Mathematics: Core Topics SL

Chapter summaries

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CHAPTER 1: STRAIGHT LINES

A The equation of a line
B Graphing a straight line
C Perpendicular bisectors
D Simultaneous equations
E Problem solving with simultaneous equations

Syllabus reference: SL 2.1, (AI)SL 3.5

We start the book with the study of straight lines. This should be a relatively gentle introduction for most students as they start the school year. Perpendicular bisectors is included in this common chapter, even though it is only listed in the Applications and Interpretation syllabus, as it is a very sensible application of straight lines, and is something the Analysis and Approaches students would benefit from being familiar with.

The syllabus defines the general form equation of a line as $ax + by + d = 0$. However, we prefer to use the form $ax + by = d$. There are several reasons for this. For example, it is easier to find the intercepts of a line written in this form, and it is more logical to express equations in problem solving this way (for example, 2 bats and 5 balls cost $\$30 \Rightarrow 2x + 5y = 30$).

Therefore, we predominantly use the form $ax + by = d$, but we mention that, in an exam, students may be asked to write their answer in the form $ax + by + d = 0$. In several exercise questions, we also ask students to write their answer in the form $ax + by + d = 0$.

The solution of simultaneous linear equations provides a good platform for solving more complicated equations graphically later in the course. The algebraic solution of simultaneous equations is also used in other contexts throughout the course, such as in arithmetic sequences.

Students who need extra help with the gradients of parallel and perpendicular lines should complete the relevant sections of Chapter 5 in the Background Knowledge.

CHAPTER 2: SETS AND VENN DIAGRAMS

A Sets
B Intersection and union
C Complement of a set
D Special number sets
E Interval notation
F Venn diagrams
G Venn diagram regions
H Problem solving with Venn diagrams

This chapter gives students access to much of the notation associated with probability (complement, intersection and union), as well as an introduction to Venn diagrams. This will allow students to focus on its application to probability once it is encountered again in Chapter 10. Students also get introduced to notation for special number sets that will be used throughout the book. The work on interval notation here will allow students to familiarise themselves with this notation before it is used in, for example, the domain and range of functions.

When discussing the union of two sets, we are careful not to describe the union of A and B as the elements that are "in A or B or both". We feel that this wording gives the implication that, if the "or both" were excluded, we would only be talking about the exclusive union of A and B.

CHAPTER 3: SURDS AND EXPONENTS

- **A** Surds and other radicals
- **B** Division by surds
- **C** Exponents
- **D** Laws of exponents
- **E** Scientific notation

Syllabus reference: SL 1.1, SL 1.5

Classes should work through this chapter quickly if the students are already familiar with this content.

When studying laws of exponents, students should be able to distinguish between laws which are a direct result of the original definition of exponents (such as $a^m \times a^n = a^{m+n}$), and those which have been defined in a certain way in order to be consistent with the existing laws (such as $a^0 = 1$).

Writing expressions such as $\frac{4}{x} - \frac{5}{x^3}$ with negative indices is practised, as it is an important skill to develop before students encounter calculus.

The chapter ends with a Discussion about how very large values could be evaluated. Students should consider other forms of technology, as well as things like cancelling common factors that exist in the numerator and denominator, and the order in which the evaluation is performed. For example, do you need to completely evaluate the numerator first, and then divide through by the denominator?

Rational indices are only required for Analysis and Approaches students, so it appears at the start of the Exponential functions chapter in the Mathematics: Analysis and Approaches SL book.

CHAPTER 4: EQUATIONS

- **A** Equations of the form $x^2 = k$
- **B** Power equations
- **C** Equations in factored form
- **D** Quadratic equations
- **E** Solving polynomial equations using technology
- **F** Solving other equations using technology

Syllabus reference: SL 2.4, (AA)SL 2.7, (AA)SL 2.10, (AI)SL 1.8

This chapter covers quadratic equations, power equations of the form $x^n = k$, and solving equations by technology. Students will benefit from studying this material early in the course, as solving these types of equations comes up in many topics, such as sequences.

Solving quadratic equations algebraically is not explicitly in the Applications and Interpretation syllabus, so these students may skip this material. However, an understanding of quadratic equations is important in the study of quadratic functions. For example, solving by completing the square is useful for writing quadratic functions in vertex form, and the quadratic formula is useful in understanding why the axis of symmetry is $x = -\frac{b}{2a}$. We therefore feel it is sensible to have quadratic equations in the Mathematics: Core Topics SL book. Solving quadratic equations analytically may prove to be a useful test for students unsure of which course to take.

We have introduced the notion of "zeros" here, even though they more appropriately apply to expressions or functions, as this is where they are referenced in the syllabus.

In the Discussion at the end of Section F, regarding the merits of using the solver versus graphical methods on the calculator to solve equations, students should talk about the work required to rearrange the equation into a suitable form to use the solver, as well as the potential to miss solutions when using the graphical method due to an inappropriate viewing window.

The subsection of the Quadratic equations section regarding the discriminant of a quadratic is for Analysis and Approaches students only.

This chapter also details how to solve quadratic equations using technology, which is only for the Applications and Interpretation students.

CHAPTER 5: SEQUENCES AND SERIES

- **A** Number sequences
- **B** Arithmetic sequences
- **C** Geometric sequences
- **D** Growth and decay
- **E** Financial mathematics
- **F** Series
- **G** Arithmetic series
- **H** Finite geometric series
- **I** Infinite geometric series

Syllabus reference: SL 1.2, SL 1.3, SL 1.4, (AA)SL 1.8

In Section B.2, we consider real-life examples which can be modelled approximately, but not exactly, by an arithmetic sequence. We therefore use an exact arithmetic sequence as an *approximation* of the real-world quantity.

When finding the general term of a geometric sequence given two of its terms, students should be aware that there will be two possible answers if the difference between the given terms is even. This is where solving equations of the form $x^n = k$ in the previous chapter will prove valuable.

When using sequences in contexts such as compound interest and depreciation, we define the initial conditions with the "zeroth" term u_0. This allows us to make more sensible deductions about the context. For example, the value of the investment after 4 years is u_4.

The simpler finance questions in this chapter are to be solved using formulae based on that for geometric sequences. More complex problems will be solved using technology. Using technology to solve finance problems will be expanded upon in the Mathematics: Applications and Interpretation SL book.

The final section (Infinite geometric series) is for Analysis and Approaches students only.

CHAPTER 6: MEASUREMENT

- **A** Circles, arcs, and sectors
- **B** Surface area
- **C** Volume
- **D** Capacity

Syllabus reference: SL 3.1, SL 3.4

Much of the material in this chapter will be familiar to many students, and classes should skip through it quickly if this is the case. In this chapter we have attempted to provide some questions about measurement that students may not be as familiar with, such as finding the radius of a cylinder given its volume and height.

We have also included some investigations detailing the derivation of some surface area and volume formulae, which use the sigma notation developed in the previous chapter. These investigations should provide an interesting challenge for the more able student.

The chapter contains an investigation on density, which students can complete if they wish, but can be skipped if need be.

CHAPTER 7: RIGHT ANGLED TRIANGLE TRIGONOMETRY

- **A** The trigonometric ratios
- **B** Finding side lengths
- **C** Finding angles
- **D** Right angles in geometric figures
- **E** Problem solving with trigonometry
- **F** True bearings
- **G** The angle between a line and a plane

Syllabus reference: SL 3.1, SL 3.2, SL 3.3

In this chapter, students will build on the work done in the previous chapter, using trigonometry to find areas of shapes, and volumes of solids. The early sections of this chapter should be worked through quickly if students are comfortable with the content. The last question of Section E provides an interesting application of trigonometry, involving measuring the parallax of the star 61 Cygni. Teachers should use questions like this as an opportunity to promote the historical interest of the work the students are doing.

The final section on finding an angle between a line and a plane may present a challenge for some students. This work will provide the platform for using trigonometry in 3-dimensional space in Chapter 9.

CHAPTER 8: NON-RIGHT ANGLED TRIANGLE TRIGONOMETRY

- **A** The unit circle
- **B** The area of a triangle
- **C** The cosine rule
- **D** The sine rule
- **E** Problem solving with trigonometry
- **F** The ambiguous case of the sine rule

Syllabus reference: SL 3.2, SL 3.3, (AA)SL 3.5

In this chapter we extend our understanding of the trigonometric ratios to angles up to 180 degrees, as that is what is required for the study of non-right angled triangles. Angles greater than 180 degrees will be considered in the study of trigonometric functions.

It is likely that this is not the first time students have studied non-right angled triangle trigonometry, so it is important that classes should not linger on this chapter if the students are comfortable with the content.

When the sine rule is introduced in Section D, it is mentioned that this rule does not always produce a unique answer so that all students are aware of this factor. However, in this section, all of the questions presented have unique answers. Section F (The ambiguous case of the sine rule) is for Analysis and Approaches students only.

CHAPTER 9: POINTS IN SPACE

- **A** Points in space
- **B** Measurement
- **C** Trigonometry

Syllabus reference: SL 3.1

In this chapter we extend the coordinate system to consider points and lines in 3-dimensional space. This work may be challenging for students who struggle with 3D spatial awareness.

Many of the techniques involving calculating volumes and surface areas of solids, and the calculation of angles, were established in Chapters 6 and 7. However, instead of being given the side lengths of figures, students will be given the coordinates of the vertices, and must first calculate the relevant distances.

There is no non-right angled triangle trigonometry in this chapter, as is specified by the syllabus. However, we have still placed this chapter after the chapter on non-right angled triangle trigonometry, so it is in the same order as the Mathematics: Core Topics HL book, for classes trying to teach SL and HL together.

CHAPTER 10: PROBABILITY

- **A** Experimental probability
- **B** Two-way tables
- **C** Sample space and events
- **D** Theoretical probability
- **E** The addition law of probability
- **F** Independent events
- **G** Dependent events
- **H** Conditional probability
- **I** Formal definition of independence
- **J** Making predictions using probability

Syllabus reference: SL 4.5, SL 4.6, (AA)SL 4.11

From Section C onwards, there is a strong emphasis on the concepts studied in the Sets and Venn diagrams chapter. Because of this, we have put the addition law of probability *before* independent/dependent events to reinforce the use of Venn diagrams in probability.

In previous books, we had dedicated sections for tree diagrams and 2-dimensional grids. However, doing this disrupted the flow of the chapter as it meant reintroducing the same concepts (for example, sample space and theoretical probability) multiple times in slightly different ways. Instead, tree diagrams and 2-dimensional grids are spread throughout the entire chapter and their usage is generally introduced via worked examples.

Section I (Formal definition of independence) is about using conditional probability notation to establish independent or dependent events. This section is intended for students doing the Mathematics: Analysis and Approaches SL course. It is highly recommended that students who are undecided on which course they want to do should work through this section as well.

We have kept Section J (Making predictions using probability) in the Probability chapter rather than putting it with expectation for discrete random variables. It is important to make a distinction between "the number of times we expect an event to occur out of many trials" and the "expected/average outcome of one trial of an experiment". Thus we have avoided the use of the word "expectation" and its variations in this section as the latter concept will be dealt with in the chapter on discrete random variables. The Discussion in this chapter should lead students to understand that the number of times we *expect* an event to occur may not be an integer, and this is not a problem, as this value is an indication of what we expect to occur *on average* in the long-term, rather than what will actually happen in a particular instance.

CHAPTER 11: SAMPLING AND DATA

- **A** Errors in sampling
- **B** Sampling methods
- **C** Types of data
- **D** Simple discrete data
- **E** Grouped discrete data
- **F** Continuous data

Syllabus reference: SL 4.1, SL 4.2

In this chapter we focus on data collection, organisation, and the display of data. These are the first three steps in a statistical investigation.

In the Discussion at the end of Section A, students should find that the companies offer incentives for completing surveys to encourage more people to do so, thus providing a larger sample size and more meaningful results. However, students should also ponder whether this will affect the integrity of the results, as people may complete the survey as quickly as possible, possibly untruthfully, just so they can receive the incentive.

The Discussion at the end of Section B invites students to consider the merits of the "Brexit" referendum of 2016. In their discussions, students should recognise that the referendum was voluntary, and consider the potential for bias in the responses which this brings.

CHAPTER 12: STATISTICS

A Measuring the centre of data
B Choosing the appropriate measure
C Using frequency tables
D Grouped data
E Measuring the spread of data
F Box and whisker diagrams
G Outliers
H Parallel box and whisker diagrams
I Cumulative frequency graphs
J Variance and standard deviation

Syllabus reference: SL 4.2, SL 4.3

At the end of Section E, there is a Discussion regarding the different interpretations of the interquartile range from different sources of technology. It is hoped that students conclude that it is important to understand what the quartiles Q_1 and Q_3 represent, however the exact method for computing these values is not as important, as the values are simply an estimate anyway. The interpretation of these values will not change.

In Section J, we briefly mention the sample formulae for variance and standard deviation as justification for the multiple values that most calculator models show when calculating summary statistics. In all exercise questions however, we explicitly ask for the *population* variance or standard deviation.