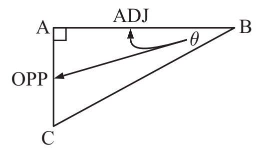
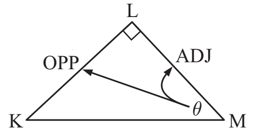
Chapter 12
TRIGONOMETRY

EXERCISE 12A

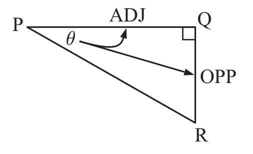
1 **a** **i** The hypotenuse is [BC].
ii The side opposite θ is [AC].
iii The side adjacent to θ is [AB].



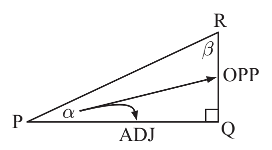
b **i** The hypotenuse is [KM].
ii The side opposite θ is [KL].
iii The side adjacent to θ is [LM].



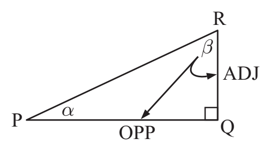
c **i** The hypotenuse is [PR].
ii The side opposite θ is [QR].
iii The side adjacent to θ is [PQ].



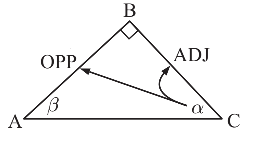
2 **a** **i** The hypotenuse is [PR].
ii The side opposite α is [QR].
iii The side adjacent to α is [PQ].



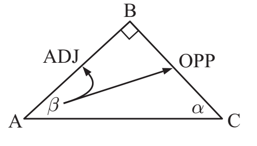
iv The side opposite β is [PQ].
v The side adjacent to β is [QR].



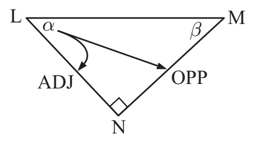
b **i** The hypotenuse is [AC].
ii The side opposite α is [AB].
iii The side adjacent to α is [BC].



iv The side opposite β is [BC].
v The side adjacent to β is [AB].

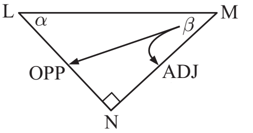


c **i** The hypotenuse is [LM].
ii The side opposite α is [MN].
iii The side adjacent to α is [LN].



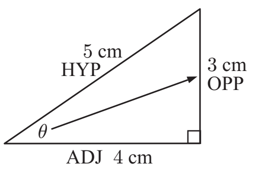
iv The side opposite β is [LN].

v The side adjacent to β is [MN].



EXERCISE 12B

1 **a**

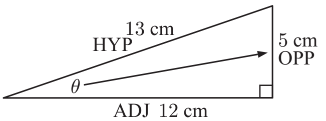


i $\sin\theta = \frac{\text{OPP}}{\text{HYP}} = \frac{3}{5}$

ii $\cos\theta = \frac{\text{ADJ}}{\text{HYP}} = \frac{4}{5}$

iii $\tan\theta = \frac{\text{OPP}}{\text{ADJ}} = \frac{3}{4}$

b

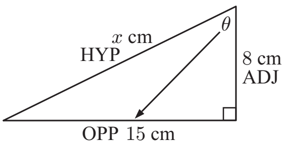


i $\sin\theta = \frac{\text{OPP}}{\text{HYP}} = \frac{5}{13}$

ii $\cos\theta = \frac{\text{ADJ}}{\text{HYP}} = \frac{12}{13}$

iii $\tan\theta = \frac{\text{OPP}}{\text{ADJ}} = \frac{5}{12}$

c



Let the hypotenuse have length x cm.

$x^2 = 8^2 + 15^2$ {Pythagoras}

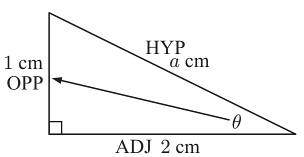
$\therefore\ x^2 = 289$

$\therefore\ x = \sqrt{289} = 17$ {as $x > 0$}

$\therefore$ the hypotenuse is 17 cm long.

i $\sin\theta = \frac{\text{OPP}}{\text{HYP}} = \frac{15}{17}$ **ii** $\cos\theta = \frac{\text{ADJ}}{\text{HYP}} = \frac{8}{17}$ **iii** $\tan\theta = \frac{\text{OPP}}{\text{ADJ}} = \frac{15}{8}$

d



Let the hypotenuse have length a cm.

$a^2 = 1^2 + 2^2$ {Pythagoras}

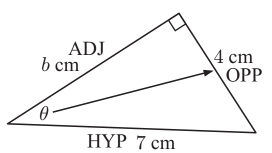
$\therefore\ a^2 = 5$

$\therefore\ a = \sqrt{5}$ {as $a > 0$}

$\therefore$ the hypotenuse is $\sqrt{5}$ cm long.

i $\sin\theta = \frac{\text{OPP}}{\text{HYP}} = \frac{1}{\sqrt{5}}$ **ii** $\cos\theta = \frac{\text{ADJ}}{\text{HYP}} = \frac{2}{\sqrt{5}}$ **iii** $\tan\theta = \frac{\text{OPP}}{\text{ADJ}} = \frac{1}{2}$

e



Let the side adjacent to θ have length b cm.

$b^2 + 4^2 = 7^2$ {Pythagoras}

$\therefore\ b^2 + 16 = 49$

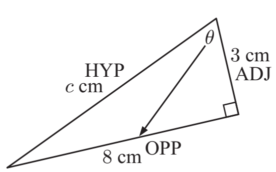
$\therefore\ b^2 = 33$

$\therefore\ b = \sqrt{33}$ {as $b > 0$}

$\therefore$ the side adjacent to θ is $\sqrt{33}$ cm long.

i $\sin\theta = \frac{\text{OPP}}{\text{HYP}} = \frac{4}{7}$ **ii** $\cos\theta = \frac{\text{ADJ}}{\text{HYP}} = \frac{\sqrt{33}}{7}$ **iii** $\tan\theta = \frac{\text{OPP}}{\text{ADJ}} = \frac{4}{\sqrt{33}}$

f



Let the hypotenuse have length c cm.

$c^2 = 3^2 + 8^2$ {Pythagoras}

$\therefore\ c^2 = 73$

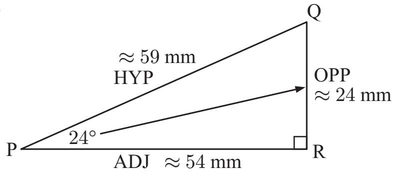
$\therefore\ c = \sqrt{73}$ {as $c > 0$}

$\therefore$ the hypotenuse is $\sqrt{73}$ cm long.

i $\sin\theta = \frac{\text{OPP}}{\text{HYP}} = \frac{8}{\sqrt{73}}$ **ii** $\cos\theta = \frac{\text{ADJ}}{\text{HYP}} = \frac{3}{\sqrt{73}}$ **iii** $\tan\theta = \frac{\text{OPP}}{\text{ADJ}} = \frac{8}{3}$

2 **a** PQ $\approx$ 59 mm, PR $\approx$ 54 mm, QR $\approx$ 24 mm

b



i $\sin 24^\circ = \dfrac{\text{OPP}}{\text{HYP}} \approx \dfrac{24 \text{ mm}}{59 \text{ mm}} \approx 0.407$

ii $\cos 24^\circ = \dfrac{\text{ADJ}}{\text{HYP}} \approx \dfrac{54 \text{ mm}}{59 \text{ mm}} \approx 0.915$

iii $\tan 24^\circ = \dfrac{\text{OPP}}{\text{ADJ}} \approx \dfrac{24 \text{ mm}}{54 \text{ mm}} \approx 0.444$

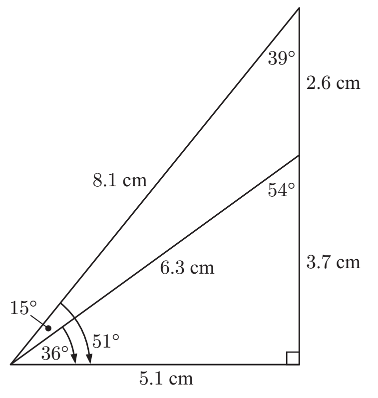
c **i** $\sin 24^\circ \approx 0.407$ **ii** $\cos 24^\circ \approx 0.914$ **iii** $\tan 24^\circ \approx 0.445$

3 **a** **i** $\sin 36^\circ = \dfrac{\text{OPP}}{\text{HYP}} = \dfrac{3.7 \text{ cm}}{6.3 \text{ cm}} \approx 0.587$

ii $\cos 51^\circ = \dfrac{\text{ADJ}}{\text{HYP}} = \dfrac{5.1 \text{ cm}}{8.1 \text{ cm}} \approx 0.630$

iii $\tan 54^\circ = \dfrac{\text{OPP}}{\text{ADJ}} = \dfrac{5.1 \text{ cm}}{3.7 \text{ cm}} \approx 1.378$

iv $\cos 39^\circ = \dfrac{\text{ADJ}}{\text{HYP}} = \dfrac{(2.6 + 3.7) \text{ cm}}{8.1 \text{ cm}} \approx 0.778$



b **i** $\sin 36^\circ \approx 0.588$

ii $\cos 51^\circ \approx 0.629$

iii $\tan 54^\circ \approx 1.376$

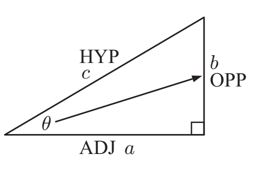
iv $\cos 39^\circ \approx 0.777$

4 **a** **i** $\cos^2 16^\circ \approx 0.9240$ **ii** $\sin^2 16^\circ \approx 0.0760$ **iii** $\cos^2 16^\circ + \sin^2 16^\circ = 1$

iv $\cos^2 65^\circ \approx 0.1786$ **v** $\sin^2 65^\circ \approx 0.8214$ **vi** $\cos^2 65^\circ + \sin^2 65^\circ = 1$

b $\cos^2 \theta + \sin^2 \theta = 1$ for any value of θ.

c



$\cos\theta = \dfrac{\text{ADJ}}{\text{HYP}} = \dfrac{a}{c}$ $\sin\theta = \dfrac{\text{OPP}}{\text{HYP}} = \dfrac{b}{c}$

$\therefore \cos^2\theta = \dfrac{a^2}{c^2}$ $\therefore \sin^2\theta = \dfrac{b^2}{c^2}$

Also, $c^2 = a^2 + b^2$ {Pythagoras}

Now, $\cos^2\theta + \sin^2\theta = \dfrac{a^2}{c^2} + \dfrac{b^2}{c^2}$

$= \dfrac{a^2 + b^2}{c^2}$

$= \dfrac{c^2}{c^2} = 1$ as required

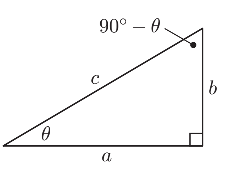
5 **a** **i** $\tan 70^\circ \times \tan 20^\circ = 1$ **ii** $\tan 63^\circ \times \tan 27^\circ = 1$ **iii** $\tan 49^\circ \times \tan 41^\circ = 1$

b $\tan\theta \times \tan(90^\circ - \theta) = 1$ for any value of θ.

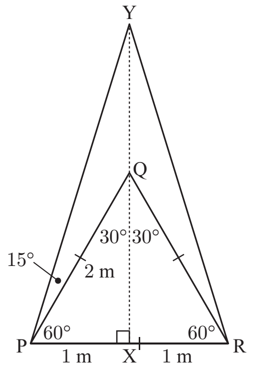
c $\tan\theta = \dfrac{\text{OPP}}{\text{ADJ}} = \dfrac{b}{a}$ $\tan(90^\circ - \theta) = \dfrac{\text{OPP}}{\text{ADJ}} = \dfrac{a}{b}$

$\tan\theta \times \tan(90^\circ - \theta) = \dfrac{b}{a} \times \dfrac{a}{b}$

$= 1$ as required



6



$\widehat{PQR} = \widehat{QRP} = \widehat{RPQ} = 60°$ {as $\triangle PQR$ is equilateral}

$PR = PX + RX = 2$ m

$\therefore$ $PX = RX = 1$ m {isosceles triangle theorem}

Also, $\widehat{PQX} = \widehat{XQR} = 30°$ {isosceles triangle theorem}

In $\triangle PQX$:

a $\cos 60° = \frac{ADJ}{HYP} = \frac{1}{2}$

b $QX^2 + PX^2 = PQ^2$ {Pythagoras}

$\therefore$ $QX^2 + 1^2 = 2^2$

$\therefore$ $QX^2 = 4 - 1 = 3$

$\therefore$ $QX = \sqrt{3}$ {as $QX > 0$}

$\sin 60° = \frac{OPP}{HYP} = \frac{QX}{PQ} = \frac{\sqrt{3}}{2}$

c $\tan 60° = \frac{OPP}{ADJ} = \frac{QX}{PX} = \frac{\sqrt{3}}{1} = \sqrt{3}$

d $\cos 30° = \frac{ADJ}{HYP} = \frac{QX}{PQ} = \frac{\sqrt{3}}{2}$

e $\sin 30° = \frac{OPP}{HYP} = \frac{PX}{PQ} = \frac{1}{2}$

f $\tan 30° = \frac{OPP}{ADJ} = \frac{PX}{QX} = \frac{1}{\sqrt{3}}$

Now, $\widehat{YPX} = 15° + 60° = 75°$

$\widehat{PQY} = 180° - 30°$ {angles on a line}

$= 150°$

$\widehat{PYQ} = 180° - 15° - 150°$ {angles in a triangle}

$= 15°$

$\therefore$ $\triangle QPY$ is isosceles {base angles equal}

$\therefore$ $YQ = PQ = 2$ m

In $\triangle YXP$:

g $\tan 75° = \frac{OPP}{ADJ}$

$= \frac{YQ + QX}{PX}$

$= \frac{2 + \sqrt{3}}{1}$

$= \sqrt{3} + 2$

h $\tan 15° = \frac{OPP}{ADJ}$

$= \frac{PX}{XY}$

$= \frac{1}{2 + \sqrt{3}}$

$= \frac{1}{2 + \sqrt{3}} \times \left(\frac{2 - \sqrt{3}}{2 - \sqrt{3}}\right)$

$= \frac{2 - \sqrt{3}}{4 - 3}$

$= 2 - \sqrt{3}$

i $PY^2 = PX^2 + XY^2$ {Pythagoras}

$= 1^2 + (2 + \sqrt{3})^2$

$= 1 + 4 + 4\sqrt{3} + 3$

$= 8 + 4\sqrt{3}$

$\therefore$ $PY = \sqrt{8 + 4\sqrt{3}}$ {as $PY > 0$}

$\cos 75° = \frac{ADJ}{HYP} = \frac{PX}{PY} = \frac{1}{\sqrt{8 + 4\sqrt{3}}}$

$\therefore$ $\cos^2 75° = \left(\frac{1}{\sqrt{8 + 4\sqrt{3}}}\right)^2$

$= \frac{1}{8 + 4\sqrt{3}}$

$= \frac{1}{8 + 4\sqrt{3}} \times \left(\frac{8 - 4\sqrt{3}}{8 - 4\sqrt{3}}\right)$

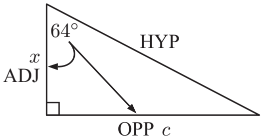
$= \frac{8 - 4\sqrt{3}}{64 - 16 \times 3}$

$= \frac{8 - 4\sqrt{3}}{16}$

$= \frac{2 - \sqrt{3}}{4}$

EXERCISE 12C

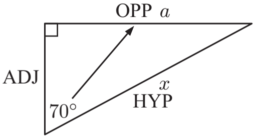
1 **a**



The relevant sides are OPP and ADJ, so we use the *tangent* ratio.

$\tan 64° = \frac{c}{x}$ $\{\tan\theta = \frac{\text{OPP}}{\text{ADJ}}\}$

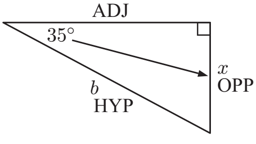
b



The relevant sides are OPP and HYP, so we use the *sine* ratio.

$\sin 70° = \frac{a}{x}$ $\{\sin\theta = \frac{\text{OPP}}{\text{HYP}}\}$

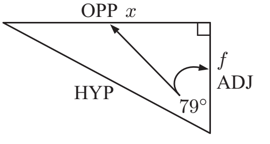
c



The relevant sides are OPP and HYP, so we use the *sine* ratio.

$\sin 35° = \frac{x}{b}$ $\{\sin\theta = \frac{\text{OPP}}{\text{HYP}}\}$

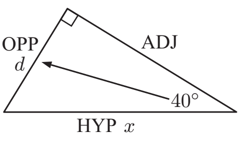
d



The relevant sides are OPP and ADJ, so we use the *tangent* ratio.

$\tan 79° = \frac{x}{f}$ $\{\tan\theta = \frac{\text{OPP}}{\text{ADJ}}\}$

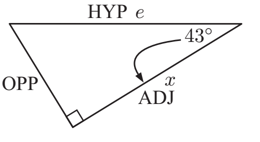
e



The relevant sides are OPP and HYP, so we use the *sine* ratio.

$\sin 40° = \frac{d}{x}$ $\{\sin\theta = \frac{\text{OPP}}{\text{HYP}}\}$

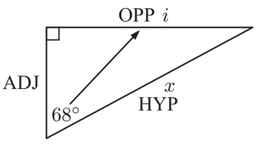
f



The relevant sides are ADJ and HYP, so we use the *cosine* ratio.

$\cos 43° = \frac{x}{e}$ $\{\cos\theta = \frac{\text{ADJ}}{\text{HYP}}\}$

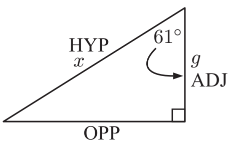
g



The relevant sides are OPP and HYP, so we use the *sine* ratio.

$\sin 68° = \frac{i}{x}$ $\{\sin\theta = \frac{\text{OPP}}{\text{HYP}}\}$

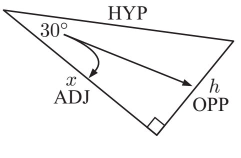
h



The relevant sides are ADJ and HYP, so we use the *cosine* ratio.

$\cos 61° = \frac{g}{x}$ $\{\cos\theta = \frac{\text{ADJ}}{\text{HYP}}\}$

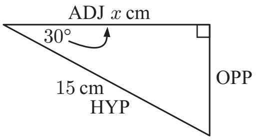
i



The relevant sides are OPP and ADJ, so we use the *tangent* ratio.

$\tan 30° = \frac{h}{x}$ $\{\tan\theta = \frac{\text{OPP}}{\text{ADJ}}\}$

2 **a**



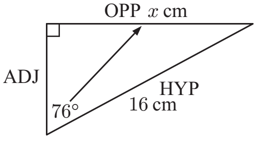
The relevant sides are ADJ and HYP, so we use the *cosine* ratio.

$$\cos 30^\circ = \frac{x}{15} \qquad \{\cos\theta = \frac{\text{ADJ}}{\text{HYP}}\}$$

$$\therefore \quad \cos 30^\circ \times 15 = x \qquad \{\text{multiplying both sides by } 15\}$$

$$\therefore \quad x \approx 12.99 \qquad \{\text{calculator}\}$$

b



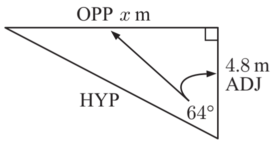
The relevant sides are OPP and HYP, so we use the *sine* ratio.

$$\sin 76^\circ = \frac{x}{16} \qquad \{\sin\theta = \frac{\text{OPP}}{\text{HYP}}\}$$

$$\therefore \quad \sin 76^\circ \times 16 = x \qquad \{\text{multiplying both sides by } 16\}$$

$$\therefore \quad x \approx 15.52 \qquad \{\text{calculator}\}$$

c



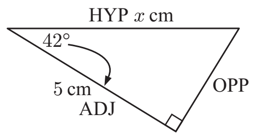
The relevant sides are OPP and ADJ, so we use the *tangent* ratio.

$$\tan 64^\circ = \frac{x}{4.8} \qquad \{\tan\theta = \frac{\text{OPP}}{\text{ADJ}}\}$$

$$\therefore \quad \tan 64^\circ \times 4.8 = x \qquad \{\text{multiplying both sides by } 4.8\}$$

$$\therefore \quad x \approx 9.84 \qquad \{\text{calculator}\}$$

d



The relevant sides are ADJ and HYP, so we use the *cosine* ratio.

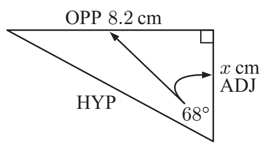
$$\cos 42^\circ = \frac{5}{x} \qquad \{\cos\theta = \frac{\text{ADJ}}{\text{HYP}}\}$$

$$\therefore \quad x \times \cos 42^\circ = 5 \qquad \{\text{multiplying both sides by } x\}$$

$$\therefore \quad x = \frac{5}{\cos 42^\circ} \qquad \{\text{dividing both sides by } \cos 42^\circ\}$$

$$\therefore \quad x \approx 6.73 \qquad \{\text{calculator}\}$$

e



The relevant sides are OPP and ADJ, so we use the *tangent* ratio.

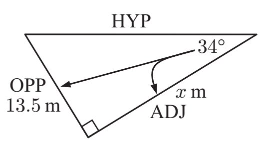
$$\tan 68^\circ = \frac{8.2}{x} \qquad \{\tan\theta = \frac{\text{OPP}}{\text{ADJ}}\}$$

$$\therefore \quad x \times \tan 68^\circ = 8.2 \qquad \{\text{multiplying both sides by } x\}$$

$$\therefore \quad x = \frac{8.2}{\tan 68^\circ} \qquad \{\text{dividing both sides by } \tan 68^\circ\}$$

$$\therefore \quad x \approx 3.31 \qquad \{\text{calculator}\}$$

f



The relevant sides are OPP and ADJ, so we use the *tangent* ratio.

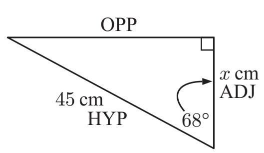
$$\tan 34^\circ = \frac{13.5}{x} \qquad \{\tan\theta = \frac{\text{OPP}}{\text{ADJ}}\}$$

$$\therefore \quad x \times \tan 34^\circ = 13.5 \qquad \{\text{multiplying both sides by } x\}$$

$$\therefore \quad x = \frac{13.5}{\tan 34^\circ} \qquad \{\text{dividing both sides by } \tan 34^\circ\}$$

$$\therefore \quad x \approx 20.01 \qquad \{\text{calculator}\}$$

g



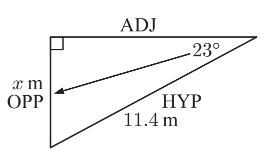
The relevant sides are ADJ and HYP, so we use the *cosine* ratio.

$$\cos 68^\circ = \frac{x}{45} \qquad \{\cos\theta = \frac{\text{ADJ}}{\text{HYP}}\}$$

$$\therefore \quad \cos 68^\circ \times 45 = x \qquad \{\text{multiplying both sides by } 45\}$$

$$\therefore \quad x \approx 16.86 \qquad \{\text{calculator}\}$$

h



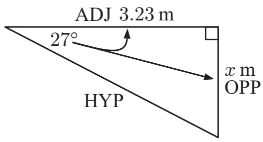
The relevant sides are OPP and HYP, so we use the *sine* ratio.

$$\sin 23^\circ = \frac{x}{11.4} \qquad \{\sin\theta = \frac{\text{OPP}}{\text{HYP}}\}$$

$$\therefore \quad \sin 23^\circ \times 11.4 = x \qquad \{\text{multiplying both sides by } 11.4\}$$

$$\therefore \quad x \approx 4.45 \qquad \{\text{calculator}\}$$

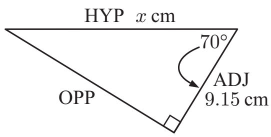
i



The relevant sides are OPP and ADJ, so we use the *tangent* ratio.

$$\tan 27^\circ = \frac{x}{3.23} \qquad \{\tan\theta = \frac{\text{OPP}}{\text{ADJ}}\}$$
$$\therefore\ \tan 27^\circ \times 3.23 = x \qquad \{\text{multiplying both sides by } 3.23\}$$
$$\therefore\ x \approx 1.65 \qquad \{\text{calculator}\}$$

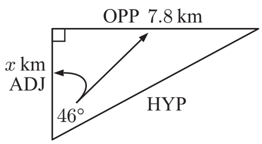
j



The relevant sides are ADJ and HYP, so we use the *cosine* ratio.

$$\cos 70^\circ = \frac{9.15}{x} \qquad \{\cos\theta = \frac{\text{ADJ}}{\text{HYP}}\}$$
$$\therefore\ x \times \cos 70^\circ = 9.15 \qquad \{\text{multiplying both sides by } x\}$$
$$\therefore\ x = \frac{9.15}{\cos 70^\circ} \qquad \{\text{dividing both sides by } \cos 70^\circ\}$$
$$\therefore\ x \approx 26.75 \qquad \{\text{calculator}\}$$

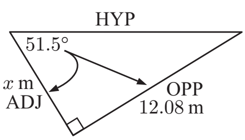
k



The relevant sides are OPP and ADJ, so we use the *tangent* ratio.

$$\tan 46^\circ = \frac{7.8}{x} \qquad \{\tan\theta = \frac{\text{OPP}}{\text{ADJ}}\}$$
$$\therefore\ x \times \tan 46^\circ = 7.8 \qquad \{\text{multiplying both sides by } x\}$$
$$\therefore\ x = \frac{7.8}{\tan 46^\circ} \qquad \{\text{dividing both sides by } \tan 46^\circ\}$$
$$\therefore\ x \approx 7.53 \qquad \{\text{calculator}\}$$

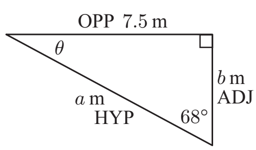
l



The relevant sides are OPP and ADJ, so we use the *tangent* ratio.

$$\tan 51.5^\circ = \frac{12.08}{x} \qquad \{\tan\theta = \frac{\text{OPP}}{\text{ADJ}}\}$$
$$\therefore\ x \times \tan 51.5^\circ = 12.08 \qquad \{\text{multiplying both sides by } x\}$$
$$\therefore\ x = \frac{12.08}{\tan 51.5^\circ} \qquad \{\text{dividing both sides by } \tan 51.5^\circ\}$$
$$\therefore\ x \approx 9.61 \qquad \{\text{calculator}\}$$

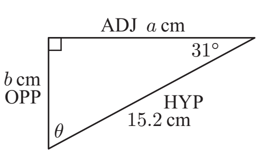
3 **a**



$$\theta + 90^\circ + 68^\circ = 180^\circ \qquad \{\text{angles in a triangle}\}$$
$$\therefore\ \theta + 158^\circ = 180^\circ$$
$$\therefore\ \theta = 22^\circ$$

$$\sin 68^\circ = \frac{7.5}{a} \qquad \{\text{the side opposite } 68^\circ \text{ is 7.5 m and the hypotenuse is } a \text{ m}\}$$
$$\therefore\ a \times \sin 68^\circ = 7.5 \qquad \{\text{multiplying both sides by } a\}$$
$$\therefore\ a = \frac{7.5}{\sin 68^\circ} \qquad \{\text{dividing both sides by } \sin 68^\circ\}$$
$$\therefore\ a \approx 8.09 \qquad \{\text{calculator}\}$$
$$\tan 68^\circ = \frac{7.5}{b} \qquad \{\text{the side opposite } 68^\circ \text{ is 7.5 m and the side adjacent to } 68^\circ \text{ is } b \text{ m}\}$$
$$\therefore\ b \times \tan 68^\circ = 7.5 \qquad \{\text{multiplying both sides by } b\}$$
$$\therefore\ b = \frac{7.5}{\tan 68^\circ} \qquad \{\text{dividing both sides by } \tan 68^\circ\}$$
$$\therefore\ b \approx 3.03 \qquad \{\text{calculator}\}$$

b



$$\theta + 90^\circ + 31^\circ = 180^\circ \qquad \{\text{angles in a triangle}\}$$
$$\therefore\ \theta + 121^\circ = 180^\circ$$
$$\therefore\ \theta = 59^\circ$$

$\cos 31° = \frac{a}{15.2}$ {the side adjacent to 31° is a cm and the hypotenuse is 15.2 cm}

$\therefore\ 15.2 \times \cos 31° = a$ {multiplying both sides by 15.2}

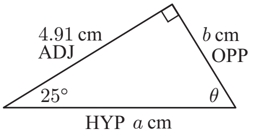
$\therefore\ a \approx 13.03$ {calculator}

$\sin 31° = \frac{b}{15.2}$ {the side opposite 31° is b cm and the hypotenuse is 15.2 cm}

$\therefore\ 15.2 \times \sin 31° = b$ {multiplying both sides by 15.2}

$\therefore\ b \approx 7.83$ {calculator}

c



$\theta + 90° + 25° = 180°$ {angles in a triangle}

$\therefore\ \theta + 115° = 180°$

$\therefore\ \theta = 65°$

$\cos 25° = \frac{4.91}{a}$ {the side adjacent to 25° is 4.91 cm and the hypotenuse is a cm}

$\therefore\ a \times \cos 25° = 4.91$ {multiplying both sides by a}

$\therefore\ a = \frac{4.91}{\cos 25°}$ {dividing both sides by $\cos 25°$}

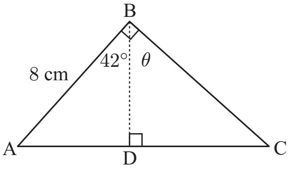
$\therefore\ a \approx 5.42$ {calculator}

$\tan 25° = \frac{b}{4.91}$ {the side opposite 25° is b cm and the side adjacent to 25° is 4.91 cm}

$\therefore\ 4.91 \times \tan 25° = b$ {multiplying both sides by 4.91}

$\therefore\ b \approx 2.29$ {calculator}

4



$\theta + 42° = 90°$

$\therefore\ \theta = 48°$

In $\triangle$ABD, $\cos 42° = \frac{\text{BD}}{8}$ {the side adjacent to 42° is BD and the hypotenuse is 8 cm}

$\therefore\ 8 \times \cos 42° = \text{BD}$ {multiplying both sides by 8}

$\therefore\ \text{BD} \approx 5.9451$ cm

Also, $\sin 42° = \frac{\text{AD}}{8}$ {the side opposite 42° is AD and the hypotenuse is 8 cm}

$\therefore\ 8 \times \sin 42° = \text{AD}$ {multiplying both sides by 8}

$\therefore\ \text{AD} \approx 5.3530$ cm

In $\triangle$BCD, $\cos 48° \approx \frac{5.9451}{\text{BC}}$ {the side adjacent to 48° is ≈ 5.9451 and the hypotenuse is BC}

$\therefore\ \text{BC} \times \cos 48° \approx 5.9451$ {multiplying both sides by BC}

$\therefore\ \text{BC} \approx \frac{5.9451}{\cos 48°}$ {dividing both sides by $\cos 48°$}

$\therefore\ \text{BC} \approx 8.8848$ cm

$\sin 48° \approx \frac{\text{CD}}{8.8848}$ {the side opposite 48° is CD and the hypotenuse is ≈ 8.8848 cm}

$\therefore\ 8.8848 \times \sin 48° \approx \text{CD}$ {multiplying both sides by ≈ 8.8848}

$\therefore\ \text{CD} \approx 6.6027$ cm

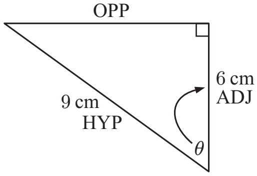
So, the perimeter of $\triangle$ABC $= \text{AB} + \text{BC} + \text{CD} + \text{AD}$

$\approx (8 + 8.8848 + 6.6027 + 5.3530)$ cm

≈ 28.8 cm

EXERCISE 12D

1 **a**

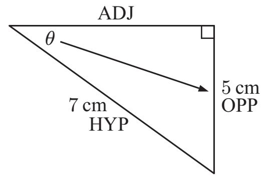


$\cos\theta = \frac{6}{9}$ $\{\cos\theta = \frac{\text{ADJ}}{\text{HYP}}\}$

$\therefore\ \theta = \cos^{-1}\left(\frac{6}{9}\right)$

$\therefore\ \theta \approx 48.2^\circ$ {calculator}

b

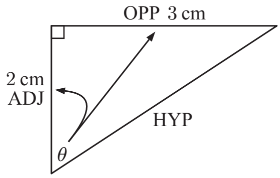


$\sin\theta = \frac{5}{7}$ $\{\sin\theta = \frac{\text{OPP}}{\text{HYP}}\}$

$\therefore\ \theta = \sin^{-1}\left(\frac{5}{7}\right)$

$\therefore\ \theta \approx 45.6^\circ$ {calculator}

c

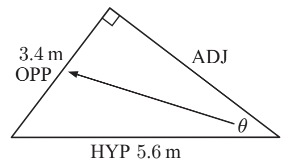


$\tan\theta = \frac{3}{2}$ $\{\tan\theta = \frac{\text{OPP}}{\text{ADJ}}\}$

$\therefore\ \theta = \tan^{-1}\left(\frac{3}{2}\right)$

$\therefore\ \theta \approx 56.3^\circ$ {calculator}

d

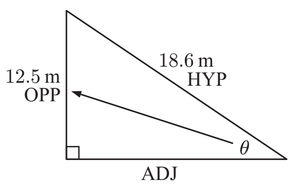


$\sin\theta = \frac{3.4}{5.6}$ $\{\sin\theta = \frac{\text{OPP}}{\text{HYP}}\}$

$\therefore\ \theta = \sin^{-1}\left(\frac{3.4}{5.6}\right)$

$\therefore\ \theta \approx 37.4^\circ$ {calculator}

e

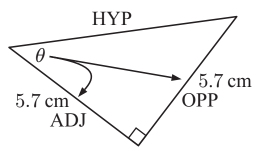


$\sin\theta = \frac{12.5}{18.6}$ $\{\sin\theta = \frac{\text{OPP}}{\text{HYP}}\}$

$\therefore\ \theta = \sin^{-1}\left(\frac{12.5}{18.6}\right)$

$\therefore\ \theta \approx 42.2^\circ$ {calculator}

f



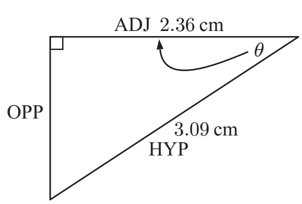
$\tan\theta = \frac{5.7}{5.7}$ $\{\tan\theta = \frac{\text{OPP}}{\text{ADJ}}\}$

$= 1$

$\therefore\ \theta = \tan^{-1}(1)$

$\therefore\ \theta = 45^\circ$ {calculator}

g

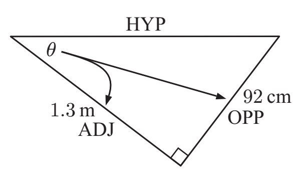


$\cos\theta = \frac{2.36}{3.09}$ $\{\cos\theta = \frac{\text{ADJ}}{\text{HYP}}\}$

$\therefore\ \theta = \cos^{-1}\left(\frac{2.36}{3.09}\right)$

$\therefore\ \theta \approx 40.2^\circ$ {calculator}

h



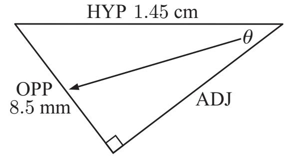
$\tan\theta = \frac{92 \text{ cm}}{1.3 \text{ m}}$ $\{\tan\theta = \frac{\text{OPP}}{\text{ADJ}}\}$

$\therefore\ \tan\theta = \frac{92}{130}$ $\{1.3 \text{ m} = 1.3 \times 100 = 130 \text{ cm}\}$

$\therefore\ \theta = \tan^{-1}\left(\frac{92}{130}\right)$

$\therefore\ \theta \approx 35.3^\circ$ {calculator}

i



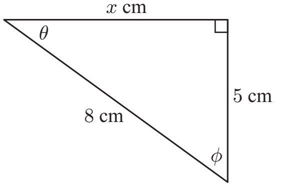
$\sin\theta = \dfrac{8.5 \text{ mm}}{1.45 \text{ cm}}$ $\{\sin\theta = \dfrac{\text{OPP}}{\text{HYP}}\}$

$\therefore\ \sin\theta = \dfrac{8.5}{14.5}$ $\{1.45 \text{ cm} = 1.45 \times 10 = 14.5 \text{ mm}\}$

$\therefore\ \theta = \sin^{-1}\left(\dfrac{8.5}{14.5}\right)$

$\therefore\ \theta \approx 35.9^\circ$ $\{\text{calculator}\}$

2 **a**



$\sin\theta = \frac{5}{8}$ $\{\sin\theta = \dfrac{\text{OPP}}{\text{HYP}}\}$

$\therefore\ \theta = \sin^{-1}(\frac{5}{8})$

$\therefore\ \theta \approx 38.7^\circ$ $\{\text{calculator}\}$

$\cos\phi = \frac{5}{8}$ $\{\cos\phi = \dfrac{\text{ADJ}}{\text{HYP}}\}$

$\therefore\ \phi = \cos^{-1}(\frac{5}{8})$

$\therefore\ \phi \approx 51.3^\circ$ $\{\text{calculator}\}$

So, $\cos\theta = \dfrac{x}{8}$ $\{\cos\theta = \dfrac{\text{ADJ}}{\text{HYP}}\}$

$\therefore\ \cos 38.6822^\circ \approx \dfrac{x}{8}$

$\therefore\ 8 \times \cos 38.6822^\circ \approx x$ {multiplying both sides by 8}

$\therefore\ x \approx 6.24$

Check:

$x^2 + 5^2 = 8^2$ {Pythagoras}

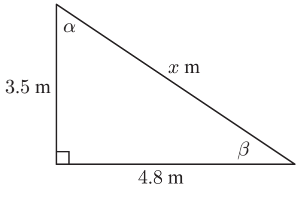
$\therefore\ x^2 + 25 = 64$

$\therefore\ x^2 = 39$

$\therefore\ x = \sqrt{39}$ $\{\text{as } x > 0\}$

≈ 6.24

b



$\tan\alpha = \dfrac{4.8}{3.5}$ $\{\tan\alpha = \dfrac{\text{OPP}}{\text{ADJ}}\}$

$\therefore\ \alpha = \tan^{-1}\left(\dfrac{4.8}{3.5}\right)$

$\therefore\ \alpha \approx 53.9^\circ$ $\{\text{calculator}\}$

$\tan\beta = \dfrac{3.5}{4.8}$ $\{\tan\beta = \dfrac{\text{OPP}}{\text{ADJ}}\}$

$\therefore\ \beta = \tan^{-1}\left(\dfrac{3.5}{4.8}\right)$

$\therefore\ \beta \approx 36.1^\circ$ $\{\text{calculator}\}$

So, $\cos\alpha = \dfrac{3.5}{x}$ $\{\cos\alpha = \dfrac{\text{ADJ}}{\text{HYP}}\}$

$\therefore\ \cos 53.9017^\circ \approx \dfrac{3.5}{x}$

$\therefore\ x \times \cos 53.9017^\circ \approx 3.5$ {multiplying both sides by x}

$\therefore\ x \approx \dfrac{3.5}{\cos 53.9017^\circ}$ {dividing both sides by $\cos 53.9017^\circ$}

$\therefore\ x \approx 5.94$

Check:

$x^2 = (3.5)^2 + (4.8)^2$ {Pythagoras}

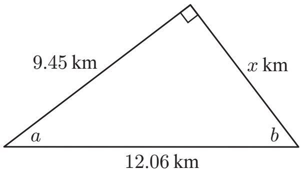
$\therefore\ x^2 = 12.25 + 23.04$

$\therefore\ x^2 = 35.29$

$\therefore\ x = \sqrt{35.29}$ $\{\text{as } x > 0\}$

$\therefore\ x \approx 5.94$

c



$\cos a = \frac{9.45}{12.06}$ $\{\cos a = \frac{\text{ADJ}}{\text{HYP}}\}$

$\therefore\ a = \cos^{-1}\left(\frac{9.45}{12.06}\right)$

$\therefore\ a \approx 38.4°$ {calculator}

$\sin b = \frac{9.45}{12.06}$ $\{\sin b = \frac{\text{OPP}}{\text{HYP}}\}$

$\therefore\ b = \sin^{-1}\left(\frac{9.45}{12.06}\right)$

$\therefore\ b \approx 51.6°$ {calculator}

So, $\sin a = \frac{x}{12.06}$ $\{\sin a = \frac{\text{OPP}}{\text{HYP}}\}$

$\therefore\ \sin 38.4103 \approx \frac{x}{12.06}$

$\therefore\ 12.06 \times \sin 38.4103 \approx x$ {multiplying both sides by 12.06}

$\therefore\ x \approx 7.49$

Check:

$x^2 + 9.45^2 = 12.06^2$ {Pythagoras}

$\therefore\ x^2 + 89.3025 = 145.4436$

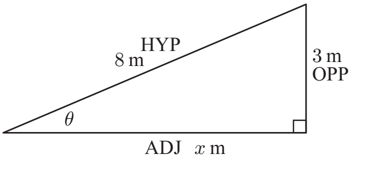
$\therefore\ x^2 = 56.1411$

$\therefore\ x = \sqrt{56.1411}$ $\{$as $x > 0\}$

$\therefore\ x \approx 7.49$

3 **a** **i** $\sin\theta = \frac{3}{8}$ $\{\sin\theta = \frac{\text{OPP}}{\text{HYP}}\}$

ii Let the side adjacent to θ have length x m.



$x^2 + 3^2 = 8^2$ {Pythagoras}

$\therefore\ x^2 + 9 = 64$

$\therefore\ x^2 = 55$

$\therefore\ x = \sqrt{55}$ $\{$as $x > 0\}$

$\therefore$ the side adjacent to θ is $\sqrt{55}$ m long.

$\cos\theta = \frac{\sqrt{55}}{8}$ $\{\cos\theta = \frac{\text{ADJ}}{\text{HYP}}\}$

iii $\tan\theta = \frac{3}{\sqrt{55}}$ $\{\tan\theta = \frac{\text{OPP}}{\text{ADJ}}\}$

b **i** $\sin\theta = \frac{3}{8}$

$\therefore\ \theta = \sin^{-1}\left(\frac{3}{8}\right)$

$\therefore\ \theta \approx 22.0°$

ii $\cos\theta = \frac{\sqrt{55}}{8}$

$\therefore\ \theta = \cos^{-1}\left(\frac{\sqrt{55}}{8}\right)$

$\therefore\ \theta \approx 22.0°$

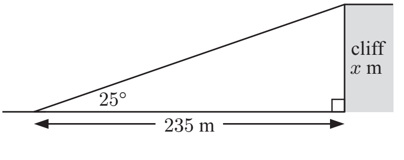
iii $\tan\theta = \frac{3}{\sqrt{55}}$

$\therefore\ \theta = \tan^{-1}\left(\frac{3}{\sqrt{55}}\right)$

$\therefore\ \theta \approx 22.0°$

EXERCISE 12E

1



Let the height of the cliff be x m.

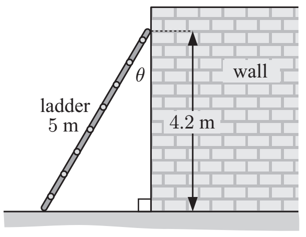
$\tan 25° = \frac{x}{235}$ $\{\tan\theta = \frac{\text{OPP}}{\text{ADJ}}\}$

$\therefore\ x = 235 \times \tan 25°$

$\therefore\ x \approx 109.6$ {calculator}

The cliff is about 110 m high.

2



Let the angle be θ.

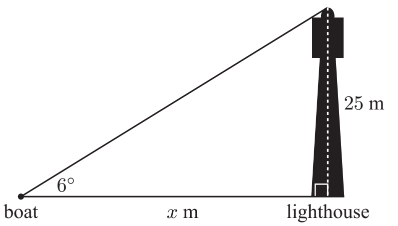
$\cos\theta = \frac{4.2}{5}$ $\{\cos\theta = \frac{\text{ADJ}}{\text{HYP}}\}$

$\therefore\ \theta = \cos^{-1}\left(\frac{4.2}{5}\right)$

$\therefore\ \theta \approx 32.9^\circ$ {calculator}

The ladder makes an angle of 32.9° with the wall.

3



Suppose the boat is x m from the lighthouse.

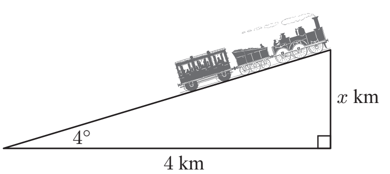
$\tan 6^\circ = \frac{25}{x}$ $\{\tan\theta = \frac{\text{OPP}}{\text{ADJ}}\}$

$\therefore\ x = \frac{25}{\tan 6^\circ}$

$\therefore\ x \approx 237.9$ {calculator}

The boat is about 238 m from the lighthouse.

4



Let the altitude be x km.

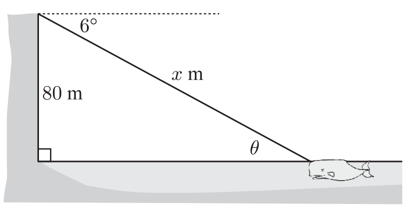
$\tan 4^\circ = \frac{x}{4}$ $\{\tan\theta = \frac{\text{OPP}}{\text{ADJ}}\}$

$\therefore\ x = 4 \times \tan 4^\circ$

$\therefore\ x \approx 0.280$ {calculator}

The train has gained an altitude of about 0.280 km or 280 m.

5



Suppose the whale is x m from the observer.

$\theta = 6^\circ$ {alternate angles}

Now $\sin 6^\circ = \frac{80}{x}$ $\{\sin\theta = \frac{\text{OPP}}{\text{HYP}}\}$

$\therefore\ x = \frac{80}{\sin 6^\circ}$

$\therefore\ x \approx 765.3$ {calculator}

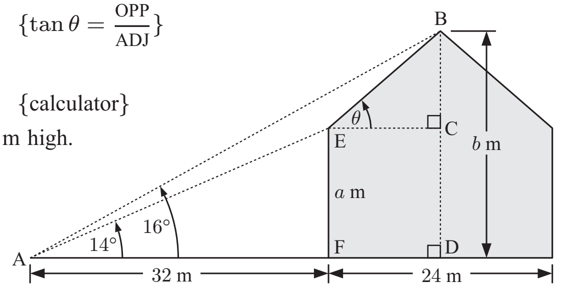
The whale is about 765 m from the observer.

6 **a** **i** In $\triangle$AEF, $\tan 14^\circ = \frac{a}{32}$ $\{\tan\theta = \frac{\text{OPP}}{\text{ADJ}}\}$

$\therefore\ 32 \times \tan 14^\circ = a$

$\therefore\ a \approx 7.978$ {calculator}

So, the side wall is about 7.98 m high.



ii In $\triangle$ABD, $\tan 16^\circ = \frac{b}{32+12}$ $\{\tan\theta = \frac{\text{OPP}}{\text{ADJ}}\}$

$= \frac{b}{44}$

$\therefore\ 44 \times \tan 16^\circ = b$

$\therefore\ b \approx 12.617$ {calculator}

So, the top of the roof is about 12.62 m high.

b In $\triangle$BCE, $\text{BC} = b - a$
$\approx 12.617 - 7.978$
≈ 4.639

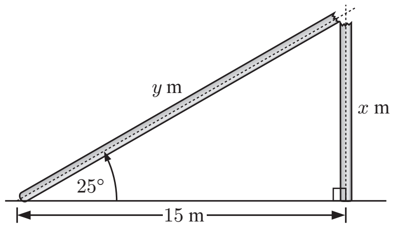
$\tan\theta \approx \frac{4.639}{12}$ $\quad \{\tan\theta = \frac{\text{OPP}}{\text{ADJ}}\}$

$\therefore \quad \theta \approx \tan^{-1}\left(\frac{4.639}{12}\right)$

$\therefore \quad \theta \approx 21.1°$ $\quad$ {calculator}

So, the pitch of the roof is approximately 21.1°.

7



Suppose the goal post snapped x m above the ground, and was $(x + y)$ m high in total.

$\tan 25° = \frac{x}{15}$ $\quad \{\tan\theta = \frac{\text{OPP}}{\text{ADJ}}\}$

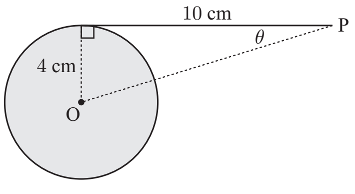
$\therefore \quad x = 15 \times \tan 25°$

$\cos 25° = \frac{15}{y}$ $\quad \{\cos\theta = \frac{\text{ADJ}}{\text{HYP}}\}$

$\therefore \quad y = \frac{15}{\cos 25°}$

The total height $= 15\tan 25° + \frac{15}{\cos 25°}$
≈ 23.5 m

8



Suppose the angle is θ.

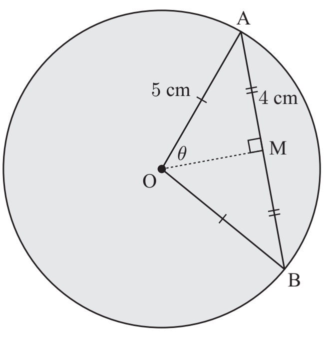
$\tan\theta = \frac{4}{10} = \frac{2}{5}$ $\quad \{\tan\theta = \frac{\text{OPP}}{\text{ADJ}}\}$

$\therefore \quad \theta = \tan^{-1}\left(\frac{2}{5}\right)$

$\therefore \quad \theta \approx 21.8°$ $\quad$ {calculator}

The angle between the tangent and [OP] is about 21.8°.

9



The perpendicular from the centre of a circle to a chord, bisects the chord. So, AM = BM = 4 cm.

In $\triangle$s OAM and OBM:

- OA = OB {equal radii}
- AM = BM
- OM is common.

$\therefore$ triangles OAM and OBM are congruent {SSS}

$\therefore \quad \text{A}\widehat{\text{O}}\text{M} = \text{B}\widehat{\text{O}}\text{M} = \theta$

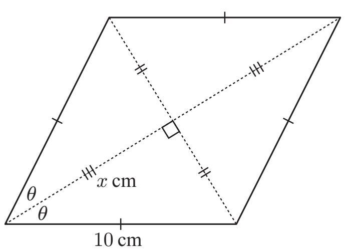
Now $\quad \sin\theta = \frac{4}{5}$ $\quad \{\sin\theta = \frac{\text{OPP}}{\text{HYP}}\}$

$\therefore \quad \theta = \sin^{-1}\left(\frac{4}{5}\right)$

$\therefore \quad \text{A}\widehat{\text{O}}\text{B} = 2\sin^{-1}\left(\frac{4}{5}\right)$

$\therefore \quad \text{A}\widehat{\text{O}}\text{B} \approx 106°$ $\quad$ {calculator}

10



The diagonals of a rhombus bisect each other at right angles. They also bisect the angles of the rhombus,

so $\quad \theta = \frac{1}{2} \times 76° = 38°$.

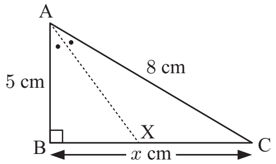
Let a half of the longer diagonal be x cm.

Now $\quad \cos 38° = \frac{x}{10}$ $\quad \{\cos\theta = \frac{\text{ADJ}}{\text{HYP}}\}$

$\therefore \quad x = 10 \times \cos 38°$

$\therefore$ the longer diagonal is $20\cos 38° \approx 15.8$ cm long.

11



Let BC be x cm.

$$x^2 + 5^2 = 8^2 \qquad \{\text{Pythagoras}\}$$
$$\therefore\ x^2 + 25 = 64$$
$$\therefore\ x^2 = 39$$
$$\therefore\ x = \sqrt{39} \qquad \{\text{as } x > 0\}$$

$$\cos B\widehat{A}C = \tfrac{5}{8} \qquad \left\{\cos\theta = \frac{\text{ADJ}}{\text{HYP}}\right\}$$
$$\therefore\ B\widehat{A}C = \cos^{-1}\left(\tfrac{5}{8}\right)$$
$$\therefore\ B\widehat{A}X = X\widehat{A}C = \tfrac{1}{2}\cos^{-1}\left(\tfrac{5}{8}\right) \approx 25.6589^\circ$$

In $\triangle$AXB, $\tan(25.6589^\circ) \approx \dfrac{\text{BX}}{5}$ $\qquad \left\{\tan\theta = \dfrac{\text{OPP}}{\text{ADJ}}\right\}$

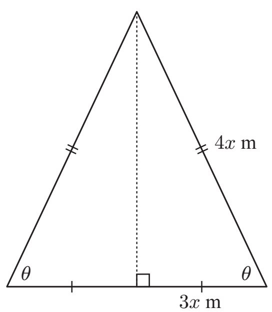
$$\therefore\ 5 \times \tan(25.6589^\circ) \approx \text{BX}$$
$$\therefore\ \text{BX} \approx 2.40 \text{ cm} \qquad \{\text{calculator}\}$$

Now, the midpoint of [BC] is $\dfrac{\sqrt{39}}{2} \approx 3.12$ cm from B and C.

So, X is not the midpoint of [BC].

The distance between X and the midpoint is approximately $3.12 - 2.40 \approx 0.72$ cm.

12



The altitude of an isosceles triangle bisects the base.

Suppose the base is $6x$ m long.

$\therefore$ half the base is $3x$ m

and the other sides are $\dfrac{2}{3} \times 6x = 4x$ m.

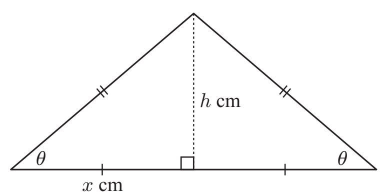
Let the base angles be θ.

Now $\cos\theta = \dfrac{3x}{4x} = \dfrac{3}{4}$ $\qquad \left\{\cos\theta = \dfrac{\text{ADJ}}{\text{HYP}}\right\}$

$$\therefore\ \theta = \cos^{-1}\left(\tfrac{3}{4}\right)$$
$$\therefore\ \theta \approx 41.4^\circ$$

The base angles are about 41.4°.

13 The altitude of an isosceles triangle bisects the base. Suppose half the base is x cm, the altitude is h cm, and the base angles are θ.



Now $\tan\theta = \dfrac{h}{x}$ $\qquad \left\{\tan\theta = \dfrac{\text{OPP}}{\text{ADJ}}\right\}$

$$\therefore\ h = x\tan\theta$$

$$\therefore\ \text{the triangle has area} = \tfrac{1}{2} \times 2x \times h = hx = x^2\tan\theta \ \text{ cm}^2$$

In this case, both triangles have base 28 cm. $\therefore\ 2x = 28$

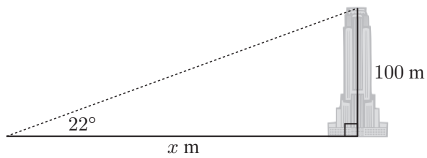
$$\therefore\ x = 14$$

The first triangle has base angles 24°, and area $\frac{1}{3}$ that of the second triangle.

$$\therefore\ 14^2 \times \tan 24^\circ = \tfrac{1}{3} \times 14^2 \times \tan\theta$$
$$\therefore\ \tan\theta = 3\tan 24^\circ$$
$$\therefore\ \theta = \tan^{-1}(3\tan 24^\circ)$$
$$\therefore\ \theta \approx 53.2^\circ$$

The base angles are about 53.2°.

14 **a**



Let the distance be x m.

$\tan 22° = \dfrac{100}{x}$ $\quad \{\tan\theta = \dfrac{\text{OPP}}{\text{ADJ}}\}$

$\therefore\ x = \dfrac{100}{\tan 22°}$

$\therefore\ x \approx 247.5$ $\quad$ {calculator}

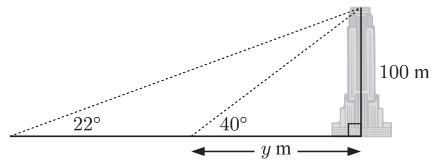
$\therefore$ the distance is about 248 m.

b Suppose the point is y m from the building.

$\tan 40° = \dfrac{100}{y}$ $\quad \{\tan\theta = \dfrac{\text{OPP}}{\text{ADJ}}\}$

$\therefore\ y = \dfrac{100}{\tan 40°}$

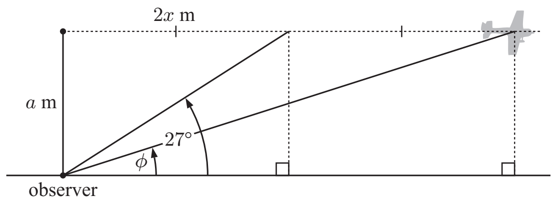
$\therefore\ y \approx 119.2$ $\quad$ {calculator}



$\therefore$ the point has moved

$\dfrac{100}{\tan 22°} - \dfrac{100}{\tan 40°} \approx 128$ m closer to the building.

15



We assume the plane is travelling with constant speed x m min^{-1}, and altitude a m.

$\therefore$ every 2 minutes the plane travels $2x$ m.

Now $\tan 27° = \dfrac{a}{2x}$ $\quad \{\tan\theta = \dfrac{\text{OPP}}{\text{ADJ}}\}$

$\therefore\ a = 2x\tan 27°$

Suppose the angle of elevation after 4 minutes is ϕ.

$\therefore\ \tan\phi = \dfrac{a}{4x}$ $\quad \{\tan\phi = \dfrac{\text{OPP}}{\text{ADJ}}\}$

So, $\tan\phi = \dfrac{2x\tan 27°}{4x}$

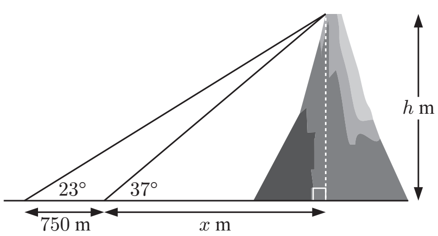
$\therefore\ \tan\phi = \frac{1}{2}\tan 27°$

$\therefore\ \phi = \tan^{-1}(\frac{1}{2}\tan 27°)$

$\therefore\ \phi \approx 14.29°$ $\quad$ {calculator}

The angle of elevation is about 14.3°.

16



Let the volcano be h m high.

Suppose the second measurement point is x m horizontally from its rim.

$\tan\theta = \dfrac{\text{OPP}}{\text{ADJ}}$

$\therefore\ \tan 23° = \dfrac{h}{x+750}$ and $\tan 37° = \dfrac{h}{x}$

$\therefore\ x + 750 = \dfrac{h}{\tan 23°}$ and $x = \dfrac{h}{\tan 37°}$

$\therefore\ \dfrac{h}{\tan 37°} + 750 = \dfrac{h}{\tan 23°}$ $\quad$ {substitute for x}

$\therefore\ \dfrac{h}{\tan 23°} - \dfrac{h}{\tan 37°} = 750$

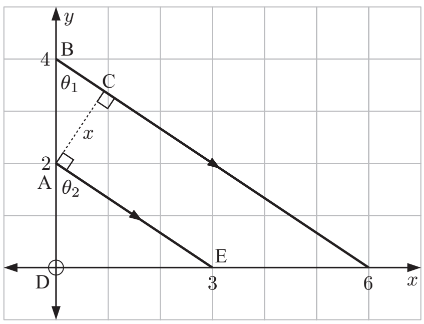
$\therefore\ h\left(\dfrac{1}{\tan 23°} - \dfrac{1}{\tan 37°}\right) = 750$

$\therefore\ h = \dfrac{750}{\left(\frac{1}{\tan 23°} - \frac{1}{\tan 37°}\right)}$

$\therefore\ h \approx 728.999$

The volcano is about 729 m high.

17



Let AC $= x$ units

$\theta_1 = \theta_2$ {corresponding angles}

In $\triangle$ADE, $\tan\theta = \dfrac{\text{DE}}{\text{AD}} = \dfrac{3}{2}$ $\{\tan\theta = \dfrac{\text{OPP}}{\text{ADJ}}\}$

$\therefore \theta = \tan^{-1}(\frac{3}{2})$ .... (1)

In $\triangle$ABC, $\sin\theta = \dfrac{\text{AC}}{\text{AB}} = \dfrac{x}{2}$ $\{\sin\theta = \dfrac{\text{OPP}}{\text{HYP}}\}$

$\therefore x = 2\sin\theta$

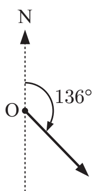
$\therefore x = 2\sin\left(\tan^{-1}(\frac{3}{2})\right)$ {using (1)}

$\therefore x \approx 1.66$

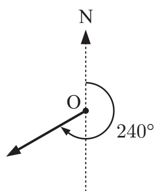
The lines are about 1.66 units apart.

EXERCISE 12F

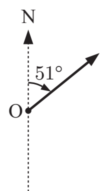
1 **a**



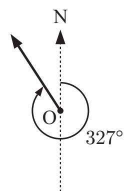
b



c



d

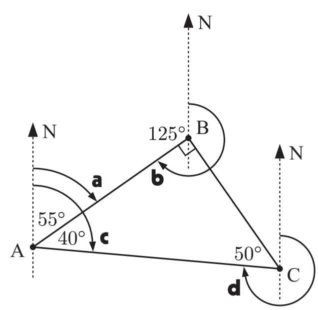


2 To find the bearing of Q from P, we start at P, facing north, and turn clockwise until we face Q.

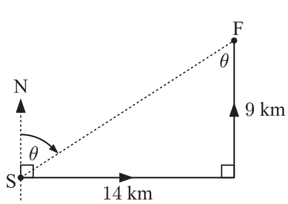
- **a** The bearing of Q from P is 040°.
- **b** The bearing of Q from P is $180° + 55° = 235°$.
- **c** The bearing of Q from P is $360° - 63° = 297°$.
- **d** The bearing of Q from P is $180° - 48° = 132°$.
- **e** The bearing of Q from P is $180° + 45° = 225°$.
- **f** The bearing of Q from P is $180° + 157° = 337°$.

3

- **a** The bearing of B from A is 055°.
- **b** Since the bearing of B from A is 055°, the bearing of A from B is $055° + 180° = 235°$.
- **c** The bearing of C from A is $055° + 040° = 095°$.
- **d** Since the bearing of C from A is 095°, the bearing of A from C is $095° + 180° = 275°$.
- **e** The bearing of C from B is $360° - 125° - 90° = 145°$.
- **f** Since the bearing of C from B is 145°, the bearing of B from C is $145° + 180° = 325°$.



4



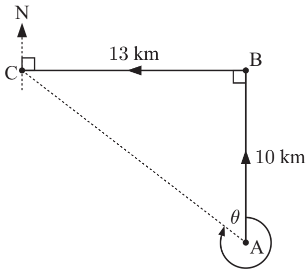
$\tan\theta = \frac{14}{9}$ $\{\tan\theta = \dfrac{\text{OPP}}{\text{ADJ}}\}$

$\therefore \theta = \tan^{-1}\left(\frac{14}{9}\right)$

$\therefore \theta \approx 57.3°$

So, the bearing of the finishing point from the starting point is approximately 057°.

5



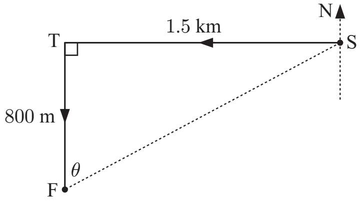
$\tan\theta = \frac{13}{10}$ $\{\tan\theta = \frac{\text{OPP}}{\text{ADJ}}\}$

$\therefore\ \theta = \tan^{-1}\left(\frac{13}{10}\right)$

$\therefore\ \theta \approx 52.4°$

So, the bearing of C from A is approximately $360° - 52.4° \approx 308°$.

6 **a**



b $\text{TF} = 800\text{ m} = (800 \div 1000)\text{ km} = 0.8\text{ km}$

In $\triangle$TSF, $\text{SF}^2 = 0.8^2 + 1.5^2$ {Pythagoras}

$= 2.89$

$\therefore\ \text{SF} = \sqrt{2.89}$ $\{\text{SF} > 0\}$

$\therefore\ \text{SF} = 1.7$

So, the canoeist is 1.7 km from his starting point.

c Let $\widehat{\text{TFS}}$ be θ.

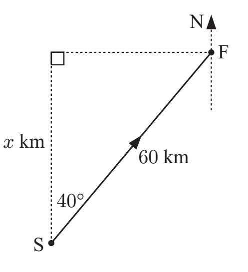
Now, $\tan\theta = \frac{1.5}{0.8}$ $\{\tan\theta = \frac{\text{OPP}}{\text{ADJ}}\}$

$\therefore\ \theta = \tan^{-1}\left(\frac{1.5}{0.8}\right)$

$\therefore\ \theta \approx 61.9°$

So, the canoeist needs to travel on a bearing of approximately 062° from F to S.

7



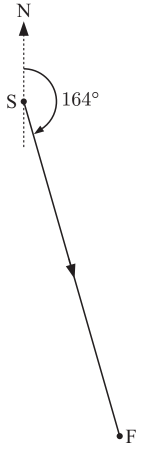
$\cos 40° = \frac{x}{60}$ $\{\cos\theta = \frac{\text{ADJ}}{\text{HYP}}\}$

$\therefore\ x = 60 \times \cos 40°$

$\therefore\ x \approx 46.0$

So, the ship is about 46.0 km north of its starting point.

8 **a**



b distance = speed × time

$= 14 \times 2.5$

$= 35$ km

c **i** $\sin 16° = \frac{a}{35}$ $\{\sin\theta = \frac{\text{OPP}}{\text{HYP}}\}$

$\therefore\ a = 35 \times \sin 16°$

$\therefore\ a \approx 9.65$

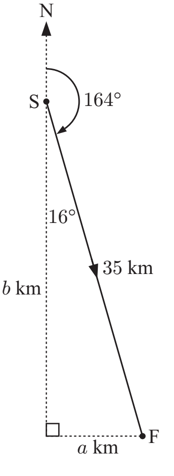
So, the athlete is about 9.65 km east of the starting point.

ii $\cos 16° = \frac{b}{35}$ $\{\cos\theta = \frac{\text{ADJ}}{\text{HYP}}\}$

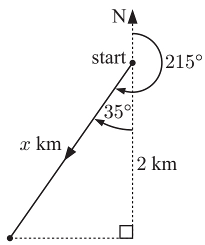
$\therefore\ b = 35 \times \cos 16°$

$\therefore\ b \approx 33.6$

So, the athlete is about 33.6 km south of the starting point.



9



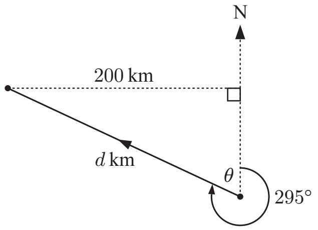
$\cos 35° = \dfrac{2}{x}$ $\quad \{\cos\theta = \dfrac{\text{ADJ}}{\text{HYP}}\}$

$\therefore\ x = \dfrac{2}{\cos 35°}$

$\therefore\ x \approx 2.44$

So, the hiker walked about 2.44 km.

10



Suppose the aeroplane travelled d km.

$\theta = 360° - 295° = 65°$

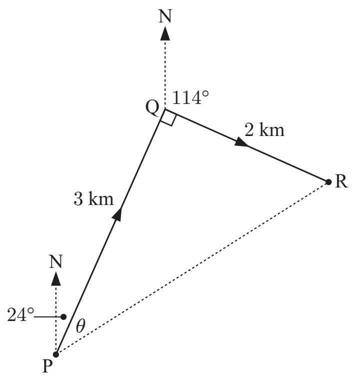
$\sin 65° = \dfrac{200}{d}$ $\quad \{\sin\theta = \dfrac{\text{OPP}}{\text{HYP}}\}$

$\therefore\ d = \dfrac{200}{\sin 65°}$

$\therefore\ d \approx 220.7$

The aeroplane travelled about 221 km.

11



$\text{N}\widehat{\text{Q}}\text{P} = 180° - 24°$ $\quad$ {cointerior angles}

$= 156°$

$\text{P}\widehat{\text{Q}}\text{R} = 360° - 156° - 114°$ $\quad$ {angles at a point}

$= 90°$

a $\text{PR}^2 = 2^2 + 3^2$ $\quad$ {Pythagoras}

$\therefore\ \text{PR} = \sqrt{2^2 + 3^2}$ $\quad$ {as $\text{PR} > 0$}

$\therefore\ \text{PR} \approx 3.61$

So, the finishing point is about 3.61 km from the starting point.

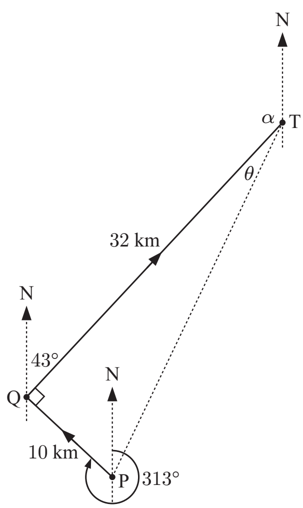
b $\tan\theta = \frac{2}{3}$ $\quad \{\tan\theta = \dfrac{\text{OPP}}{\text{ADJ}}\}$

$\therefore\ \theta = \tan^{-1}\left(\frac{2}{3}\right)$

$\therefore\ \theta \approx 33.7°$

So, the bearing of the finishing point from the starting point is $24° + 33.7° \approx 057.7°$.

12



$\text{N}\widehat{\text{P}}\text{Q} = 360° - 313° = 47°$ $\quad$ {angles at a point}

$\text{T}\widehat{\text{Q}}\text{P} = 180° - 43° - 47°$ $\quad$ {cointerior angles}

$= 90°$

a $\text{TP}^2 = 10^2 + 32^2$ $\quad$ {Pythagoras}

$\therefore\ \text{TP} = \sqrt{10^2 + 32^2}$ $\quad$ {as $\text{TP} > 0$}

$\therefore\ \text{TP} \approx 33.5$

So, the trawler is about 33.5 km from P.

b $\tan\theta = \frac{10}{32}$ $\quad \{\tan\theta = \dfrac{\text{OPP}}{\text{ADJ}}\}$

$\therefore\ \theta = \tan^{-1}\left(\frac{10}{32}\right)$

$\therefore\ \theta \approx 17.4°$

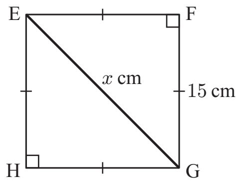
$\alpha = 180° - 43°$ $\quad$ {cointerior angles}

$= 137°$

So, the trawler must sail on a bearing of $360° - 137° - 17.4° \approx 206°$.

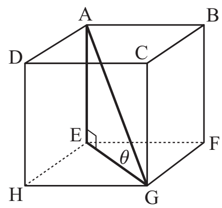
EXERCISE 12G

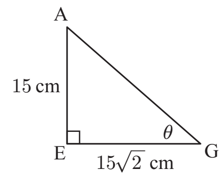
1 **a**



$$x^2 = 15^2 + 15^2 \quad \{\text{Pythagoras}\}$$
$$\therefore \; x^2 = 450$$
$$\therefore \; x = \sqrt{450} = 15\sqrt{2}$$
$$\therefore \; x \approx 21.21$$
So, $\text{EG} \approx 21.2$ cm

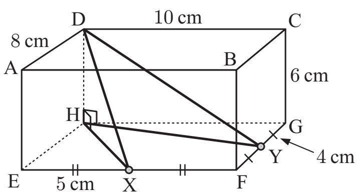
b



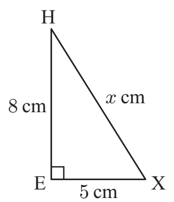


$$\tan\theta = \frac{15}{15\sqrt{2}} = \frac{1}{\sqrt{2}} \quad \left\{\tan\theta = \frac{\text{OPP}}{\text{ADJ}}\right\}$$
$$\therefore \; \theta = \tan^{-1}\left(\tfrac{1}{\sqrt{2}}\right)$$
$$\therefore \; \theta \approx 35.3^\circ$$
So, $\text{A}\widehat{\text{G}}\text{E} \approx 35.3^\circ$

2

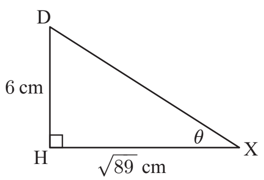


a



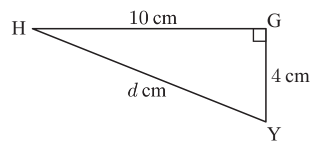
$$x^2 = 5^2 + 8^2 \quad \{\text{Pythagoras}\}$$
$$\therefore \; x^2 = 25 + 64 = 89$$
$$\therefore \; x = \sqrt{89} \quad \{\text{as } x > 0\}$$
$$\therefore \; x \approx 9.43$$
$$\therefore \; \text{HX} \approx 9.43 \text{ cm}$$

b



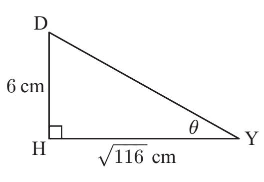
$$\tan\theta = \frac{6}{\sqrt{89}} \quad \left\{\tan\theta = \frac{\text{OPP}}{\text{ADJ}}\right\}$$
$$\therefore \; \theta = \tan^{-1}\left(\frac{6}{\sqrt{89}}\right)$$
$$\therefore \; \theta \approx 32.5^\circ$$
So, $\text{D}\widehat{\text{X}}\text{H} \approx 32.5^\circ$

c



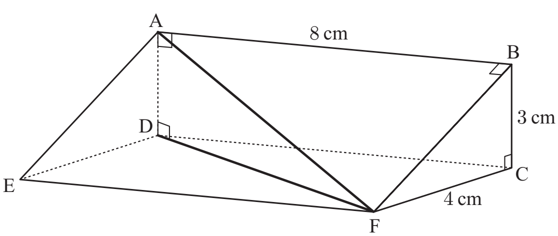
$$d^2 = 4^2 + 10^2 \quad \{\text{Pythagoras}\}$$
$$\therefore \; d^2 = 16 + 100 = 116$$
$$\therefore \; d = \sqrt{116} \quad \{\text{as } d > 0\}$$
$$\therefore \; d \approx 10.8$$
$$\therefore \; \text{HY} \approx 10.8 \text{ cm}$$

d

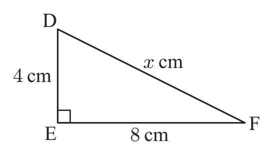


$$\tan\theta = \frac{6}{\sqrt{116}} \quad \left\{\tan\theta = \frac{\text{OPP}}{\text{ADJ}}\right\}$$
$$\therefore \; \theta = \tan^{-1}\left(\frac{6}{\sqrt{116}}\right)$$
$$\therefore \; \theta \approx 29.1^\circ$$
So, $\text{D}\widehat{\text{Y}}\text{H} \approx 29.1^\circ$

3

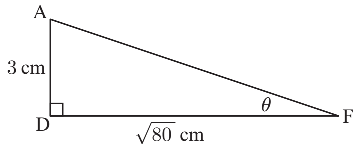


a



$$x^2 = 4^2 + 8^2 \quad \{\text{Pythagoras}\}$$
$$\therefore \; x^2 = 16 + 64 = 80$$
$$\therefore \; x = \sqrt{80} \quad \{\text{as } x > 0\}$$
$$\therefore \; x \approx 8.94$$
$$\therefore \; \text{DF} \approx 8.94 \text{ cm}$$

b



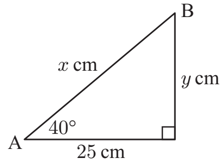
$\tan\theta = \dfrac{3}{\sqrt{80}}$ $\quad\{\tan\theta = \dfrac{\text{OPP}}{\text{ADJ}}\}$

$\therefore\ \theta = \tan^{-1}(\frac{3}{\sqrt{80}})$

$\therefore\ \theta \approx 18.5^\circ$

So, $\widehat{\text{AFD}} \approx 18.5^\circ$

4

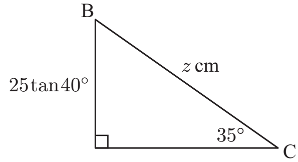


$\cos 40^\circ = \dfrac{25}{x}$ $\quad\{\cos\theta = \dfrac{\text{ADJ}}{\text{HYP}}\}$

$\therefore\ x = \dfrac{25}{\cos 40^\circ}$

Also, $\tan 40^\circ = \dfrac{y}{25}$ $\quad\{\tan\theta = \dfrac{\text{OPP}}{\text{ADJ}}\}$

$\therefore\ y = 25\tan 40^\circ$



$\sin 35^\circ = \dfrac{25\tan 40^\circ}{z}$ $\quad\{\sin\theta = \dfrac{\text{OPP}}{\text{HYP}}\}$

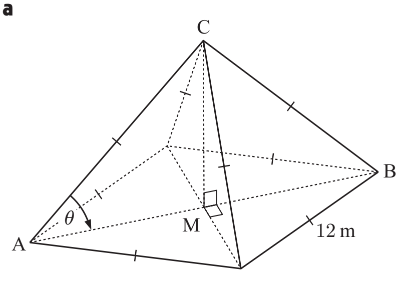
$\therefore\ z = \dfrac{25\tan 40^\circ}{\sin 35^\circ}$

The total length of wood = AB + BC

$= \dfrac{25}{\cos 40^\circ} + \dfrac{25\tan 40^\circ}{\sin 35^\circ}$

≈ 69.2 cm

5 **a**



$\text{AB}^2 = 12^2 + 12^2$ $\quad$ {Pythagoras}

$\therefore\ \text{AB}^2 = 144 \times 2$

$\therefore\ \text{AB} = 12\sqrt{2}$ m $\quad$ {as AB > 0}

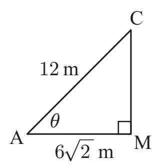
$\therefore\ \text{AM} = 6\sqrt{2}$ m

$\cos\theta = \dfrac{6\sqrt{2}}{12}$ $\quad\{\cos\theta = \dfrac{\text{ADJ}}{\text{HYP}}\}$

$\therefore\ \cos\theta = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$

$\therefore\ \theta = \cos^{-1}(\frac{1}{\sqrt{2}})$

$\therefore\ \theta = 45^\circ$



The angle between the slant edge and a base diagonal is 45°.

b Suppose the sides of the pyramid are x m long.

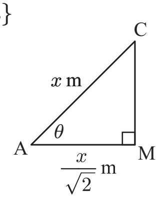
Then $\text{AB}^2 = x^2 + x^2$ $\quad$ {Pythagoras}

$= 2x^2$

$\therefore\ \text{AB} = x\sqrt{2}$ m $\quad$ {AB > 0}

$\therefore\ \text{AM} = \dfrac{x\sqrt{2}}{2}$ m

$= \dfrac{x}{\sqrt{2}}$ m



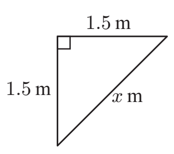
$\cos\theta = \dfrac{\frac{x}{\sqrt{2}}}{x} = \dfrac{1}{\sqrt{2}}$ $\quad\{\cos\theta = \dfrac{\text{ADJ}}{\text{HYP}}\}$

$\therefore\ \theta = \cos^{-1}(\frac{1}{\sqrt{2}})$

$\therefore\ \theta = 45^\circ$

So, the angle is always 45°.

6

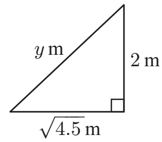


$x^2 = 1.5^2 + 1.5^2$ $\quad$ {Pythagoras}

$= 2.25 + 2.25$

$= 4.5$

$\therefore\ x = \sqrt{4.5}$ $\quad$ {as $x > 0$}

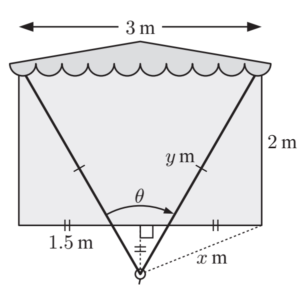


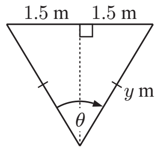
$y^2 = 2^2 + (\sqrt{4.5})^2$ $\quad$ {Pythagoras}

$= 4 + 4.5$

$= 8.5$

$\therefore\ y = \sqrt{8.5}$ $\quad$ {as $y > 0$}





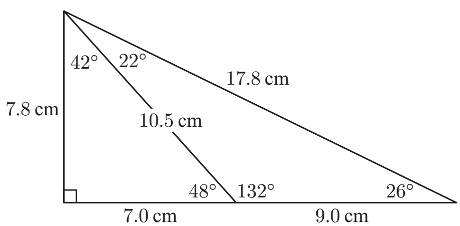
$\sin\left(\frac{\theta}{2}\right) = \frac{1.5}{\sqrt{8.5}}$ $\quad\left\{\sin\theta = \frac{\text{OPP}}{\text{HYP}}\right\}$

$\therefore \quad \frac{\theta}{2} = \sin^{-1}\left(\frac{1.5}{\sqrt{8.5}}\right)$

$\therefore \quad \theta \approx 61.9°$

REVIEW SET 12

1 **a**



i $\cos 42° = \frac{\text{ADJ}}{\text{HYP}} \approx \frac{7.8 \text{ cm}}{10.5 \text{ cm}} \approx 0.743$

ii $\sin 64° = \frac{\text{OPP}}{\text{HYP}} \approx \frac{(7.0 + 9.0) \text{ cm}}{17.8 \text{ cm}}$

≈ 0.899

iii $\tan 48° = \frac{\text{OPP}}{\text{ADJ}} \approx \frac{7.8 \text{ cm}}{7.0 \text{ cm}} \approx 1.114$

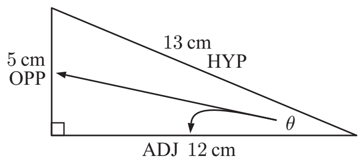
iv $\sin 26° = \frac{\text{OPP}}{\text{HYP}} \approx \frac{7.8 \text{ cm}}{17.8 \text{ cm}} \approx 0.438$

b **i** $\cos 42° \approx 0.743$ **ii** $\sin 64° \approx 0.899$ **iii** $\tan 48° \approx 1.111$ **iv** $\sin 26° \approx 0.438$

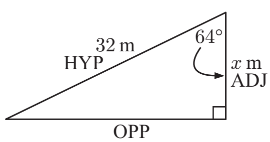
2 $\sin\theta = \frac{\text{OPP}}{\text{HYP}} = \frac{5}{13}$

$\cos\theta = \frac{\text{ADJ}}{\text{HYP}} = \frac{12}{13}$

$\tan\theta = \frac{\text{OPP}}{\text{ADJ}} = \frac{5}{12}$



3 **a**

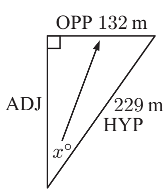


$\cos 64° = \frac{x}{32}$ $\quad\left\{\cos\theta = \frac{\text{ADJ}}{\text{HYP}}\right\}$

$\therefore \quad \cos 64° \times 32 = x$ $\quad$ {multiplying both sides by 32}

$\therefore \quad x \approx 14.0$ $\quad$ {calculator}

b

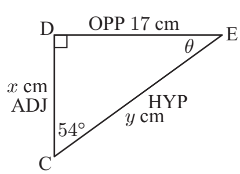


$\sin x° = \frac{132}{229}$ $\quad\left\{\sin\theta = \frac{\text{OPP}}{\text{HYP}}\right\}$

$\therefore \quad x° = \sin^{-1}\left(\frac{132}{229}\right)$

$\therefore \quad x \approx 35.2$ $\quad$ {calculator}

4



$\theta + 90° + 54° = 180°$ $\quad$ {angles in a triangle}

$\therefore \quad \theta + 144° = 180°$

$\therefore \quad \theta = 36°$

$\tan 54° = \frac{17}{x}$ $\quad$ {the side opposite 54° is 17 cm and the side adjacent to 54° is x cm}

$\therefore \quad x \times \tan 54° = 17$ $\quad$ {multiplying both sides by x}

$\therefore \quad x = \frac{17}{\tan 54°}$ $\quad$ {dividing both sides by $\tan 54°$}

$\therefore \quad x \approx 12.4$ $\quad$ {calculator}

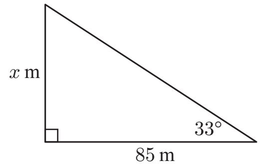
$\sin 54^\circ = \dfrac{17}{y}$ {the side opposite 54° is 17 cm and the hypotenuse is y cm}

$\therefore\ y \times \sin 54^\circ = 17$ {multiplying both sides by y}

$\therefore\ y = \dfrac{17}{\sin 54^\circ}$ {dividing both sides by $\sin 54^\circ$}

$\therefore\ y \approx 21.0$ {calculator}

5



Let the height of the cathedral be x m.

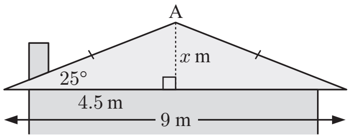
$\tan 33^\circ = \dfrac{x}{85}$ $\{\tan\theta = \dfrac{\text{OPP}}{\text{ADJ}}\}$

$\therefore\ x = 85 \times \tan 33^\circ$

$\therefore\ x \approx 55.2$ {calculator}

So, the height of the cathedral is about 55.2 m.

6



Let point A be x m above the ceiling.

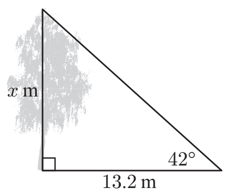
$\tan 25^\circ = \dfrac{x}{4.5}$ $\{\tan\theta = \dfrac{\text{OPP}}{\text{ADJ}}\}$

$\therefore\ x = 4.5 \times \tan 25^\circ$

$\therefore\ x \approx 2.10$ {calculator}

So, point A is about 2.10 m above the ceiling.

7



Let the height of the tree be x m.

$\tan 42^\circ = \dfrac{x}{13.2}$ $\{\tan\theta = \dfrac{\text{OPP}}{\text{ADJ}}\}$

$\therefore\ x = 13.2 \times \tan 42^\circ$

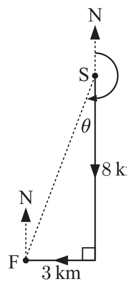
$\therefore\ x \approx 11.89$ {calculator}

As the tree is 12 m from the house, it will miss the house by about $12 - 11.89 \approx 0.11$ m ≈ 11 cm.

8 **a** The bearing of Q from P is $180^\circ - 23^\circ = 157^\circ$.

b The bearing of Q from P is $360^\circ - 79^\circ = 281^\circ$.

9



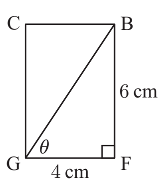
$\tan\theta = \frac{3}{8}$ $\{\tan\theta = \dfrac{\text{OPP}}{\text{ADJ}}\}$

$\therefore\ \theta = \tan^{-1}\left(\frac{3}{8}\right)$

$\therefore\ \theta \approx 20.6^\circ$

So, the bearing of the taxi's finishing point from its starting point is approximately $180^\circ + 20.6^\circ \approx 201^\circ$.

10 **a**



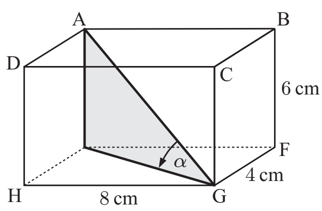
$\tan\theta = \dfrac{6}{4} = \dfrac{3}{2}$ $\{\tan\theta = \dfrac{\text{OPP}}{\text{ADJ}}\}$

$\therefore\ \theta = \tan^{-1}(\frac{3}{2})$

$\therefore\ \theta \approx 56.3^\circ$

$\therefore\ \text{B}\widehat{\text{G}}\text{F} \approx 56.3^\circ$

b



$(\text{EG})^2 = 4^2 + 8^2$ {Pythagoras}

$\therefore\ \text{EG} = \sqrt{80}$

$\tan\alpha = \dfrac{6}{\sqrt{80}}$ $\{\tan\alpha = \dfrac{\text{OPP}}{\text{ADJ}}\}$

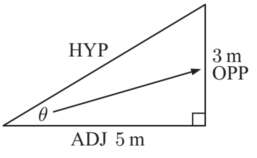
$\therefore\ \alpha = \tan^{-1}(\frac{6}{\sqrt{80}})$

$\therefore\ \alpha \approx 33.9^\circ$

$\therefore\ \text{A}\widehat{\text{G}}\text{E} \approx 33.9^\circ$

PRACTICE TEST 12A

1



Let the hypotenuse have length a m.

$a^2 = 3^2 + 5^2$ {Pythagoras}

$\therefore\ a^2 = 34$

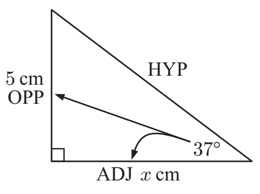
$\therefore\ a = \sqrt{34}$

$\therefore$ the hypotenuse is $\sqrt{34}$ m long.

Now, $\sin\theta = \dfrac{\text{OPP}}{\text{HYP}} = \dfrac{3}{\sqrt{34}}$

$\therefore$ the answer is **C**.

2



$\tan 37^\circ = \dfrac{5}{x}$ $\quad\{\tan\theta = \dfrac{\text{OPP}}{\text{ADJ}}\}$

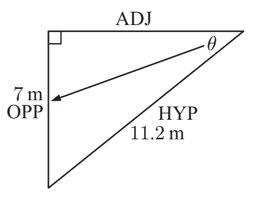
$\therefore\ x \times \tan 37^\circ = 5$ $\quad$ {multiplying both sides by x}

$\therefore\ x = \dfrac{5}{\tan 37^\circ}$ $\quad$ {dividing both sides by $\tan 37^\circ$}

$\therefore\ x \approx 6.64$ $\quad$ {calculator}

$\therefore$ the answer is **A**.

3



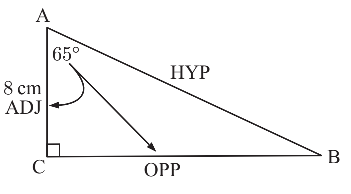
$\sin\theta = \dfrac{7}{11.2}$ $\quad\{\sin\theta = \dfrac{\text{OPP}}{\text{HYP}}\}$

$\therefore\ \theta = \sin^{-1}\left(\dfrac{7}{11.2}\right)$

$\therefore\ \theta \approx 38.7^\circ$ $\quad$ {calculator}

$\therefore$ the answer is **D**.

4



$\tan 65^\circ = \dfrac{\text{BC}}{8}$ $\quad\{\tan\theta = \dfrac{\text{OPP}}{\text{ADJ}}\}$

$\therefore\ 8 \times \tan 65^\circ = \text{BC}$ $\quad$ {multiplying both sides by 8}

$\therefore\ \text{BC} \approx 17.2$ cm $\quad$ {calculator}

$\cos 65^\circ = \dfrac{8}{\text{AB}}$ $\quad\{\cos\theta = \dfrac{\text{ADJ}}{\text{HYP}}\}$

$\therefore\ \text{AB} \times \cos 65^\circ = 8$ $\quad$ {multiplying both sides by AB}

$\therefore\ \text{AB} = \dfrac{8}{\cos 65^\circ}$ $\quad$ {dividing both sides by $\cos 65^\circ$}

$\therefore\ \text{AB} \approx 18.9$ cm

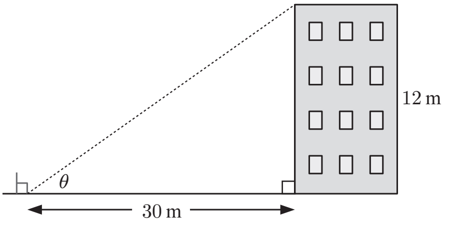
The perimeter of the triangle $= \text{AB} + \text{BC} + \text{CA}$

$\approx (18.9 + 17.2 + 8)$ cm

≈ 44.1 cm

$\therefore$ the answer is **E**.

5



Let the angle of elevation be θ.

$\tan\theta = \frac{12}{30}$ $\quad\{\tan\theta = \dfrac{\text{OPP}}{\text{ADJ}}\}$

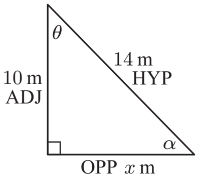
$\therefore\ \theta = \tan^{-1}\left(\frac{12}{30}\right)$

$\therefore\ \theta \approx 21.8^\circ$

So, the angle of elevation is about 21.8°.

$\therefore$ the answer is **C**.

6



Let the two unknown angles be θ and α.

$\cos\theta = \frac{10}{14}$ $\{\cos\theta = \frac{\text{ADJ}}{\text{HYP}}\}$

$\therefore\ \theta = \cos^{-1}\left(\frac{10}{14}\right)$

$\therefore\ \theta \approx 44.4^\circ$

Now, $\theta + \alpha + 90^\circ = 180^\circ$ {angles in a triangle}

$\therefore\ \alpha = 90^\circ - \theta$

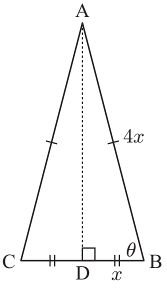
$\therefore\ \alpha \approx 90^\circ - 44.4^\circ$

$\therefore\ \alpha \approx 45.6^\circ$

So, the smallest angle is θ which is $\approx 44.4^\circ$.

$\therefore$ the answer is **B**.

7



Let the base of $\triangle$ABC have length $2x$.

$\therefore$ AB = AC = $2 \times 2x = 4x$

Now, in $\triangle$ABD, $\cos\theta = \frac{x}{4x}$ $\{\cos\theta = \frac{\text{ADJ}}{\text{HYP}}\}$

$= \frac{1}{4}$

$\therefore\ \theta = \cos^{-1}\left(\frac{1}{4}\right)$

$\therefore\ \theta \approx 75.5^\circ$

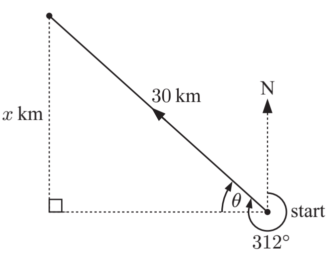
So, the equal angles of $\triangle$ABC are each $\approx 75.5^\circ$.

$\therefore$ the answer is **B**.

8 The bearing of Q from P is $180^\circ + 31^\circ = 211^\circ$.

$\therefore$ the answer is **E**.

9



Suppose the car travels x km north.

$\theta = 312^\circ - 270^\circ$

$= 42^\circ$

$\sin 42^\circ = \frac{x}{30}$ $\{\sin\theta = \frac{\text{OPP}}{\text{HYP}}\}$

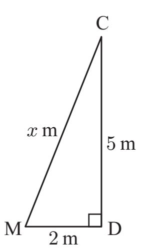
$\therefore\ x = 30 \times \sin 42^\circ$

$\therefore\ x \approx 20.1$

So, the car travels about 20.1 km north.

$\therefore$ the answer is **D**.

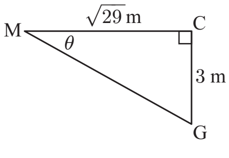
10



$x^2 = 2^2 + 5^2$ {Pythagoras}

$\therefore\ x = \sqrt{2^2 + 5^2}$ $\{\text{as } x > 0\}$

$\therefore\ x = \sqrt{29}$



$\tan\theta = \frac{3}{\sqrt{29}}$ $\{\tan\theta = \frac{\text{OPP}}{\text{ADJ}}\}$

$\therefore\ \theta = \tan^{-1}\left(\frac{3}{\sqrt{29}}\right)$

$\therefore\ \theta \approx 29.1^\circ$

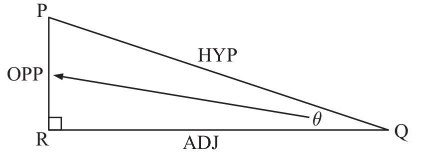
$\therefore\ \widehat{\text{CMG}} \approx 29.1^\circ$

$\therefore$ the answer is **C**.

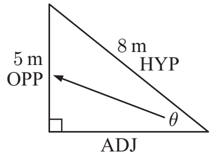
PRACTICE TEST 12B

1 **a** The hypotenuse is [PQ].

b The side opposite angle θ is [PR].



2 **a**

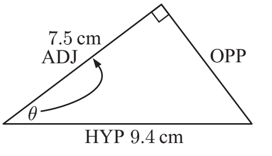


$\sin\theta = \frac{5}{8}$ $\quad \{\sin\theta = \frac{\text{OPP}}{\text{HYP}}\}$

$\therefore\ \theta = \sin^{-1}\left(\frac{5}{8}\right)$

$\therefore\ \theta \approx 38.7^\circ$ $\quad$ {calculator}

b

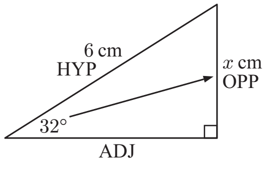


$\cos\theta = \frac{7.5}{9.4}$ $\quad \{\cos\theta = \frac{\text{ADJ}}{\text{HYP}}\}$

$\therefore\ \theta = \cos^{-1}\left(\frac{7.5}{9.4}\right)$

$\therefore\ \theta \approx 37.1^\circ$ $\quad$ {calculator}

3 **a**

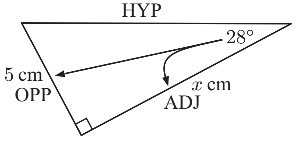


$\sin 32^\circ = \frac{x}{6}$ $\quad \{\sin\theta = \frac{\text{OPP}}{\text{HYP}}\}$

$\therefore\ 6 \times \sin 32^\circ = x$ $\quad$ {multiplying both sides by 6}

$\therefore\ x \approx 3.18^\circ$ $\quad$ {calculator}

b



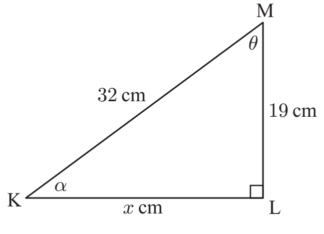
$\tan 28^\circ = \frac{5}{x}$ $\quad \{\tan\theta = \frac{\text{OPP}}{\text{ADJ}}\}$

$\therefore\ x \times \tan 28^\circ = 5$ $\quad$ {multiplying both sides by x}

$\therefore\ x = \frac{5}{\tan 28^\circ}$ $\quad$ {dividing both sides by $\tan 28^\circ$}

$\therefore\ x \approx 9.40$ $\quad$ {calculator}

4



$\cos\theta = \frac{19}{32}$ $\quad \{\cos\theta = \frac{\text{ADJ}}{\text{HYP}}\}$

$\therefore\ \theta = \cos^{-1}\left(\frac{19}{32}\right)$

$\therefore\ \theta \approx 53.6^\circ$ $\quad$ {calculator}

$\sin\alpha = \frac{19}{32}$ $\quad \{\sin\alpha = \frac{\text{OPP}}{\text{HYP}}\}$ *or* use angle sum of triangle

$\therefore\ \alpha = \sin^{-1}\left(\frac{19}{32}\right)$

$\therefore\ \alpha \approx 36.4^\circ$ $\quad$ {calculator}

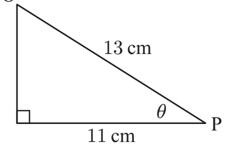
So, $\sin\theta = \frac{x}{32}$ $\quad \{\sin\theta = \frac{\text{OPP}}{\text{HYP}}\}$ *or* use Pythagoras

$\therefore\ \sin 53.5764^\circ \approx \frac{x}{32}$

$\therefore\ 32 \times \sin 53.5764^\circ \approx x$ $\quad$ {multiplying both sides by 32}

$\therefore\ x \approx 25.7$

5



Suppose the angle is θ.

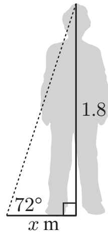
$\cos\theta = \frac{11}{13}$ $\quad \{\cos\theta = \frac{\text{ADJ}}{\text{HYP}}\}$

$\therefore\ \theta = \cos^{-1}\left(\frac{11}{13}\right)$

$\therefore\ \theta \approx 32.2^\circ$ $\quad$ {calculator}

So, the angle is about 32.2°.

6



Let the length of the shadow be x m.

$$\tan 72° = \frac{1.8}{x} \qquad \{\tan\theta = \frac{\text{OPP}}{\text{ADJ}}\}$$

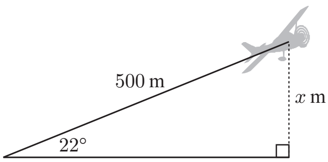
$$\therefore\ x \times \tan 72° = 1.8 \qquad \{\text{multiplying both sides by } x\}$$

$$\therefore\ x = \frac{1.8}{\tan 72°} \qquad \{\text{dividing both sides by } \tan 72°\}$$

$$\therefore\ x \approx 0.585 \qquad \{\text{calculator}\}$$

So, the boy's shadow is about 0.585 m long.

7



Let the altitude of the plane be x m.

$$\sin 22° = \frac{x}{500} \qquad \{\sin\theta = \frac{\text{OPP}}{\text{HYP}}\}$$

$$\therefore\ 500 \times \sin 22° = x \qquad \{\text{multiplying both sides by } 500\}$$

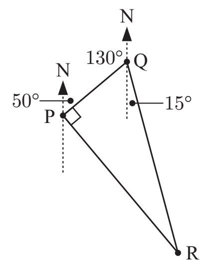
$$\therefore\ x = 187$$

So, the altitude of the plane is about 187 m.

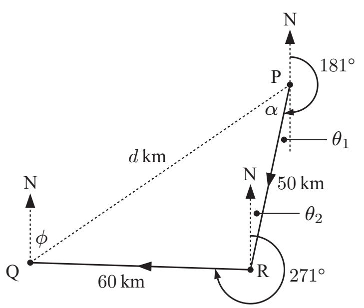
8 **a** The bearing of P from Q is $360° - 130° = 230°$.

b The bearing of R from Q is $180° - 15° = 165°$.

c The bearing of R from P is $50° + 90° = 140°$.



9 **a**



$$\theta_1 = 181° - 180° = 1°$$

$$\theta_2 = \theta_1 \qquad \{\text{alternate angles}\}$$

$$\therefore\ \theta_2 = 1°$$

$$\therefore\ Q\widehat{R}P = 360° - 271° + 1°$$
$$= 90°$$

Let PQ be d km.

Now, $d^2 = 50^2 + 60^2 \qquad \{\text{Pythagoras}\}$

$$\therefore\ d = \sqrt{50^2 + 60^2} \qquad \{\text{as } d > 0\}$$

$$\therefore\ d \approx 78.1$$

So, Q is about 78.1 km from P.

b The required angle is ϕ.

Let $Q\widehat{P}R = \alpha$.

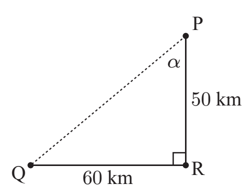
$$\tan\alpha = \tfrac{60}{50} \qquad \{\tan\alpha = \frac{\text{OPP}}{\text{ADJ}}\}$$

$$\therefore\ \alpha = \tan^{-1}\left(\tfrac{60}{50}\right)$$

$$\therefore\ \alpha \approx 50.2°$$

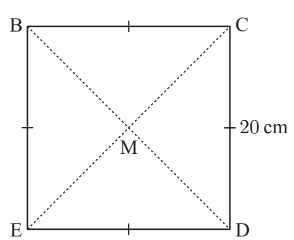
$$\therefore\ \alpha + \theta_1 \approx 50.2° + 1° \approx 51.2°$$

$$\therefore\ \phi \approx 51.2° \qquad \{\text{alternate angles}\}$$



So, the ship must sail on the bearing of $\approx 051°$.

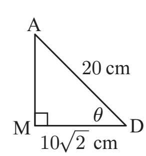
10 **a**



$$BD^2 = 20^2 + 20^2 \qquad \{\text{Pythagoras}\}$$

$$\therefore\ BD = 20\sqrt{2}\text{ cm}$$

$$\therefore\ MD = 10\sqrt{2}\text{ cm}$$



$$\cos\theta = \frac{10\sqrt{2}}{20} = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}} \qquad \{\cos\theta = \frac{\text{ADJ}}{\text{HYP}}\}$$

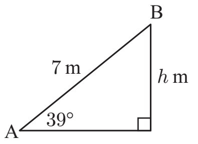
$$\therefore\ \theta = 45°$$

$$\therefore\ A\widehat{D}M = 45°$$

b Triangle ACD is equilateral, so $\widehat{ACD} = 60°$.

PRACTICE TEST 12C

1 **a**



$\sin 39° = \frac{h}{7}$ $\{\sin\theta = \frac{\text{OPP}}{\text{HYP}}\}$

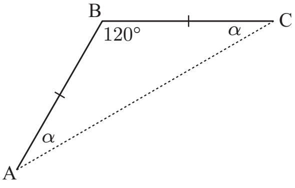
$\therefore\ h = 7\sin 39°$ {multiplying both sides by 7}

b Area of $\triangle ABC = \frac{1}{2} \times \text{base} \times \text{height}$

$= \frac{1}{2} \times 9 \times 7\sin 39°$

$\approx 19.82\ \text{m}^2$

2 **a**



$\widehat{ABC} = 120°$ {given}

$\widehat{BAC} = \widehat{BCA}$ {base angles of isosceles triangle}

Let $\widehat{BCA} = \alpha$

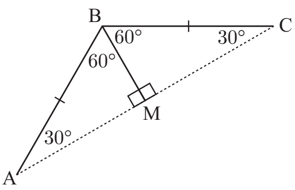
$2\alpha + 120° = 180°$ {angles in a triangle}

$\therefore\ 2\alpha = 60°$

$\therefore\ \alpha = 30°$

$\therefore\ \widehat{BCA} = 30°$

b



In $\triangle BMC$, $\cos 30° = \frac{\text{MC}}{\text{BC}}$ $\{\cos\theta = \frac{\text{ADJ}}{\text{HYP}}\}$

$\therefore\ \frac{\sqrt{3}}{2} = \frac{\text{MC}}{\text{BC}}$

$\therefore\ \text{MC} = \frac{\sqrt{3}}{2} \times \text{BC}$

Now, $\triangle ABM \cong \triangle CBM$ {SAS or RHS}

$\therefore\ \text{AM} = \text{MC}$

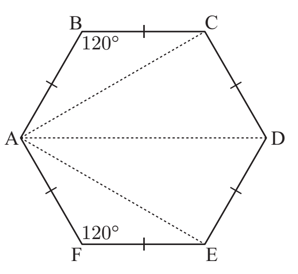
$\therefore\ \text{AC} = 2 \times \text{MC}$

$= 2 \times \frac{\sqrt{3}}{2} \times \text{BC}$

$= \sqrt{3} \times \text{BC}$

So, the length of [AC] is $\sqrt{3}$ times the side length of the hexagon.

c



i In triangles ABC and AFE:

AB = BC = AF = EF {given}

$\widehat{ABC} = \widehat{AFE} = 120°$ {equal angles of regular hexagon}

$\therefore\ \triangle ABC \cong \triangle AFE$ {SAS}

$\therefore\ \text{AC} = \text{AE}$ {corresponding sides}

ii In triangles ADC and ADE:

AC = AE {from **i**}

CD = ED {given}

AD is common to both.

$\therefore\ \triangle ADC \cong \triangle ADE$ {SSS}

$\therefore\ \widehat{ADC} = \widehat{ADE}$ {corresponding angles}

d From **a**, $\widehat{BCA} = 30°$ and since $\widehat{BCD} = 120°$ {angles of regular hexagon}

then $\widehat{ACD} = 120° - 30° = 90°$

So, $\triangle ACD$ is right angled at C.

From **b**, $\text{AC} = \sqrt{3} \times \text{CD}$ {as BC = CD}

Using Pythagoras, $AD^2 = AC^2 + CD^2$

$\therefore\ AD^2 = (\sqrt{3} \times CD)^2 + CD^2$

$= 3 \times CD^2 + CD^2$

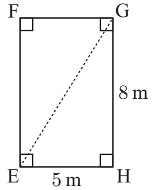
$= 4 \times CD^2$

$\therefore\ AD = \sqrt{4 \times CD^2}$ {as $AD > 0$}

$\therefore\ AD = 2 \times CD$

So, the length of [AD] is twice the side length of the hexagon.

3 **a**



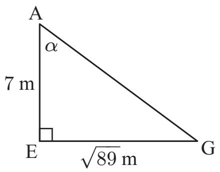
$EG^2 = 5^2 + 8^2$ {Pythagoras}

$= 89$

$\therefore\ EG = \sqrt{89}$ {as $EG > 0$}

$\therefore\ EG \approx 9.43$

b



Let $G\widehat{A}E$ be α.

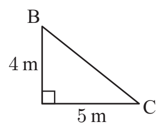
$\tan\alpha = \frac{\sqrt{89}}{7}$ $\{\tan\alpha = \frac{\text{OPP}}{\text{ADJ}}\}$

$\therefore\ \alpha = \tan^{-1}\left(\frac{\sqrt{89}}{7}\right)$

$\therefore\ \alpha \approx 53.4°$

$\therefore\ G\widehat{A}E \approx 53.4°$

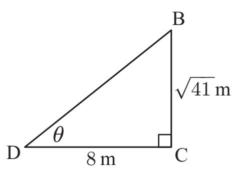
c



$BC^2 = 4^2 + 5^2$ {Pythagoras}

$= 41$

$\therefore\ BC = \sqrt{41}$ m {as $BC > 0$}



Let $B\widehat{D}C$ be θ.

$\tan\theta = \frac{\sqrt{41}}{8}$ $\{\tan\theta = \frac{\text{OPP}}{\text{ADJ}}\}$

$\therefore\ \theta = \tan^{-1}\left(\frac{\sqrt{41}}{8}\right)$

$\therefore\ \theta \approx 38.7°$

$\therefore\ B\widehat{D}C \approx 38.7°$

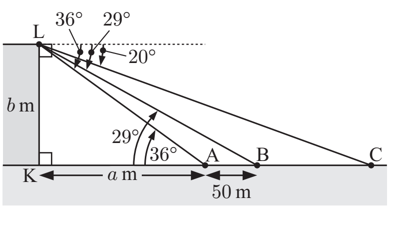
4 **a** Since the angle of depression from L to B is 29°, then the angle of elevation from B to L is also 29°.

b In $\triangle LAK$, $\tan 36° = \frac{b}{a}$ $\{\tan\theta = \frac{\text{OPP}}{\text{ADJ}}\}$

$\therefore\ b = a\tan 36°$ .... (1)

In $\triangle LBK$, $\tan 29° = \frac{b}{a+50}$ $\{\tan\theta = \frac{\text{OPP}}{\text{ADJ}}\}$

$\therefore\ b = (a+50)\tan 29°$



So, equating bs, $a\tan 36° = (a+50)\tan 29°$

$= a\tan 29° + 50\tan 29°$

$\therefore\ a\tan 36° - a\tan 29° = 50\tan 29°$

$\therefore\ a(\tan 36° - \tan 29°) = 50\tan 29°$

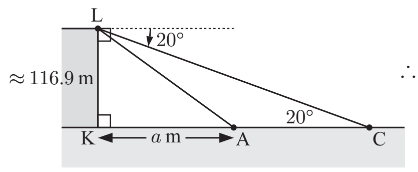
$\therefore\ a = \frac{50\tan 29°}{\tan 36° - \tan 29°}$

$\therefore\ b = \frac{50\tan 29° \times \tan 36°}{\tan 36° - \tan 29°}$ {using (1)}

$\therefore\ b \approx 116.9$

So, the height of the lookout is about 117 m.

c



$\tan 20° = \frac{LK}{KC}$ $\{\tan\theta = \frac{OPP}{ADJ}\}$

$\therefore \tan 20° \approx \frac{116.913}{KC}$

$\therefore KC \approx \frac{116.913}{\tan 20°}$

$\therefore KC \approx 321.2$ m

Now, $AC = KC - KA$

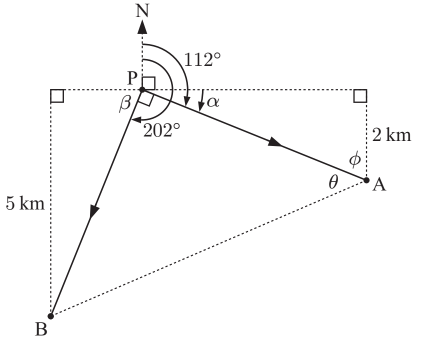
$\approx 321.2 - a$

$\approx 321.2 - \frac{50\tan 29°}{\tan 36° - \tan 29°}$ {from **b**}

≈ 160.3 m

So, the car is about 160 m from Ariel.

5



a **i** $\alpha = 112° - 90° = 22°$

$\sin 22° = \frac{2}{PA}$ $\{\sin\alpha = \frac{OPP}{HYP}\}$

$\therefore PA = \frac{2}{\sin 22°}$

$\therefore PA \approx 5.34$ km

So, boat A travelled about 5.34 km.

ii $\beta = 270° - 202° = 68°$

$\sin 68° = \frac{5}{PB}$ $\{\sin\beta = \frac{OPP}{HYP}\}$

$\therefore PB = \frac{5}{\sin 68°}$

$\therefore PB \approx 5.39$ km

So, boat B travelled about 5.39 km.

b **i** $A\widehat{P}B = 202° - 112° = 90°$

$AB^2 = PB^2 + PA^2$ {Pythagoras}

$\therefore AB^2 = \left(\frac{5}{\sin 68°}\right)^2 + \left(\frac{2}{\sin 22°}\right)^2$

$\therefore AB = \sqrt{\left(\frac{5}{\sin 68°}\right)^2 + \left(\frac{2}{\sin 22°}\right)^2}$ {as $AB > 0$}

$\therefore AB \approx 7.59$

So, the boats are about 7.59 km apart.

ii $\phi = 180° - 90° - 22°$ {angles in a △}

$\therefore \phi = 68°$

$\tan\theta = \frac{PB}{PA}$ $\{\tan\theta = \frac{OPP}{ADJ}\}$

$\therefore \tan\theta = \frac{\left(\frac{5}{\sin 68°}\right)}{\left(\frac{2}{\sin 22°}\right)}$

$\therefore \theta = \tan^{-1}\left(\frac{\left(\frac{5}{\sin 68°}\right)}{\left(\frac{2}{\sin 22°}\right)}\right)$

$\therefore \theta \approx 45.3°$

So, the bearing of B from A is about $360° - 45.3° - 68° \approx 247°$.