $2(2)^3 + b(2)^2 + c(2) = 6$

$\therefore\ 16 + 4b + 2c = 6$

$\therefore\ 4b + 2c = -10$

$\therefore\ 2b + c = -5\quad .... (1)$

Using C, $2(-2)^3 + b(-2)^2 + c(-2) = -6$

$\therefore\ -16 + 4b - 2c = -6$

$\therefore\ 4b - 2c = 10$

$\therefore\ 2b - c = 5\quad .... (2)$

Adding (1) and (2) gives $4b = 0$

$\therefore\ b = 0$

Using (1), $c = -5$

b From **a**, $f(x) = 2x^3 - 5x$
But $f(k) = -2$, so $2k^3 - 5k = -2$ and we need a solution of this equation between $k = 0$ and $k = 2$.
Using technology, $k \approx 1.32$.

7 **a** As x increases, y increases also, so the correlation is positive.

b $y = 2.12x + 0.34$

c $r \approx 0.999\,79$
There is a very strong positive correlation between x and y.

d When $x = 7,\ y \approx 15.2$.

e $x = 12$ is outside the domain of the data used to find the regression line.
So, the result would be an extrapolation and therefore would not be reliable.

Section B

8 **a** Using technology, $a \approx -2.828$, $b \approx 2.828$

b $c \approx 2.27\quad$ {technology}

c From the graph, the asymptotes appear to be $x = 3$ and $x = -3$.
As $x \to \pm 3,\ \ 9 - x^2 \to 0$
Now $\ln 0$ is undefined, but as $\theta \to 0,\ \ \ln\theta \to -\infty$.
$\therefore$ as $x \to \pm 3,\ \ f(x) \to -\infty$
$\therefore$ $x = \pm 3$ are the vertical asymptotes.

d Shaded area $= \int_{-1}^{0} x^2 \ln(9 - x^2)\,dx$

$\approx 0.709\ \text{units}^2\quad$ {technology}

e Volume $= \pi\int_{-1}^{0}\left[x^2\ln(9 - x^2)\right]^2\,dx$

$\approx 2.81\ \text{units}^3$

9 **a** $2r + 3 = 123\quad$ {max height}

$\therefore\ 2r = 120$

$\therefore\ r = 60$

So, the radius is 60 m.

b The amplitude of the function is the radius of the wheel, so $a = 60$.
The average height corresponds to the principal axis, and this is the height of the centre of the wheel.
$\therefore\ d = 63$.

c The period $= \dfrac{2\pi}{b} = 20$

$\therefore\ b = \frac{2\pi}{20}$

$\therefore\ b = \frac{\pi}{10}$

d When $t = 0$, $H(t)$ is at its minimum.

$\therefore\ 60\sin\left(\frac{\pi}{10}(0-c)\right) + 63 = 3$

$\therefore\ \sin\left(-\frac{c\pi}{10}\right) = -1$

$\therefore\ -\frac{c\pi}{10} = -\frac{\pi}{2}$

$\therefore\ c = 5$

e Thus $H(t) = 60\sin\left(\frac{\pi}{10}(t-5)\right) + 63$

$\therefore\ H(8) = 60\sin\left(\frac{3\pi}{10}\right) + 63$

$\therefore\ H(8) \approx 111.5$

The seat is 111.5 m above the ground.

f When $H(t) = 100$,

$60\sin\left(\frac{\pi}{10}(t-5)\right) + 63 = 100$

The first positive solution for t is

$t \approx 7.115$ {technology}

The seat is 100 m above the ground about 7 min 7 s after the start.

10 **a** $y = 0$ appears to be the only asymptote.

b When $y = 0$, $\frac{1}{2}e^{-\frac{x}{2}}\left(4x - x^2\right) = 0$

$\therefore\ 4x - x^2 = 0$

$\therefore\ x(4-x) = 0$

$\therefore\ x = 0$ or 4

The x-intercepts are 0 and 4.

c Using technology:

A is a local maximum at (1.17, 0.922).

C is a local minimum at (6.83, −0.318).

d If $y = x^2e^{-\frac{x}{2}}$, $\frac{dy}{dx} = 2xe^{-\frac{x}{2}} + x^2e^{-\frac{x}{2}}\left(-\frac{1}{2}\right)$

$= \frac{1}{2}e^{-\frac{x}{2}}\left(4x - x^2\right)$

e From **d**, $\int \frac{1}{2}e^{-\frac{x}{2}}\left(4x-x^2\right)\,dx = x^2e^{-\frac{x}{2}} + c_1$

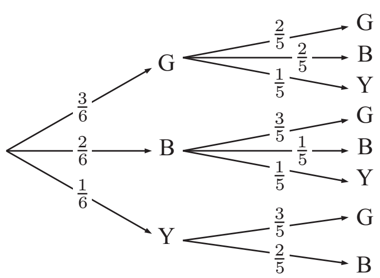
$\therefore\ \int e^{-\frac{x}{2}}\left(4x-x^2\right)\,dx = 2x^2e^{-\frac{x}{2}} + c$

SOLUTIONS TO TRIAL EXAMINATION 2

Paper 1 - No calculators

Section A

1 **a**



b P(G second) = P(GG or BG or YG)

$= \left(\frac{3}{6}\right)\left(\frac{2}{5}\right) + \left(\frac{2}{6}\right)\left(\frac{3}{5}\right) + \left(\frac{1}{6}\right)\left(\frac{3}{5}\right)$

$= \frac{1}{2}$

c $\text{P(B first} \mid \text{G second)} = \frac{\text{P(B first and G second)}}{\text{P(G second)}}$

$= \frac{\frac{6}{30}}{\frac{1}{2}}$

$= \frac{2}{5}$

2 **a** $(f \circ g)(x) = f(g(x))$

$= 3^{g(x)}$

$= 3^{x+1}$

b The inverse of $y = x + 1$ is $x = y + 1$

which is $y = x - 1$

$\therefore\ g^{-1}(x) = x - 1$

$(g^{-1} \circ f)(x) = g^{-1}(f(x))$

$= g^{-1}(3^x)$

$= 3^x - 1$

c $3^x - 1 = 8$

$\therefore\ 3^x = 9$

$\therefore\ x = 2$

3 **a** Let $u = x^2$

$\therefore\ du = 2x\,dx$

$\int xe^{-x^2}\,dx = \int \frac{1}{2}e^{-x^2}(2x)\,dx$

$= \int \frac{1}{2}e^{-u}\,du$

$= -\frac{1}{2}e^{-u} + c$

$= -\frac{1}{2}e^{-x^2} + c$

b $\int_0^m xe^{-x^2}\,dx = \frac{e-1}{2e}$

$\therefore\ \left[-\frac{1}{2}e^{-x^2}\right]_0^m = \frac{e-1}{2e}$

$\therefore\ -\frac{1}{2}e^{-m^2} + \frac{1}{2} = \frac{1}{2} - \frac{1}{2}e^{-1}$

$\therefore\ e^{-m^2} = e^{-1}$

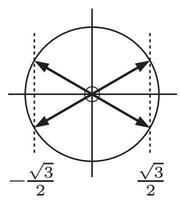
$\therefore\ m^2 = 1$

$\therefore\ m = 1$ {as $m > 0$}

4 **a** $4\cos^2 x = 3$

$\therefore\ \cos^2 x = \frac{3}{4}$

$\therefore\ \cos x = \pm\frac{\sqrt{3}}{2}$



But $\frac{\pi}{2} < x < \pi$, so $x = \frac{5\pi}{6}$

b **i** $\tan x = \tan\left(\frac{5\pi}{6}\right)$

$= -\frac{1}{\sqrt{3}}$

ii $\sin 2x = \sin\left(\frac{5\pi}{3}\right)$

$= -\frac{\sqrt{3}}{2}$

5 **a** **i** Since 12, a, 3, ... is a geometric sequence,

$\frac{a}{12} = \frac{3}{a}$

$\therefore\ a^2 = 36$

$\therefore\ a = 6$ {as $a > 0$}

ii The common ratio $r = \frac{a}{12} = \frac{1}{2}$.

iii $u_{12} = u_1r^{11}$

$= 12 \times \left(\frac{1}{2}\right)^{11}$

$= \frac{3 \times 2^2}{2^{11}}$

$= \frac{3}{2^9}$

$= \frac{3}{512}$

b $S_\infty = \frac{u_1}{1-r} = \frac{12}{\frac{1}{2}} = 24$