Chapter 8

THE GEOMETRY OF POLYGONS

EXERCISE 8A

1 **a** $88°$ is between $0°$ and $90°$, so $88°$ is an acute angle.
$\therefore$ the statement is true.

b $92°$ is between $90°$ and $180°$, so $92°$ is an obtuse angle.
$\therefore$ the statement is true.

c When two lines meet, an angle is formed *between* them, which is the same regardless of the length of the lines.
$\therefore$ the statement is false.

d When a vertical line crosses a horizontal line, the angle formed is $90°$, which is a right angle.
$\therefore$ the statement is false.

e A straight angle measures $180°$.

So, $\frac{2}{3}$ of a straight angle is
$$\frac{2}{3} \times 180°$$
$$= \frac{2}{\cancel{3}_1} \times \frac{\cancel{180}^{60}}{1}$$
$$= 120°$$

Since $120°$ is between $90°$ and $180°$, $120°$ is an obtuse angle.
$\therefore$ the statement is false.

f A right angle measures $90°$.
An acute angle measures *between* $0°$ and $90°$, so $90°$ is not an acute angle.
An obtuse angle measures *between* $90°$ and $180°$, so $90°$ is not an obtuse angle.
$\therefore$ the statement is true.

2 **a**
$$A\widehat{O}B = A\widehat{O}P + P\widehat{O}B$$
$$\therefore\ 90° = 76° + b°$$
$$\therefore\ b = 90 - 76$$
$$\therefore\ b = 14$$

b
$$A\widehat{O}B = A\widehat{O}P + P\widehat{O}B$$
$$\therefore\ 90° = a° + 55°$$
$$\therefore\ a = 90 - 55$$
$$\therefore\ a = 35$$

c
$$A\widehat{O}B = A\widehat{O}P + P\widehat{O}B$$
$$\therefore\ 90° = 0° + b°$$
$$\therefore\ b = 90$$

d
$$A\widehat{O}B = A\widehat{O}P + P\widehat{O}B$$
$$\therefore\ 90° = a° + 89°$$
$$\therefore\ a = 90 - 89$$
$$\therefore\ a = 1$$

e
$$A\widehat{O}B = A\widehat{O}P + P\widehat{O}B$$
$$\therefore\ 90° = a° + b°$$
$$\therefore\ 90 = a + a \quad \{\text{since } a = b\}$$
$$\therefore\ 2a = 90$$
$$\therefore\ a = 45$$
$$\therefore\ b = 45$$

3 **a** Complementary angles add to $90°$, so the angle complementary to $28°$ is $90° - 28° = 62°$.

b Complementary angles add to $90°$, so the angle complementary to $77°$ is $90° - 77° = 13°$.

c Complementary angles add to $90°$, so the angle complementary to $y°$ is $(90 - y)°$.

d Complementary angles add to $90°$, so the angle complementary to $(90 - x)°$ is
$$90° - (90 - x)° = 90° - 90° + x°$$
$$= x°$$

4 **a** Supplementary angles add to 180°, so the angle supplementary to 85° is $180° - 85° = 95°$.

b Supplementary angles add to 180°, so the angle supplementary to 127° is $180° - 127° = 53°$.

c Supplementary angles add to 180°, so the angle supplementary to $q°$ is $(180 - q)°$.

d Supplementary angles add to 180°, so the angle supplementary to $(180 - n)°$ is

$$\begin{aligned} 180° - (180 - n)° &= 180° - 180° + n° \\ &= n° \end{aligned}$$

e Supplementary angles add to 180°, so the angle supplementary to $(x + 90)°$ is

$$\begin{aligned} 180° - (x + 90)° &= 180° - x° - 90° \\ &= (90 - x)° \end{aligned}$$

5 **a**
$$\begin{aligned} a + 29 &= 90 && \{\text{complementary angles}\} \\ \therefore\ a + 29 - \mathbf{29} &= 90 - \mathbf{29} && \{\text{subtracting 29 from both sides}\} \\ \therefore\ a &= 61 \end{aligned}$$

b
$$\begin{aligned} b + 135 &= 180 && \{\text{angles on a straight line}\} \\ \therefore\ b + 135 - \mathbf{135} &= 180 - \mathbf{135} && \{\text{subtracting 135 from both sides}\} \\ \therefore\ b &= 45 \end{aligned}$$

c
$$\begin{aligned} c + 42 &= 360 && \{\text{angles at a point}\} \\ \therefore\ c + 42 - \mathbf{42} &= 360 - \mathbf{42} && \{\text{subtracting 42 from both sides}\} \\ \therefore\ c &= 318 \end{aligned}$$

d $x = 100$ {vertically opposite angles are equal}

e $x = 115$ {equal corresponding angles on parallel lines}

f
$$\begin{aligned} f + 139 &= 180 && \{\text{co-interior angles on parallel lines are supplementary}\} \\ \therefore\ f + 139 - \mathbf{139} &= 180 - \mathbf{139} && \{\text{subtracting 139 from both sides}\} \\ \therefore\ f &= 41 \end{aligned}$$

g
$$\begin{aligned} g + 330 &= 360 && \{\text{angles at a point}\} \\ \therefore\ g + 330 - \mathbf{330} &= 360 - \mathbf{330} && \{\text{subtracting 330 from both sides}\} \\ \therefore\ g &= 30 \end{aligned}$$

h $h = 58$ {equal alternate angles on parallel lines}

i
$$\begin{aligned} n + 87 + 63 &= 360 && \{\text{angles at a point}\} \\ \therefore\ n + 150 &= 360 && \{\text{simplifying}\} \\ \therefore\ n + 150 - \mathbf{150} &= 360 - \mathbf{150} && \{\text{subtracting 150 from both sides}\} \\ \therefore\ n &= 210 \end{aligned}$$

j
$$\begin{aligned} j + 27 &= 180 && \{\text{co-interior angles on parallel lines are supplementary}\} \\ \therefore\ j + 27 - \mathbf{27} &= 180 - \mathbf{27} && \{\text{subtracting 27 from both sides}\} \\ \therefore\ j &= 153 \end{aligned}$$

k
$$\begin{aligned} n + 175 + 120 &= 360 && \{\text{angles at a point}\} \\ \therefore\ n + 295 &= 360 && \{\text{simplifying}\} \\ \therefore\ n + 295 - \mathbf{295} &= 360 - \mathbf{295} && \{\text{subtracting 295 from both sides}\} \\ \therefore\ n &= 65 \end{aligned}$$

l $t = 155$ {equal alternate angles on parallel lines}

m
$$\begin{aligned} m + m &= 180 && \{\text{angles on a straight line}\} \\ \therefore\ 2m &= 180 && \{\text{simplifying}\} \\ \therefore\ \frac{2m}{\mathbf{2}} &= \frac{180}{\mathbf{2}} && \{\text{dividing both sides by 2}\} \\ \therefore\ m &= 90 \end{aligned}$$

n $x = 48$ {equal corresponding angles on parallel lines}
$\therefore\ y = 48$ {vertically opposite angles are equal}

o $a + 115 = 180$ {angles on a straight line}
$\therefore\ a + 115 - \mathbf{115} = 180 - \mathbf{115}$ {subtracting 115 from both sides}
$\therefore\ a = 65$ {simplifying}
$\therefore\ b = 65$ {equal corresponding angles on parallel lines}

6 **a** $a + a + 90 = 360$ {angles at a point}
$\therefore\ 2a + 90 = 360$ {simplifying}
$\therefore\ 2a + 90 - \mathbf{90} = 360 - \mathbf{90}$ {subtracting 90 from both sides}
$\therefore\ 2a = 270$ {simplifying}
$\therefore\ a = 135$ {dividing both sides by 2}

b $3b + 48 = 90$ {complementary angles}
$\therefore\ 3b + 48 - \mathbf{48} = 90 - \mathbf{48}$ {subtracting 48 from both sides}
$\therefore\ 3b = 42$ {simplifying}
$\therefore\ b = 14$ {dividing both sides by 3}

c $c + 2c = 180$ {angles on a straight line; equal corresponding angles on parallel lines}
$\therefore\ 3c = 180$ {simplifying}
$\therefore\ c = 60$ {dividing both sides by 3}

d $d + (d + 80) = 180$ {co-interior angles on parallel lines are supplementary}
$\therefore\ 2d + 80 = 180$ {simplifying}
$\therefore\ 2d + 80 - \mathbf{80} = 180 - \mathbf{80}$ {subtracting 80 from both sides}
$\therefore\ 2d = 100$ {simplifying}
$\therefore\ d = 50$ {dividing both sides by 2}

e $(3x + 5) + 2x = 180$ {angles on a straight line}
$\therefore\ 5x + 5 = 180$ {simplifying}
$\therefore\ 5x + 5 - \mathbf{5} = 180 - \mathbf{5}$ {subtracting 5 from both sides}
$\therefore\ 5x = 175$ {simplifying}
$\therefore\ x = 35$ {dividing both sides by 5}

f $2x - 25 = x + 10$ {equal alternate angles on parallel lines}
$\therefore\ 2x - 25 - \boldsymbol{x} = x + 10 - \boldsymbol{x}$ {subtracting x from both sides}
$\therefore\ x - 25 = 10$ {simplifying}
$\therefore\ x - 25 + \mathbf{25} = 10 + \mathbf{25}$ {adding 25 to both sides}
$\therefore\ x = 35$ {simplifying}

g $2x = 156 - x$ {equal corresponding angles on parallel lines}
$\therefore\ 2x + \boldsymbol{x} = 156 - x + \boldsymbol{x}$ {adding x to both sides}
$\therefore\ 3x = 156$ {simplifying}
$\therefore\ x = 52$ {dividing both sides by 3}

h $(2x - 34) + (x + 100) = 180$ {angles on a straight line}
$\therefore\ 3x + 66 = 180$ {simplifying}
$\therefore\ 3x + 66 - \mathbf{66} = 180 - \mathbf{66}$ {subtracting 66 from both sides}
$\therefore\ 3x = 114$ {simplifying}
$\therefore\ x = 38$ {dividing both sides by 3}

i $(2x - 10) + (x + 30) + x + 28 = 360$ {angles at a point}
$\therefore\ 4x + 48 = 360$ {simplifying}
$\therefore\ 4x + 48 - \mathbf{48} = 360 - \mathbf{48}$ {subtracting 48 from both sides}
$\therefore\ 4x = 312$ {simplifying}
$\therefore\ x = 78$ {dividing both sides by 4}

7 **a** $150°$ and $39°$ are co-interior angles, and $150° + 39° = 189°$.
Since these co-interior angles do not add to $180°$, [AB] is *not* parallel to [CD].

b The two angles marked $120°$ are corresponding angles.
Since these corresponding angles are equal, [AB] is parallel to [CD].

c The two angles marked $76°$ are alternate angles.
Since these alternate angles are equal, [AB] is parallel to [CD].

EXERCISE 8B

1 **a** Two sides are equal in length.
$\therefore$ the triangle is isosceles.

b Each side has a different length.
$\therefore$ the triangle is scalene.

c All three sides are equal in length.
$\therefore$ the triangle is equilateral.

2 **a** All angles are between $0°$ and $90°$.
$\therefore$ the triangle is acute angled.

b One of the angles is $90°$.
$\therefore$ the triangle is right angled.

c One of the angles is between $90°$ and $180°$.
$\therefore$ the triangle is obtuse angled.

3 **a**
$$\begin{aligned} a + 30 + 20 &= 180 && \{\text{angle sum of triangle}\} \\ \therefore\ a + 50 &= 180 && \{\text{simplifying}\} \\ \therefore\ a + 50 - \mathbf{50} &= 180 - \mathbf{50} && \{\text{subtracting 50 from both sides}\} \\ \therefore\ a &= 130 && \{\text{simplifying}\} \end{aligned}$$

b
$$\begin{aligned} b + 57 + 33 &= 180 && \{\text{angle sum of triangle}\} \\ \therefore\ b + 90 &= 180 && \{\text{simplifying}\} \\ \therefore\ b + 90 - \mathbf{90} &= 180 - \mathbf{90} && \{\text{subtracting 90 from both sides}\} \\ \therefore\ b &= 90 && \{\text{simplifying}\} \end{aligned}$$

c
$$\begin{aligned} c + 47 + 90 &= 180 && \{\text{angle sum of triangle}\} \\ \therefore\ c + 137 &= 180 && \{\text{simplifying}\} \\ \therefore\ c + 137 - \mathbf{137} &= 180 - \mathbf{137} && \{\text{subtracting 137 from both sides}\} \\ \therefore\ c &= 43 && \{\text{simplifying}\} \end{aligned}$$

d
$$\begin{aligned} d &= 41 + 88 && \{\text{exterior angle of a triangle}\} \\ \therefore\ d &= 129 \end{aligned}$$

e
$$\begin{aligned} e &= 28 + 25 && \{\text{exterior angle of a triangle}\} \\ \therefore\ e &= 53 \end{aligned}$$

f
$$\begin{aligned} x + 36 &= 167 && \{\text{exterior angle of a triangle}\} \\ \therefore\ x + 36 - \mathbf{36} &= 167 - \mathbf{36} && \{\text{subtracting 36 from both sides}\} \\ \therefore\ x &= 131 && \{\text{simplifying}\} \end{aligned}$$

4 Let the third angle be $x°$.
$$\begin{aligned} x + 58 + 72 &= 180 && \{\text{angle sum of triangle}\} \\ \therefore\ x + 130 &= 180 && \{\text{simplifying}\} \\ \therefore\ x + 130 - \mathbf{130} &= 180 - \mathbf{130} && \{\text{subtracting 130 from both sides}\} \\ \therefore\ x &= 50 && \{\text{simplifying}\} \end{aligned}$$
So, the third angle in Nancy's spinach triangle is $50°$.

5 **a** The sum of the angles in a triangle is $180°$.
Two right angles have a sum of $90° + 90° = 180°$.
$\therefore$ the statement is true.

b A right angled triangle contains a $90°$ angle, which is not obtuse.

$\therefore$ the other two angles must add to $90°$ {the angle sum of a triangle is $180°$}.

$\therefore$ the other two angles must be acute.

$\therefore$ the statement is false.

c In a right angled triangle, one angle is $90°$.

$\therefore$ the other two angles must add to $90°$ {the angle sum of a triangle is $180°$}.

$\therefore$ in this case, the sum of two angles of a triangle is *not* greater than the third angle.

$\therefore$ the statement is false.

d In a right angled triangle, one angle is $90°$.

$\therefore$ the other two angles must add to $90°$ {the angle sum of a triangle is $180°$}.

$\therefore$ the two smaller angles are complementary.

$\therefore$ the statement is false.

6 **a** The largest angle is $72°$, which is at C.

$\therefore$ the longest side is [AB].

b The third angle is $180° - 118° - 26° = 36°$ {angle sum of a triangle}

$\therefore$ the largest angle is $118°$, which is at B.

$\therefore$ the longest side is [AC].

c The third angle is $180° - 33° - 33° = 114°$ {angle sum of a triangle}

$\therefore$ the largest angle is $114°$, which is at A.

$\therefore$ the longest side is [BC].

d The third angle is $180° - 31° - 28° = 121°$ {angle sum of a triangle}

$\therefore$ the largest angle is $121°$, which is at C.

$\therefore$ the longest side is [AB].

e The third angle is $180° - 88° - 81° = 11°$ {angle sum of triangle}

$\therefore$ the largest angle is $88°$, which is at A.

$\therefore$ the longest side is [BC].

f The third angle is $180° - 108° - 26° = 46°$ {angle sum of triangle}

$\therefore$ the largest angle is $108°$, which is at C.

$\therefore$ the longest side is [AB].

g The third angle is $180° - 50° - 65° = 65°$ {angle sum of triangle}

$\therefore$ the two largest angles are each $65°$, and are at A and C.

$\therefore$ the two longest sides are [AB] and [BC].

h In any right angled triangle, the largest angle is the right angle ($90°$).
So, the longest side on this triangle is opposite the right angle at B.

$\therefore$ the longest side is [AC].

i The third angle is $180° - 77° - 27° = 76°$ {angle sum of triangle}

$\therefore$ the largest angle is $77°$, which is at B.

$\therefore$ the longest side is [AC].

7 **a**
$$\begin{aligned}(a+25)+a+(a-10)&=180 &&\text{\{angles sum of triangle\}}\\ \therefore\ 3a+15&=180 &&\text{\{collecting like terms\}}\\ \therefore\ 3a&=165\\ \therefore\ a&=55\end{aligned}$$

b
$$\begin{aligned}b+b+(2b-40)&=180 &&\text{\{angles sum of triangle\}}\\ \therefore\ 4b-40&=180 &&\text{\{collecting like terms\}}\\ \therefore\ 4b&=220\\ \therefore\ b&=55\end{aligned}$$

c $x + 80 = 140 - 2x$ {exterior angle of a triangle}
$\therefore\ 3x + 80 = 140$ {adding $2x$ to both sides}
$\therefore\ 3x = 60$
$\therefore\ x = 20$

d $a = 55$ {equal corresponding angles on parallel lines}
$b + 55 + 48 = 180$ {angle sum of triangle}
$\therefore\ b + 103 = 180$ {collecting like terms}
$\therefore\ b = 77$

e $3x - 90 = x$ {vertically opposite angles}
$\therefore\ 2x - 90 = 0$ {subtracting x from both sides}
$\therefore\ 2x = 90$
$\therefore\ x = 45$

$y + 65 + 45 = 180$ {angle sum of triangle}
$\therefore\ y + 110 = 180$
$\therefore\ y = 70$

f $a = 45$ {vertically opposite angles}
$b = 85$ {vertically opposite angles}
$c = 45 + 85$ {exterior angle of a triangle}
$\therefore\ c = 130$

$d + 130 = 180$ {angles on a line}
$\therefore\ d = 50$

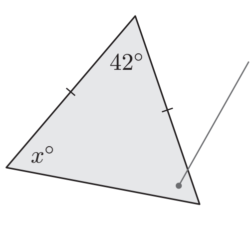
EXERCISE 8C

1 **a** In an isosceles triangle, the line joining the apex to the midpoint of the base meets the base at right angles.
So, if the line joining one vertex to the midpoint of the opposite side is perpendicular to that side, then the triangle must be isosceles. (The vertex is the apex of the triangle, and the opposite side is its base.)
$\therefore$ yes, the statement is true.

b In an isosceles triangle, the line joining the apex to the midpoint of the base bisects the vertical angle.
So, if the line joining one vertex to the midpoint of the opposite side bisects the angle at the vertex, then the triangle must be isosceles. (The vertex is the apex of the triangle, and the opposite side is its base.)
$\therefore$ yes, the statement is true.

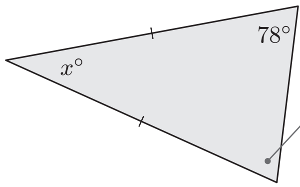
c In an isosceles triangle, the line joining the apex to the midpoint of the base bisects the vertical angle and meets the base at right angles.
So, if the perpendicular to one side of the triangle passes through a vertex and bisects the angle at that vertex, then the triangle must be isosceles. (The vertex is the apex of the triangle, and the side with the perpendicular is the base.)
$\therefore$ yes, the statement is true.

2 **a**



Two of the sides are equal in length, so the triangle is isosceles.
So, this angle is also $x°$. {isosceles triangle theorem}
$\therefore\ x + x + 42 = 180$ {angle sum of triangle}
$\therefore\ 2x + 42 = 180$
$\therefore\ 2x = 138$
$\therefore\ x = 69$

b



Two of the sides are equal in length, so the triangle is isosceles.

So, this angle is also 78°. {isosceles triangle theorem}

$\therefore \quad x + 78 + 78 = 180$ {angle sum of triangle}

$\therefore \quad x + 156 = 180$

$\therefore \quad x = 24$

c Two of the sides are equal in length, so the triangle is isosceles.

$\therefore \quad x = 90 - x$ {isosceles triangle theorem}

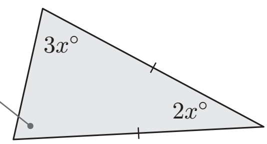
$\therefore \quad 2x = 90$

$\therefore \quad x = 45$

d Two of the sides are equal in length, so the triangle is isosceles.

So, this angle is also $3x°$.

{isosceles triangle theorem}

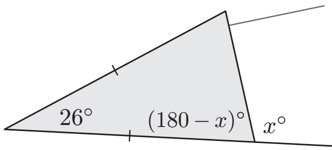


$\therefore \quad 3x + 3x + 2x = 180$ {angle sum of triangle}

$\therefore \quad 8x = 180$

$\therefore \quad x = 22.5$

e Two of the sides are equal in length, so the triangle is isosceles.



So, this angle is also $(180 - x)°$. {isosceles triangle theorem}

$\therefore \quad 26 + (180 - x) + (180 - x) = 180$

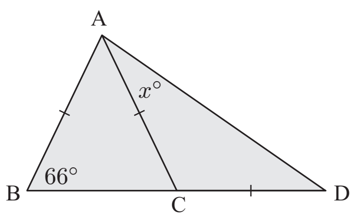
{angles sum of triangle}

$\therefore \quad 386 - 2x = 180$

$\therefore \quad -2x = -206$

$\therefore \quad x = 103$

f



Since AB = AC, △ABC is isosceles.

$\therefore$ A$\widehat{C}$B = 66° {isosceles triangle theorem}

Since AC = CD, △ACD is isosceles as well.

$\therefore$ A$\widehat{D}$C = $x°$ {isosceles triangle theorem}

$\therefore \quad 66 = x + x$ {exterior angle of a triangle}

$\therefore \quad 2x = 66$

$\therefore \quad x = 33$

3 **a** Since A$\widehat{B}$C = A$\widehat{C}$B, △ABC is isosceles. {converse of isosceles triangle theorem}

$\therefore$ AB = AC

$\therefore \quad x = 12$

b Since Q$\widehat{S}$R = Q$\widehat{R}$S, △QRS is isosceles. {converse of isosceles triangle theorem}

$\therefore$ QS = QR

$\therefore$ QS = 8 cm

Now, since S$\widehat{Q}$P = S$\widehat{P}$Q, △SPQ is also isosceles. {converse of isosceles triangle theorem}

$\therefore$ QS = PS

$\therefore \quad x = 8$

c Since XZ = XY, △XZY is isosceles.

Now ZM = MY, which means M is the midpoint of the base of △XZY.

$\therefore$ [XM] ⊥ [ZY] {isosceles triangle theorem}

$\therefore \quad x = 90$

d Since $K\widehat{J}M = K\widehat{L}M$, $\triangle KJL$ is isosceles. {converse of isosceles triangle theorem}

Now $JM = ML$, which means M is the midpoint of the base of $\triangle KJL$.

$\therefore$ $[KM] \perp [JL]$ {isosceles triangle theorem}

$\therefore$ $J\widehat{M}K = 90°$

$\therefore$ $40 + 90 + x = 180$ {angle sum of $\triangle JMK$}

$\therefore$ $x + 130 = 180$

$\therefore$ $x = 50$

e $R\widehat{P}Q + 57° + 66° = 180°$ {angle sum of triangle}

$\therefore$ $R\widehat{P}Q + 123° = 180°$

$\therefore$ $R\widehat{P}Q = 57°$

So, $R\widehat{P}Q = R\widehat{Q}P = 57°$, which means $\triangle RQP$ is isosceles {converse of isosceles triangle theorem}

$\therefore$ $RQ = RP$

$\therefore$ $x = 5$

f $BC = AC$, so $\triangle CAB$ is isosceles.

Now, since $D\widehat{C}A = D\widehat{C}B$, DC is the angle bisector of the apex of this triangle.

$\therefore$ $[DC] \perp [AB]$ {converse of isosceles triangle theorem}

$\therefore$ $x = 90$

g Since $WM = MY$, M is the midpoint of [WY].

Now, $[ZM] \perp [WY]$, and connects the apex of the triangle to the midpoint of the base.

$\therefore$ $\triangle ZWY$ is isosceles with $ZW = ZY$ {converse of isosceles triangle theorem}

$\therefore$ $Z\widehat{W}M = Z\widehat{Y}M = 20°$ {isosceles triangle theorem}

$\therefore$ $x + 20 + 20 = 180$ {angle sum of $\triangle ZWY$}

$\therefore$ $x + 40 = 180$

$\therefore$ $x = 140$

h Since $ED = EF$, $\triangle EDF$ is isosceles.

$\therefore$ $E\widehat{F}D = E\widehat{D}F = 60°$ {isosceles triangle theorem}

$\therefore$ $D\widehat{E}F + 60° + 60° = 180°$ {angle sum of $\triangle EDF$}

$\therefore$ $D\widehat{E}F + 120° = 180°$

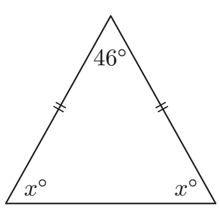
$\therefore$ $D\widehat{E}F = 60°$

So, all angles of $\triangle EDF$ are equal.

$\therefore$ $\triangle EDF$ is equilateral.

$\therefore$ $x = 4$

4



Since two of the sides are equal, the triangle is isosceles.

$\therefore$ the base angles are equal. {isosceles triangle theorem}

Let the base angles each be $x°$.

$\therefore$ $x + x + 46 = 180$ {angle sum of triangle}

$\therefore$ $2x + 46 = 180$

$\therefore$ $2x = 134$

$\therefore$ $x = 67$

So, the other two angles each measure 67°.

5 **a** $x + 48 + (2x - 66) = 180$ {angle sum of triangle}

$\therefore$ $3x - 18 = 180$

$\therefore$ $3x = 198$

$\therefore$ $x = 66$

b If $x = 66$, then $A\widehat{B}C = 66°$, $B\widehat{C}A = 48°$, and $C\widehat{A}B = 2 \times 66 - 66$
$= 66°$

$\therefore$ $\triangle ABC$ is isosceles with $BC = AC$.

6 **a** Since $BC = CD$, $\triangle CBD$ is isosceles.

$\therefore$ $C\widehat{B}X = C\widehat{D}X$ {isosceles triangle theorem}

$\therefore$ $C\widehat{B}X + C\widehat{D}X + 20° + 20° = 180°$ {angle sum of $\triangle CBD$}

$\therefore$ $2 \times C\widehat{B}X + 40° = 180°$ {base angles are equal}

$\therefore$ $2 \times C\widehat{B}X = 140°$

$\therefore$ $C\widehat{B}X = 70°$

$\therefore$ $C\widehat{D}X = 70°$ also

Now $48° + A\widehat{B}D + 70° + 20° + 20° + 70° + 84° = 360°$ {angle sum of quadrilateral ABCD}

$\therefore$ $A\widehat{B}D + 312° = 360°$

$\therefore$ $A\widehat{B}D = 48°$

b Since $A\widehat{B}D = B\widehat{A}D = 48°$, $\triangle ABD$ is isosceles. {converse of isosceles triangle theorem}

c From **a**, $\triangle CBD$ is isosceles, with $BC = CD$ and [CX] is the angle bisector of the apex of this triangle.

$\therefore$ [CX] bisects the base [BD] at right angles. {converse of isosceles triangle theorem}

$\therefore$ X is the midpoint of [BD].

$\therefore$ $XD = x$ cm and $BD = x + x$
$= 2x$ cm

Since $AD = BD$ {$\triangle ABD$ is isosceles, from **b**}

then $2x = 6$

$\therefore$ $x = 3$

7 The base angles of an isosceles triangle are equal. {isosceles triangle theorem}

$\therefore$ the angles of the triangle are $(5x - 12)°$, $(5x - 12)°$, and $(4x + 8)°$.

$\therefore$ $(5x - 12) + (5x - 12) + (4x + 8) = 180$ {angle sum of triangle}

$\therefore$ $14x - 16 = 180$

$\therefore$ $14x = 196$

$\therefore$ $x = 14$

If $x = 14$, then $5x - 12 = 5 \times 14 - 12 = 58$

and $4x + 8 = 4 \times 14 + 8 = 64$

So, the base angles are 58° and the apex angle is 64°.

EXERCISE 8D.1

1 **a** $x + 110 + 105 + 80 = 360$ {angle sum of quadrilateral}

$\therefore$ $x + 295 = 360$

$\therefore$ $x = 65$

b $x + 90 + 121 + 56 = 360$ {angle sum of quadrilateral}

$\therefore$ $x + 267 = 360$

$\therefore$ $x = 93$

c $x + 235 + 40 + 50 = 360$ {angle sum of quadrilateral}

$\therefore$ $x + 325 = 360$

$\therefore$ $x = 35$

d $x + 3x + 90 + 110 = 360$ {angle sum of quadrilateral}

$\therefore$ $4x + 200 = 360$

$\therefore$ $4x = 160$

$\therefore$ $x = 40$

e $x + (2x + 10) + 130 + (x + 20) = 360$ {angle sum of quadrilateral}

$\therefore \quad 4x + 160 = 360$

$\therefore \quad 4x = 200$

$\therefore \quad x = 50$

f $x + x + x + x = 360$ {angle sum of quadrilateral}

$\therefore \quad 4x = 360$

$\therefore \quad x = 90$

2 **a** $a + 115 = 180$ {angles on a line}

$\therefore \quad a = 65$

In the quadrilateral, the fourth angle is $360° - 65° - 100° - 106° = 89°$.
{angles of a quadrilateral add to 360°}

$\therefore \quad b + 89 = 180$ {angles on a line}

$\therefore \quad b = 91$

b $(2m - 5) + 90 + m + 95 = 360$ {angle sum of quadrilateral}

$\therefore \quad 3m + 180 = 360$

$\therefore \quad 3m = 180$

$\therefore \quad m = 60$

and so $2m - 5 = 2 \times 60 - 5$
$= 115$

$\therefore \quad n + 115 = 180$ {angles on a line}

$\therefore \quad n = 65$

c The fourth angle in the quadrilateral is $(180 - a)°$. {angles on a line}

$\therefore \quad (180 - a) + (a - 15) + (a + 5) + (2a - 20) = 360$ {angle sum of quadrilateral}

$\therefore \quad 3a + 150 = 360$

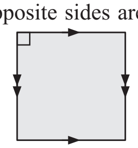
$\therefore \quad 3a = 210$

$\therefore \quad a = 70$

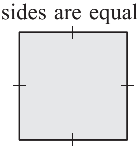
EXERCISE 8D.2

1 **a** In a square:

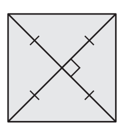
- opposite sides are parallel



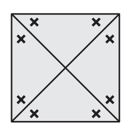
- all sides are equal in length



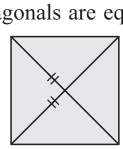
- diagonals bisect each other at right angles



- diagonals bisect the angles at each vertex

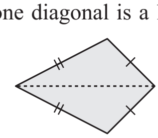


- diagonals are equal in length

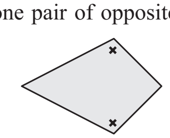


b In a kite:

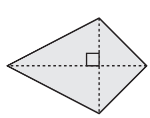
- one diagonal is a line of symmetry



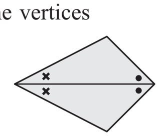
- one pair of opposite angles are equal



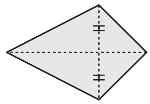
- diagonals cut each other at right angles



- **one** diagonal bisects **one** pair of angles at the vertices

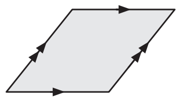


- one of the diagonals bisects the other

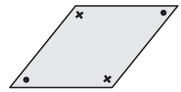


c In a rhombus:

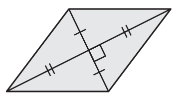
- opposite sides are parallel



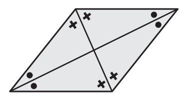
- opposite angles are equal in size



- diagonals bisect each other at right angles



- diagonals bisect the angles at each vertex



2 **a** Two pairs of adjacent sides are equal in length, so the figure is a kite.

$\therefore\ x = 90$ {diagonals of a kite intersect at right angles}

$\therefore\ y = 5$

b Opposite sides are parallel, so the figure is a parallelogram.

$\therefore\ 2x + (x + 30) = 180$ {co-interior angles on parallel lines}

$\therefore\ 3x + 30 = 180$

$\therefore\ 3x = 150$

$\therefore\ x = 50$

c All sides are equal in length, so the figure is a rhombus.

$\therefore\ x + 60 = 4x$ {opposite angles of a rhombus are equal in size}

$\therefore\ 3x = 60$

$\therefore\ x = 20$

d All sides are equal, so the figure is a rhombus.

$\therefore\ y = 90$ {diagonals of a rhombus bisect each other at right angles}

$\therefore\ x + 30 = 90$ {exterior angle of a triangle}

$\therefore\ x = 60$

e Opposite sides are parallel, so the figure is a parallelogram.

$\therefore\ a = 6,\ \ b = 8$ {diagonals of a parallelogram bisect each other}

f All angles are 90°, so the figure is a rectangle.

$\therefore\ x = 14$ {diagonals of a rectangle are equal in length}

3 **a** A square has four equal sides.

$\therefore$ the statement is true.

b There are *two* quadrilaterals in which all sides are equal: the rhombus, and the square.

$\therefore$ the statement is false.

c The diagonals of a parallelogram are not always equal in length (they are only equal in the special cases of the rectangle and the square).

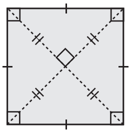
$\therefore$ the statement is false.

d The diagonals of a kite cut each other at right angles.

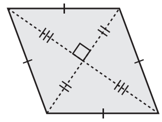
$\therefore$ the statement is true.

4 **a** The diagonals intersect at right angles, so the three possibilities are:

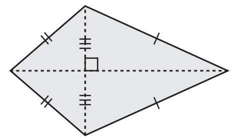
a square



a rhombus



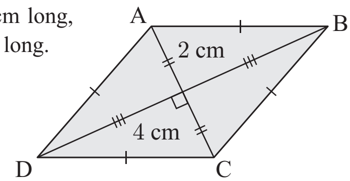
a kite



b If one diagonal is twice the length of the other then the quadrilateral cannot be a square (which has diagonals equal in length).

The diagonals bisect each other, so the quadrilateral must be a rhombus.

c If the shorter diagonal [AC] is 4 cm long, then diagonal [BD] must be 8 cm long.



5 **a** One pair of sides is parallel, so the figure is a trapezium.

$\therefore\ a + 130 = 180$ {co-interior angles on parallel lines}

$\therefore\ a = 50$

$\therefore\ 2a - 40 = 2 \times 50 - 40$

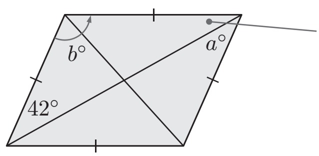
$= 60$

$\therefore\ b + 60 = 180$ {co-interior angles on parallel lines}

$\therefore\ b = 120$

b All sides are equal in length, so the figure is a rhombus.

$\therefore\ a = 42$ {opposite angles of a rhombus are equal; diagonals of a rhombus bisect the angles}



this angle is also 42°

{diagonals of a rhombus bisect the angles}

$\therefore\ b + 42 + 42 = 180$ {angle sum of triangle}

$\therefore\ b + 84 = 180$

$\therefore\ b = 96$

c Opposite sides are parallel, so the figure is a parallelogram.

$\therefore\ 3x + 2 = 10 - x$ {opposite sides of a parallelogram are equal in length}

$\therefore\ 4x + 2 = 10$

$\therefore\ 4x = 8$

$\therefore\ x = 2$

and $a = 40$ {alternate angles on parallel lines}

6 Opposite sides are parallel, so the figure is a parallelogram.

$\therefore\ x = 180 - x$ {opposite angles of a parallelogram are equal in size}

$\therefore\ 2x = 180$

$\therefore\ x = 90$

So, ABCD is a parallelogram with angles of 90°.

$\therefore$ ABCD is a rectangle.

7 A square is a parallelogram with all sides equal in length and all angles equal in size.

A square is also a rhombus with all angles equal in size.

$\therefore$ the statement is true.

8 $x + x + 2x = 180$ {angle sum of triangle}

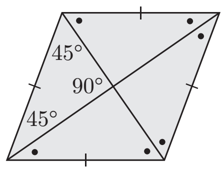
$\therefore\ 4x = 180$

$\therefore\ x = 45$

The figure is a parallelogram, since opposite sides are parallel.

Since $x = 45$, $2x = 2 \times 45 = 90$, so the diagonals intersect at right angles.

$\therefore$ the figure is also a rhombus.



$\therefore$ the angles marked ● are all 45°

{diagonals of a rhombus bisect the angles, opposite angles of a rhombus are equal in size}

$\therefore$ all angles of the rhombus are 90°.

$\therefore$ the special name for this quadrilateral is a square.

REVIEW SET 8

1 **a** $a = 142$ {vertically opposite angles are equal}

b $b + 50 = 180$ {angles on a straight line}

$\therefore\ b = 130$

c $c + c = 90$ {complementary angles}

$\therefore\ 2c = 90$

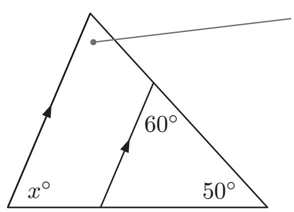
$\therefore\ c = 45$

2 **a** $a = 120$ {corresponding angles on parallel lines}

$b + 120 = 180$ {angles on a straight line}

$\therefore\ b = 60$

b



This angle is 60°. {corresponding angles on parallel lines}

$\therefore\ x + 60 + 50 = 180$ {angle sum of triangle}

$\therefore\ x + 110 = 180$

$\therefore\ x = 70$

c $a = 55$ {vertically opposite angles are equal}

$b + 100 = 180$ {angles on a straight line}

$\therefore\ b = 80$

$c + 55 + 80 = 180$ {angle sum of triangle}

$\therefore\ c + 135 = 180$

$\therefore\ c = 45$

3 **a** The third angle is $180° - 52° - 38° = 90°$ {angle sum of triangle}

$\therefore$ the triangle is right angled.

Since all of the sides are different lengths, the triangle is scalene.

b The third angle is $180° - 25° - 130° = 25°$ {angle sum of triangle}

Two angles are equal, so the triangle is isosceles. {converse of isosceles triangle theorem}

Since one angle is greater than 90°, the triangle is obtuse angled.

c All of the angles are equal in size, so the triangle is equilateral.

Now $x + x + x = 180$ {angle sum of triangle}

$\therefore\ 3x = 180$

$\therefore\ x = 60$

Since all of the angles are acute, the triangle is acute angled.

4 **a** All of the sides are the same length, so the figure is a rhombus.

$\therefore\ x = 100$ {opposite angles of a rhombus are equal in size}

$y + 100 = 180$ {opposite sides of a rhombus are parallel, co-interior angles on parallel lines}

$\therefore\ y = 80$

b $x + 90 = 180$ {co-interior angles on parallel lines}

$\therefore\ x = 90$

$y + 2y = 180$ {co-interior angles on parallel lines}

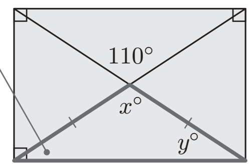
$\therefore\ 3y = 180$

$\therefore\ y = 60$

c $x = 110$ {vertically opposite angles are equal}

Since the diagonals of a rectangle are equal in length and bisect each other, we have an isosceles triangle.

$\therefore$ this angle is also $y°$ {isosceles triangle theorem}



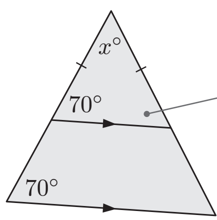
$\therefore\ 110 + y + y = 180$ {angle sum of triangle}

$\therefore\ 2y + 110 = 180$

$\therefore\ 2y = 70$

$\therefore\ y = 35$

d $y = 70°$ {corresponding angles on parallel lines}



Now, in the smaller triangle two sides are the same length, so the triangle is isosceles.

$\therefore$ this angle is also 70° {isosceles triangle theorem}

$\therefore\ x + 70 + 70 = 180$ {angle sum of triangle}

$\therefore\ x + 140 = 180$

$\therefore\ x = 40$

e Adjacent pairs of sides are equal, so the figure is a kite.

Since the two known opposite angles are different, this means $x = y$.

{In a kite, one pair of opposite angles is equal in size.}

$\therefore\ 116 + x + 48 + x = 360$ {angle sum of quadrilateral}

$\therefore\ 2x + 164 = 360$

$\therefore\ 2x = 196$

$\therefore\ x = 98$

$\therefore\ y = 98$

f Two sides are equal in length, so the triangle is isosceles.

The line connecting the apex and the base bisects the angle at the apex,

$\therefore$ it bisects the base at right angles.

$\therefore\ x + x = 8$

$\therefore\ 2x = 8$

$\therefore\ x = 4$

5 **a** The diagonals bisect each other at right angles, and are equal in length.

$\therefore$ the figure is a square.

b All sides are equal in length.

No other information is given, so the figure is a rhombus.

c Opposite sides are parallel, and one angle is 90°.

$\therefore$ all angles must be 90°. {corresponding angles on parallel lines}

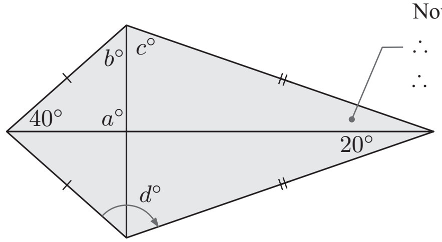
$\therefore$ the figure is a rectangle.

6 **a** Two of the angles are equal in size, so the triangle is isosceles.
{converse 1 of the isosceles triangle theorem}
$\therefore \quad x = 12$ {equal sides of an isosceles triangle}

b Two of the sides are equal in length, so the triangle is isosceles.
$\therefore \quad x = 90$ {isosceles triangle theorem}
$\therefore \quad y = 42$ {isosceles triangle theorem}

c $116 + 93 + (x + 11) + x = 360$ {angle sum of quadrilateral}
$\therefore \quad 2x + 220 = 360$
$\therefore \quad 2x = 140$
$\therefore \quad x = 70$

7 Two pairs of adjacent sides are equal in length, so the figure is a kite.
$\therefore \quad a = 90$ {diagonals of a kite intersect at right angles}
$b + 40 + 90 = 180$ {angle sum of triangle}
$\therefore \quad b + 130 = 180$
$\therefore \quad b = 50$



Now, the longer diagonal of a kite is a line of symmetry.
$\therefore$ this angle is also 20°.
$\therefore \quad 90 + c + 20 = 180$ {angle sum of triangle}
$\therefore \quad c + 110 = 180$
$\therefore \quad c = 70$
$d = 50 + 70$
{one pair of opposite angles are equal in size}
$\therefore \quad d = 120$

PRACTICE TEST 8A

1 182° is between 180° and 360°.
$\therefore$ 182° is a reflex angle.
$\therefore$ the answer is **E**.

2 $x + x + 16 = 90$ {complementary angles}
$\therefore \quad 2x + 16 = 90$
$\therefore \quad 2x = 74$
$\therefore \quad x = 37$
$\therefore$ the answer is **C**.

3 $y + (y + 40) = 180$ {co-interior angles on parallel lines}
$\therefore \quad 2y + 40 = 180$
$\therefore \quad 2y = 140$
$\therefore \quad y = 70$
$\therefore$ the answer is **D**.

4 Triangle **B** has angles of 90° and 60°.
$\therefore$ the third angle is $180° - 90° - 60° = 30°$ {angle sum of triangle}
Since none of the angles are the same size, the triangle cannot be isosceles. {isosceles triangle theorem}
$\therefore$ the answer is **B**.

5 $a + 35 = 120$ {exterior angle of a triangle}
$\therefore \quad a = 85$
$\therefore$ the answer is **C**.

6 One pair of opposite sides is parallel.
Now, the two marked angles are co-interior, and have a sum of $60° + 120° = 180°$.
$\therefore$ the other pair of sides is parallel also.
So, the figure is a parallelogram.
$\therefore$ the answer is **A**.

7 $p + 3p + (p + 5) = 180$ {angle sum of triangle}
$\therefore\ 5p + 5 = 180$
$\therefore\ 5p = 175$
$\therefore\ p = 35$
$\therefore$ the answer is **B**.

8 The angles shown are corresponding angles.
$\therefore$ the answer is **D**.

9 Opposite sides are equal in length, and one of the angles is 90°.
$\therefore$ the figure is a rectangle.
$\therefore\ x = 15 + 15$ {diagonals of a rectangle are equal in length}
$\therefore\ x = 30$
$\therefore$ the answer is **E**.

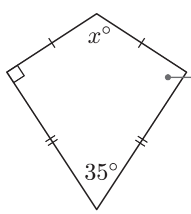
10 $x + 90 + 70 + x + 120 = 360$ {angles at a point}
$\therefore\ 2x + 280 = 360$
$\therefore\ 2x = 80$
$\therefore\ x = 40$
$\therefore$ the answer is **B**.

PRACTICE TEST 8B

1 $a + 140 + 120 = 360$ {angles at a point}
$\therefore\ a + 260 = 360$
$\therefore\ a = 100$

2 **a** $150 - x = x + 40$ {vertically opposite angles are equal}
$\therefore\ 2x + 40 = 150$
$\therefore\ 2x = 110$
$\therefore\ x = 55$

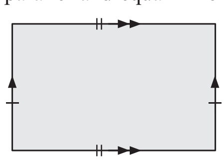
b



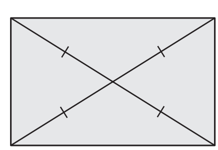
Two adjacent pairs of sides are equal in length.
$\therefore$ the figure is a kite.
$\therefore$ this angle is also 90°
{in a kite, one pair of opposite angles is equal in size}
$\therefore\ x + 90 + 35 + 90 = 360$ {angle sum of quadrilateral}
$\therefore\ x + 215 = 360$
$\therefore\ x = 145$

3 In a rectangle:

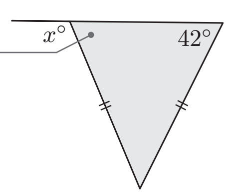
- all angles are equal in size, and opposing sides are parallel and equal in length



- the diagonals bisect each other and are equal in length

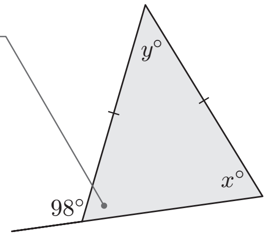


4 **a** Two sides are equal, so the triangle is isosceles.
$\therefore$ this angle is also 42°
{isosceles triangle theorem}
$\therefore\ x + 42 = 180$ {angles on a straight line}
$\therefore\ x = 138$

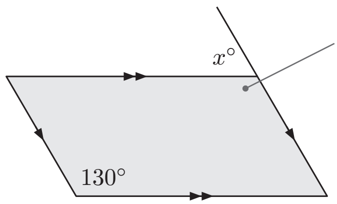


b $2x + 6x + x = 180$ {angle sum of triangle}
$\therefore\ 9x = 180$
$\therefore\ x = 20$

5 Two sides are equal, so the triangle is isosceles.
$\therefore$ this angle is also $x°$
{isosceles triangle theorem}
$\therefore\ x + 98 = 180$ {angles on a straight line}
$\therefore\ x = 82$
$\therefore\ 82 + y + 82 = 180$ {angle sum of triangle}
$\therefore\ y + 164 = 180$
$\therefore\ y = 16$



6



Opposite sides are parallel, so the figure is a parallelogram.
$\therefore$ this angle is also 130°
{opposite angles of a parallelogram are equal}
$\therefore\ x + 130 = 180$ {angles on a straight line}
$\therefore\ x = 50$

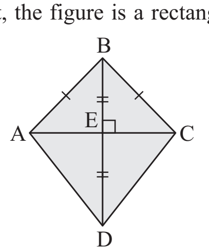
7 **a** Line l is a transversal.
b The two marked angles are co-interior angles.
c If the marked co-interior angles are supplementary, then [AB] $\parallel$ [CD].

8 $A\widehat{D}B + 50° + 40° = 180°$ {angles on a straight line}
$\therefore\ A\widehat{D}B + 90° = 180°$
$\therefore\ A\widehat{D}B = 90°$
Now, $90 + x + x = 180$ {angle sum of $\triangle$ADB}
$\therefore\ 2x + 90 = 180$
$\therefore\ 2x = 90$
$\therefore\ x = 45$

$\triangle$ADE is isosceles, since $A\widehat{E}D = A\widehat{D}E$ {converse of isosceles triangle theorem}
$\therefore$ AD = 4 cm {equal sides of isosceles triangle}
$\triangle$ADB is also isosceles, since $A\widehat{B}D = B\widehat{A}D$ {converse of isosceles triangle theorem}
$\therefore$ BD = 4 cm {equal sides of isosceles triangle}
$\triangle$BDC is also isosceles, since $B\widehat{D}C = B\widehat{C}D$ {converse of isosceles triangle theorem}
$\therefore\ y = 4$ {equal sides of isosceles triangle}

9 **a** Opposite sides are parallel, so the figure is a parallelogram
and $x + x = 180$ {co-interior angles on parallel lines}
$\therefore\ 2x = 180$
$\therefore\ x = 90$
In fact, the figure is a rectangle.

b



Two sides are the same length, so $\triangle$ABC is isosceles.
$\therefore$ AE = BC {isosceles triangle theorem}
So, the diagonals bisect each other at right angles.
$\therefore$ the figure is a rhombus.

c There is one pair of equal alternate angles, which means one pair of opposite sides is parallel. No other information is given, so the figure is a trapezium.

10 Opposite sides are parallel, so the figure is a parallelogram.

$\therefore\ 110 - x = x$ {opposite angles in a parallelogram are equal in size}

$\therefore\ 2x = 110$

$\therefore\ x = 55$

PRACTICE TEST 8C

1 **a** BCDE has one pair of parallel sides.

$\therefore$ BCDE is a trapezium.

b $A\widehat{B}E = A\widehat{C}D$ {equal corresponding angles on parallel lines}

c Let $A\widehat{D}C = x^\circ$

$\therefore\ C\widehat{B}E = (180 - x)^\circ$ {$A\widehat{D}C$ and $C\widehat{B}E$ are supplementary}

and $A\widehat{C}D + C\widehat{B}E = 180^\circ$ {co-interior angles on parallel lines are supplementary}

$\therefore\ A\widehat{C}D + (180 - x)^\circ = 180^\circ$

$\therefore\ A\widehat{C}D = x^\circ$

So, in $\triangle$ACD two of the angles are equal.

$\therefore$ $\triangle$ACD is isosceles. {converse of isosceles triangle theorem}

2 **a** **i** $C\widehat{E}B = 70^\circ$ {equal alternate angles on parallel lines}

ii $C\widehat{B}E = 70^\circ$ {$\triangle$CBE is isosceles, with $C\widehat{E}B = C\widehat{B}E$}

iii $B\widehat{C}E + 70^\circ + 70^\circ = 180^\circ$ {angle sum of triangle}

$\therefore\ B\widehat{C}E + 140^\circ = 180^\circ$

$\therefore\ B\widehat{C}E = 40^\circ$

b If [AB] and [BE] are equal in length, then $\triangle$ABE is isosceles.

c The base angles of isosceles triangle ABE are $B\widehat{A}E$ and $B\widehat{E}A$ {since AB = BE}

and $B\widehat{A}E = B\widehat{E}A$ {isosceles triangle theorem}

Let the base angles both be x°.

$\therefore\ x + x + 40 = 180$ {angle sum of triangle}

$\therefore\ 2x + 40 = 180$

$\therefore\ 2x = 140$

$\therefore\ x = 70$

So, the base angles of $\triangle$ABE are both 70°.

d $C\widehat{B}E = B\widehat{E}A = 70^\circ$ {using **a ii** and **c**}

$\therefore$ [BC] $\parallel$ [AE] {equal alternate angles}

e ABCE has one pair of opposite sides parallel. {from **d**}

$\therefore$ ABCE is a trapezium.

3 **a** If ABEF and BCDE are identical rectangles, then [BF] and [CE] are corresponding diagonals. So, [BF] and [CE] are parallel and equal in length.

b BCEF has one pair of opposite sides which are parallel and equal in length. {from **a**}

$\therefore$ BCEF is a parallelogram.

c ABEF and BCDE are identical rectangles, so all their diagonals ([AE], [BF], [BD], [CE]) are equal in length.

$\therefore$ in triangle BDF, BF = BD.

$\therefore$ triangle BDF is isosceles.

d **i** BCEF is a parallelogram. {from **b**}
Now, since ABEF is a rectangle, diagonals [AE] and [BF] bisect each other.
Similarly, in rectangle BCDE, diagonals [BD] and [CE] bisect each other.
So, X and Y are the midpoints of [BF] and [CE] respectively.
$\therefore$ [BX] $\parallel$ [EY] and BX = EY.
$\therefore$ BYEX is a parallelogram, so Pat is correct.

ii The diagonals of a rectangle bisect each other and are equal in length.
$\therefore$ BX = XE and BY = YE.
But ABEF and BCDE are identical, so their diagonals are equal in length.
$\therefore$ BX = XE = BY = YE.
$\therefore$ BYEX is also a rhombus. {all sides are equal in length}

4 **a** AX = FX {X is just the corner of the paper, folded down}
$\therefore$ $\triangle$AFX is isosceles.
Similarly, EX = FX.
$\therefore$ $\triangle$EFX is also isosceles.

b From **a**, $\triangle$AFX and $\triangle$EFX are isosceles.
Since [XF] is common to both, AX = EX.
$\therefore$ X is the midpoint of [AE] and $F\widehat{X}A = 90^\circ$.
$\therefore$ the line [XF] connects the apex of $\triangle$AFE to the midpoint of the base, and is perpendicular to the base.
$\therefore$ $\triangle$AFE is right angled and isosceles. {converse of isosceles triangle theorem}

c $F\widehat{X}A = 90^\circ$ {from **b**}
and $D\widehat{C}X = 90^\circ$ {given}
Now, $F\widehat{X}A$ and $D\widehat{C}X$ are corresponding angles and are equal.
$\therefore$ [AE] $\parallel$ [BD].

d $BC = CD = \dfrac{21}{2}$
$= 10.5$ cm
and $AB = 29.7 \text{ cm} - 10.5 \text{ cm}$
$= 19.2$ cm
So, the dimensions of ABDE are 19.2 cm by 21 cm.
$\therefore$ ABDE is not a square. Lisa is incorrect.

5 **a** AB = BC and $A\widehat{B}C = 90^\circ$ {ABCD is a square}
$\therefore$ $\triangle$ABC is right angled and isosceles.
$\therefore$ the base angles $B\widehat{A}C$ and $B\widehat{C}A$ are equal {isosceles triangle theorem}
$\therefore$ $B\widehat{A}C + B\widehat{C}A + 90^\circ = 180^\circ$ {angle sum of triangle}
$\therefore$ $2 \times B\widehat{A}C + 90^\circ = 180^\circ$
$\therefore$ $2 \times B\widehat{A}C = 90^\circ$
$\therefore$ $B\widehat{A}C = 45^\circ$

b $B\widehat{A}F = 90^\circ$ {angle of a square}
$\therefore$ $G\widehat{A}F + 45^\circ = 90^\circ$ {using **a**, complementary angles}
$\therefore$ $G\widehat{A}F = 45^\circ$

c [BE] and [AC] are the diagonals of a square.
Now, J lies on [AC], and since B, J, and E are collinear, J lies on [BE] as well.
$\therefore$ [BE] and [AC] intersect at J.
$\therefore$ $B\widehat{J}C = 90^\circ$ {diagonals of a square bisect each other at right angles}

d $\widehat{GJI} = \widehat{BJC}$ {vertically opposite angles are equal}

$\therefore$ $\widehat{GJI} = 90^\circ$ {using **c**}

If $\triangle GJI$ is isosceles, then the base angles $\widehat{JGI}$ and $\widehat{JIG}$ are equal. {isosceles triangle theorem}

$\therefore$ $\widehat{JGI} + \widehat{JIG} + 90^\circ = 180^\circ$ {angle sum of triangle}

$\therefore$ $2 \times \widehat{JGI} + 90^\circ = 180^\circ$

$\therefore$ $2 \times \widehat{JGI} = 90^\circ$

$\therefore$ $\widehat{JGI} = 45^\circ$